Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

EE2S11 SIGNALS AND SYSTEMS

Midterm exam, 7 December 2020, 13:30–15:50
Block 1 (13:30-14:30)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 14:25–14:40

This block consists of three questions (20 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (8 points)

Let $\Lambda(t)$ denote the triangular pulse signal. Furthermore, let $v(t) = \Lambda(2t)$ and

$$w(t) = \sum_{k=-\infty}^{\infty} v(t-k).$$

(a) Sketch the signals $v(t)$ and $w(t)$.

(b) Compute $\frac{\mathrm{d}w}{\mathrm{d}t}$ and express this derivative in terms of (time-shifted) step functions.

(c) Is $v(t)$ a finite-energy signal? Motivative your answer.

(d) Is $w(t)$ a finite-energy signal? Motivative your answer.

(e) Is $\frac{\mathrm{d}w}{\mathrm{d}t}(t)$ a finite-energy signal? Motivative your answer.

Question 2 (6 points)

Let $r(t) = t\,u(t)$ denote the ramp signal and let $w(t) = \cos(t)u(t)$.

(a) Determine the convolution $y(t) = r(t) * w(t)$ directly using the convolution integral.

(b) Determine the convolution $y(t) = r(t) * w(t)$ using the Laplace transform.

Question 3 (6 points)

(a) The one-sided Laplace transform of a causal signal $f(t)$ is given by

$$F(s) = \frac{2(2s+7)}{(s+4)(s+2)}, \qquad \mathrm{Re}(s) > -2.$$

Determine $f(t)$.

(b) The one-sided Laplace transform of a causal signal $g(t)$ is given by

$$G(s) = e^{-2s} \frac{1}{s^2 + s - 2}, \qquad \mathrm{Re}(s) > 1.$$

Determine $g(t)$.

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Block 2 (14:50-15:50)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 15:45–16:00

This block consists of two questions (19 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 4 (7 points)

(a) The one-sided Laplace transform of a causal signal $w(t)$ is given by

$$W(s) = \frac{s^2 + 2k^2}{s(s^2 + 4k^2)}, \quad k > 0, \quad \mathrm{Re}(s) > 0.$$

Determine $w(t)$.

The Laplace transform of the signal

$$f(t) = \sin(\omega t + \varphi)u(t), \quad \omega > 0,$$

can be written as

$$F(s) = \frac{\alpha \sin(\varphi) + \beta \cos(\varphi)}{s^2 + \omega^2}.$$

(b) Determine the ROC of $F(s)$.

(c) Determine α and β.

Question 5 (12 points)

On the interval $-\pi \leq t \leq \pi$, a periodic signal $f(t)$ with a fundamental period $T_0 = 2\pi$ is given by $f(t) = \cos(at)$, where a is **not** an integer. Recall that the trigoniometric Fourier expansion of a periodic signal is given by

$$f(t) = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t).$$

(a) Determine the dc-component c_0.

(b) Show that the Fourier coefficients c_k for $k \geq 1$ are given by

$$c_k = \frac{\sin(a\pi)}{\pi}(-1)^k \frac{a}{a^2 - k^2}, \qquad k \geq 1.$$

Hints:

$$\cos(\alpha t)\cos(\beta t) = \frac{1}{2}\Big\{\cos[(\alpha+\beta)t] + \cos[(\alpha-\beta)t]\Big\} \quad \text{and} \quad \sin[(k+a)\pi] = (-1)^k \sin(a\pi).$$

(c) Determine the Fourier coefficients d_k.

(d) Use the Fourier expansion of $f(t)$ to show that

$$\frac{1}{\sin(z)} = \frac{1}{z} + \sum_{k=1}^{\infty}(-1)^k\left(\frac{1}{z-k\pi} + \frac{1}{z+k\pi}\right),$$

where z is not an integer multiple of π.