Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

Resit exam EE2S1 SIGNALS AND SYSTEMS
8 January 2026, 13:30–16:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (30 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (4 points)

Given the signal

$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ 2 & \text{for } t = 0, \\ 4 & \text{for } 0 < t < 2, \\ 3 & \text{for } t = 2, \\ 2 & \text{for } t > 2. \end{cases}$$

(a) Express $f(t)$ in terms of two unit step functions.

(b) Determine the Laplace transform of $f(t)$ including its ROC.

(c) Determine the derivative $g(t) = \frac{\mathrm{d}f}{\mathrm{d}t}$.

(d) Determine the Laplace transform of $g(t)$ including its ROC.

Solution

(a) 1p $f(t) = 4u(t) - 2u(t-2)$.

(b) 1p $F(s) = \frac{2(2-e^{-s})}{s}$. ROC $= \{s \in \mathbb{C}; \mathrm{Re}(s) > 0\}$.

(c) 1p $g(t) = 4\delta(t) - 2\delta(t-2)$.

(c) 1p $G(s) = 2(2 - e^{-s}) = sF(s), \mathrm{ROC} = \mathbb{C}$.

Question 2 (6 points)

Let

$$p(t) = \begin{cases} 1 & \text{for } 0 < t < 1, \\ 0 & \text{elsewhere} \end{cases}$$

and consider the signal $x(t) = p(t) * p(t) * p(t)$, where '$*$' denotes convolution.

(a) Determine $x(t)$ for $t > 3$. Motivate your answer.

The signal

$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ \sin(t) & \text{for } 0 < t < \pi, \\ t & \text{for } t > \pi, \end{cases}$$

can be written as

$$f(t) = \sin(t)u(t) + g(t-\pi)u(t-\pi),$$

where $u(t)$ is the unit step function.

(b) Determine the signal $g(t)$.

(c) Determine the one-sided Laplace transform of $f(t)$.

The one-sided Laplace transform of a causal signal $z(t)$ is given by

$$Z(s) = \frac{1}{(s^2+4)(s^2+9)}, \quad \mathrm{Re}(s) > 0.$$

(d) Determine $z(t)$.

Solution

(a) 1.5p $x(t) = 0$ for $t > 3$.

(b) 1.5p $g(t) = \pi + t + \sin(t)$.

(c) 1.5p

$$F(s) = \frac{1}{s^2+1} + e^{-\pi s}\left(\frac{\pi}{s} + \frac{1}{s^2} + \frac{1}{s^2+1}\right) \quad \mathrm{ROC} = \{s \in \mathbb{C}; \mathrm{Re}(s) > 0\}.$$

(d) 1.5p

$$Z(s) = \frac{1}{5}\frac{1}{s^2+4} - \frac{1}{5}\frac{1}{s^2+9} \quad \longleftrightarrow \quad z(t) = \left(\frac{1}{10}\sin 2t - \frac{1}{15}\sin 3t\right)u(t)$$

Question 3 (3 points)

A periodic signal $x(t)$ with a fundamental period T_0 has a Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\alpha}{\beta + (k\pi)^2} e^{\mathrm{j}k\pi t} \quad \text{with } \alpha > 0 \text{ and } \beta > 0.$$

(a) What is the fundamental period T_0?

(b) What is the average value of $x(t)$?

(c) Is $x(t)$ even, odd, or neither? Motivate your answer.

One of the harmonics of $x(t)$ is expressed as $a\cos(4\pi t)$.

(d) What is a?

Solution

(a) 0.5p $T_0 = 2$

(b) 0.5p Average value is $X_0 = \alpha/\beta$.

(c) 0.5p $x(t)$ is even.

(d) 1.5p Clearly, this harmonic corresponds to the $k = \pm 4$ terms:

$$a\cos(4\pi t) = X_{-4}e^{-\mathrm{j}4\pi t} + X_4 e^{\mathrm{j}4\pi t} = 2X_4\cos(4\pi t).$$

We observe that $a = 2X_4 = \frac{2\alpha}{\beta + 16\pi^2}$.

