Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

Resit exam EE2S1 SIGNALS AND SYSTEMS
8 January 2026, 13:30–16:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (30 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (4 points)

Given the signal

$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ 2 & \text{for } t = 0, \\ 4 & \text{for } 0 < t < 2, \\ 3 & \text{for } t = 2, \\ 2 & \text{for } t > 2. \end{cases}$$

(a) Express $f(t)$ in terms of two unit step functions.

(b) Determine the Laplace transform of $f(t)$ including its ROC.

(c) Determine the derivative $g(t) = \frac{\mathrm{d}f}{\mathrm{d}t}$.

(d) Determine the Laplace transform of $g(t)$ including its ROC.

Question 2 (6 points)

Let

$$p(t) = \begin{cases} 1 & \text{for } 0 < t < 1, \\ 0 & \text{elsewhere} \end{cases}$$

and consider the signal $x(t) = p(t) * p(t) * p(t)$, where '$*$' denotes convolution.

(a) Determine $x(t)$ for $t > 3$. Motivate your answer.

The signal

$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ \sin(t) & \text{for } 0 < t < \pi, \\ t & \text{for } t > \pi, \end{cases}$$

can be written as

$$f(t) = \sin(t)u(t) + g(t - \pi)u(t - \pi),$$

where $u(t)$ is the unit step function.

(b) Determine the signal $g(t)$.

(c) Determine the one-sided Laplace transform of $f(t)$.

The one-sided Laplace transform of a causal signal $z(t)$ is given by

$$Z(s) = \frac{1}{(s^2 + 4)(s^2 + 9)}, \quad \mathrm{Re}(s) > 0.$$

(d) Determine $z(t)$.

Question 3 (3 points)

A periodic signal $x(t)$ with a fundamental period T_0 has a Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\alpha}{\beta + (k\pi)^2} e^{jk\pi t} \quad \text{with } \alpha > 0 \text{ and } \beta > 0.$$

(a) What is the fundamental period T_0?

(b) What is the average value of $x(t)$?

(c) Is $x(t)$ even, odd, or neither? Motivate your answer.

One of the harmonics of $x(t)$ is expressed as $a\cos(4\pi t)$.

(d) What is a?

Question 4 (6 points)

(a) For a causal LTI system the response to an input signal $x = [\cdots, 0, \boxed{1}, 3, 3, 1, 0, \cdots]$ is given by $y = [\cdots, 0, \boxed{1}, 4, 6, 4, 1, 0, \cdots]$.

Determine the impulse response $h[n]$ of the system. *(Hint: first determine the filter length.)*

(b) Given $x[n] = (2)^n u[-n]$, compute its z-transform and specify the ROC.

(c) Given $X(z) = \dfrac{4z}{(z-1)(z+0.25)}$, ROC $= \{|z| > 1\}$.

Determine $x[n]$ using the inverse z-transform.

(d) Let $h[n]$ be the impulse response of a lowpass filter with cut-off frequency $\frac{1}{4}\pi$. Denote its DTFT by $H(e^{j\omega})$. Now consider

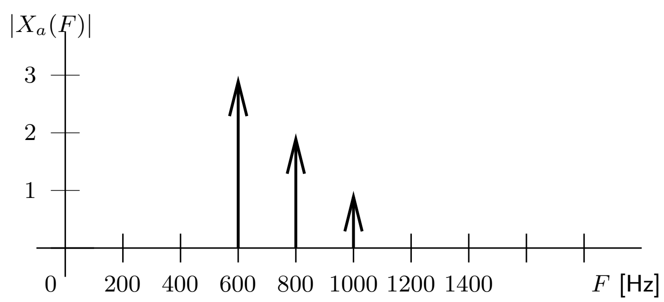
$$h_1[n] = \begin{cases} 2h[n] & n \text{ even} \\ 0 & \text{otherwise.} \end{cases}$$

Derive an expression for the DTFT of $h_1[n]$ in terms of $H(e^{j\omega})$ and point out what type of filter it is.

Hint: use $(-1)^n = e^{j\pi n}$.

Question 5 (3 points)

The real-valued continuous-time signal $x_a(t)$ has frequency components as indicated below; the spectrum is real-valued.

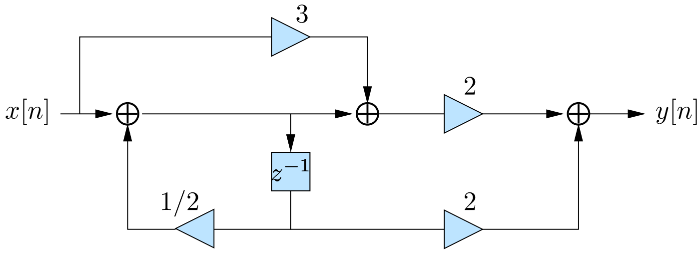


(a) What is the (minimal) sampling frequency required to avoid aliasing?

(b) The signal is sampled at $F_s = 1000$ Hz, resulting in $x[n]$, there is no filtering. What frequency components are present in the sampled signal?

(c) Draw the amplitude spectrum of $x[n]$; clearly indicate the frequencies and amplitudes.

Question 6 (4 points)



(a) Determine the transfer function $H(z)$ of the causal system shown above.

(b) Is this a minimal system? (why)

(c) Is this a stable system? (why)

(d) Draw the "Direct form II" realization.

Question 7 (4 points)

We aim to design an analog *high-pass* Chebyshev filter, with specifications

passband frequency:	6 rad/s
maximal damping in the passband:	3 dB
stopband frequency:	3 rad/s
filter order:	2

An generic expression for the frequency response (amplitude-square) of a prototype N-th order low-pass Chebyshev filter is

$$|H(\Omega)|^2 = \frac{1}{1 + \epsilon^2 T_N(\Omega/\Omega_p)^2}$$

while the 2nd order Chebyshev polynomial is $T_2(\Omega) = 2\Omega^2 - 1$.

We first design a lowpass filter, and then apply a frequency transformation.

(a) What frequency transformation do you use? What are the corresponding specifications for the lowpass filter?

(b) Determine the filter parameters. What is the resulting frequency response $|H(\Omega)|^2$ for the lowpass filter that satisfies the specifications?

(c) What is the minimal damping in the stopband of this lowpass filter?

(d) Apply the frequency transformation and specify the frequency response $|G(\Omega)|^2$ of the resulting desired highpass filter.