Question 4 (6 points)

(a) For a causal LTI system the response to an input signal $x = [\cdots, 0, \boxed{1}, 3, 3, 1, 0, \cdots]$ is given by $y = [\cdots, 0, \boxed{1}, 4, 6, 4, 1, 0, \cdots]$.

Determine the impulse response $h[n]$ of the system. *(Hint: first determine the filter length.)*

(b) Given $x[n] = (2)^n u[-n]$, compute its z-transform and specify the ROC.

(c) Given $X(z) = \dfrac{4z}{(z-1)(z+0.25)}$, ROC $= \{|z| > 1\}$.

Determine $x[n]$ using the inverse z-transform.

(d) Let $h[n]$ be the impulse response of a lowpass filter with cut-off frequency $\frac{1}{4}\pi$. Denote its DTFT by $H(e^{j\omega})$. Now consider

$$h_1[n] = \begin{cases} 2h[n] & n \text{ even} \\ 0 & \text{otherwise.} \end{cases}$$

Derive an expression for the DTFT of $h_1[n]$ in terms of $H(e^{j\omega})$ and point out what type of filter it is.

Hint: use $(-1)^n = e^{j\pi n}$.

Solution

(a) 1p The convolution sum is $y[n] = \sum_{k=0}^{L-1} h[k]x[n-k]$, where L is the filter length.

The length of $x[n]$ is $N_x = 4$, and of $y[n]$ is $N_y = 5$. Thus, the filter length is $L = N_y - Nx + 12$. Subsequently, the convolution expressions are worked out as

$$h[0]x[n] + h[1]x[n-1] = y[n] \quad \Leftrightarrow \quad \begin{cases} n=0 & h[0]\cdot 1 + h[1]\cdot 0 &=& 1 & \Rightarrow h[0]=1 \\ n=1 & h[0]\cdot 3 + h[1]\cdot 1 &=& 4 & \Rightarrow h[1]=1 \\ n=2 & h[0]\cdot 3 + h[1]\cdot 3 &=& 6 & \Rightarrow \text{(check)} \\ n=3 & h[0]\cdot 1 + h[1]\cdot 3 &=& 4 & \Rightarrow \text{(check)} \\ n=4 & h[0]\cdot 0 + h[1]\cdot 1 &=& 1 & \Rightarrow \text{(check)} \end{cases}$$

so that $h[n] = [\boxed{1}, 1, 0, \cdots]$.

(b) 1p

$$X(z) = 1 + \frac{1}{2}z + \frac{1}{4}z^2 + \cdots = \frac{1}{1-\frac{1}{2}z}\;; \text{ROC} = \{|z| < 2\}\,.$$

(c) 2p First write as function of z^{-1} and do a partial fraction expansion:

$$X(z) = \frac{4z^{-1}}{(1-z^{-1})(1+0.25z^{-1})} = \frac{16/5}{1-z^{-1}} - \frac{16/5}{1+0.25z^{-1}}$$

Check the ROC: both terms correspond to causal sequences. Hence

$$x[n] = \frac{16}{5}u[n] - \frac{16}{5}\left(-\frac{1}{4}\right)^n u[n]$$

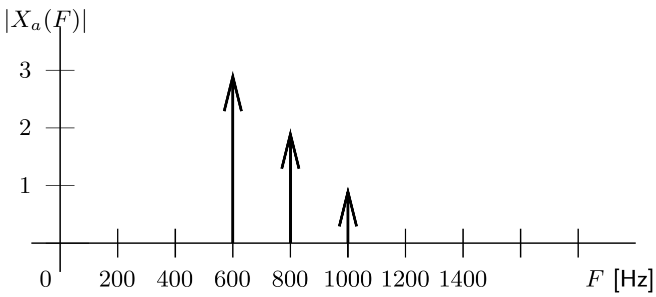
(d) 2p

$$h_1[n] = h[n] + (-1)^n h[n] = h[n] + e^{j\pi n}h[n] \quad \Rightarrow \quad H_1(e^{j\omega}) = H(e^{j\omega}) + H(e^{j(\omega+\pi)})$$

Note that the 2nd term is the spectrum of $H(e^{j\omega})$ shifted by π: a high-pass filter. As a result, $H_1(e^{j\omega})$ is a bandstop filter (in this case: it stops the band $(\frac{1}{4}\pi, \frac{3}{4}\pi)$).

Question 5 (3 points)

The real-valued continuous-time signal $x_a(t)$ has frequency components as indicated below; the spectrum is real-valued.



(a) What is the (minimal) sampling frequency required to avoid aliasing?

(b) The signal is sampled at $F_s = 1000$ Hz, resulting in $x[n]$, there is no filtering. What frequency components are present in the sampled signal?

(c) Draw the amplitude spectrum of $x[n]$; clearly indicate the frequencies and amplitudes.

Solution

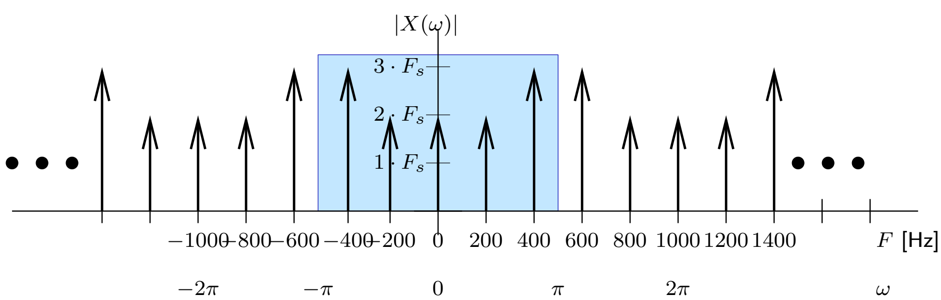
(a) 0.5p Twice the highest frequency: $F_s = 2000$ Hz.

(b) 1.5p The aliasing results in frequency components at all multiples of 1000 Hz. Also consider the negative frequencies! (The signal is real so the spectrum is symmetric.) We only need to consider components between -500 and 500 Hz, outside this fundamental interval the spectrum is repeated. This results in:

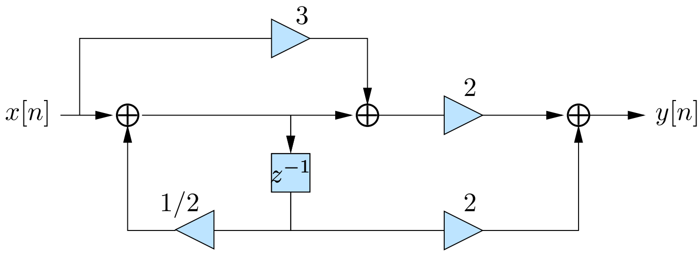
$$\begin{aligned} +600 &\Rightarrow -400 \\ -600 &\Rightarrow +400 \\ +800 &\Rightarrow -200 \\ -800 &\Rightarrow +200 \\ +1000 &\Rightarrow 0 \\ -1000 &\Rightarrow 0 \end{aligned}$$

The two components at 0 Hz add up, doubling the amplitude.

(c) 1p Regarding the amplitudes, note that they get scaled by F_s. The fundamental interval runs from -500 Hz to 500 Hz, outside this interval it is periodic.



Question 6 (4 points)



(a) Determine the transfer function $H(z)$ of the causal system shown above.

(b) Is this a minimal system? (why)

(c) Is this a stable system? (why)

(d) Draw the "Direct form II" realization.

Solution

(a) 2p Denote the signal at the input of the delay element equal to $P(z)$. Then

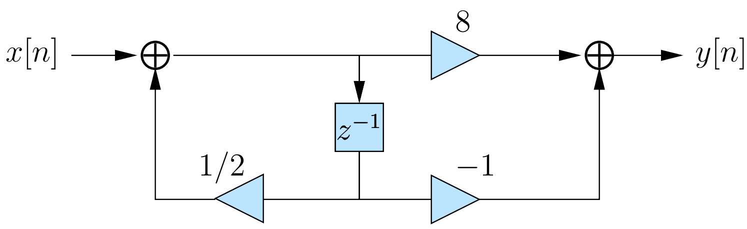
$$\begin{cases} P(z) &= X(z) + \frac{1}{2}z^{-1}P(z) \\ Y(z) &= 6X(z) + 2P(z) + 2z^{-1}P(z) \end{cases} \Rightarrow \begin{cases} P(z) &= X(z)\dfrac{1}{1-\frac{1}{2}z^{-1}} \\ Y(z) &= X(z)(6 + \dfrac{2}{1-\frac{1}{2}z^{-1}} + \dfrac{2z^{-1}}{1-\frac{1}{2}z^{-1}}) \\ &= X(z)\,\dfrac{8-z^{-1}}{1-\frac{1}{2}z^{-1}} \end{cases}$$

$$\Rightarrow \quad H(z) = \frac{8-z^{-1}}{1-\frac{1}{2}z^{-1}}$$

(b) 0.5p Yes (system order equal to the number of delays).

(d) 0.5p Yes (causal system with pole $z = 1/2$ within the unit circle)

(d) 1p



Question 7 (4 points)

We aim to design an analog *high-pass* Chebyshev filter, with specifications

passband frequency:	6 rad/s
maximal damping in the passband:	3 dB
stopband frequency:	3 rad/s
filter order:	2

An generic expression for the frequency response (amplitude-square) of a prototype N-th order low-pass Chebyshev filter is

$$|H(\Omega)|^2 = \frac{1}{1+\epsilon^2 T_N(\Omega/\Omega_p)^2}$$

while the 2nd order Chebyshev polynomial is $T_2(\Omega) = 2\Omega^2 - 1$.

We first design a lowpass filter, and then apply a frequency transformation.

(a) What frequency transformation do you use? What are the corresponding specifications for the lowpass filter?

(b) Determine the filter parameters. What is the resulting frequency response $|H(\Omega)|^2$ for the lowpass filter that satisfies the specifications?

(c) What is the minimal damping in the stopband of this lowpass filter?

(d) Apply the frequency transformation and specify the frequency response $|G(\Omega)|^2$ of the resulting desired highpass filter.

Solution

(a) 1p $\Omega \to \frac{6}{\Omega}$.

For the lowpass filter, the passband frequency becomes $\Omega_p = 1$ rad/s, with a damping of 3 dB. The stopband frequency is $\Omega_s = \frac{6}{3} = 2.0$ rad/s.

(Other transformations are possible.)

(b) 2p

$$|H(\Omega)|^2 = \frac{1}{1+\epsilon^2[T_2(\Omega)]^2} = \frac{1}{1+\epsilon^2(2\Omega^2-1)^2}$$

At $\Omega_p = 1$ we need a damping of 3 dB, i.e. $|H(1)|^2 = \frac{1}{2}$.

$$|H(1)|^2 = \frac{1}{1+\epsilon^2} = \frac{1}{2} \qquad \Rightarrow \quad \epsilon = 1$$

Hence

$$|H(\Omega)|^2 = \frac{1}{1+(2\Omega^2-1)^2} = \frac{1}{4\Omega^2 - 4\Omega^2 + 2}$$

(c) 1p

$$|H(\Omega_s)|^2 = \frac{1}{1+(2(2.0)^2-1)^2} = \frac{1}{50}$$

This corresponds to -17 dB.

(d) 1p

$$\begin{aligned} |G(\Omega)|^2 &= \frac{1}{1+(2\left(\frac{6}{\Omega}\right)^2-1)^2} \\ &= \frac{1}{1+(\frac{72}{\Omega^2}-1)^2} \\ &= \frac{\Omega^4}{2\Omega^4-2\cdot 72\Omega^2+(72)^2} \end{aligned}$$