Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

Resit exam EE2S11 SIGNALS AND SYSTEMS
8 July 2024, 13:30–16:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (35 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (6 points)

Given the signal $x(t) = e^{-t}[u(t-5) - u(t-6)]$, where $u(t)$ is the Heaviside unit step function.

(a) Is this signal causal? Motivate your answer.

(b) Determine the energy of $x(t)$.

(c) Determine the power of $x(t)$.

(d) What is the support of the signal $z(t) = x(t) * x(t)$? Motivate your answer.

(e) Determine the Laplace transform of $x(t)$, and specify its ROC.

Question 2 (6 points)

Determine the inverse Laplace transforms of

(a) $F(s) = \dfrac{s-2}{s^2-2s-3}, \quad \mathrm{Re}(s) > 3.$

(b) $G(s) = \dfrac{3s+2}{s^2+25}, \quad \mathrm{Re}(s) > 0.$

(c) $W(s) = \dfrac{5}{(s+2)^3}, \quad \mathrm{Re}(s) > -2.$

Question 3 (5 points)

Given the periodic signal $x(t)$ with fundamental period $T_0 = 2\pi$ and

$$x(t) = e^t, \quad -\pi < t < \pi.$$

(a) Determine the power P_x of this periodic signal.

(b) Determine the Fourier coefficients X_k of this periodic signal.

(c) Show that

$$\sum_{k=-\infty}^{\infty} \frac{1}{k^2+1} = \frac{\pi}{\tanh(\pi)}.$$

Question 4 (7 points)

(a) Given the signal $x[n] = [\cdots, 0, \boxed{1}, 2, 3, 0, 0, \cdots]$, where the 'box' denotes the value for $n = 0$. Determine $r[n] = x[n] * x[-n]$ using the convolution sum.

(b) Determine the z-transform for the following discrete-time signal, also specify the ROC:

$$x[n] = u[n] + 2^n\, u[-n]$$

(c) Determine the signal $x[n]$ corresponding to the z-transform:

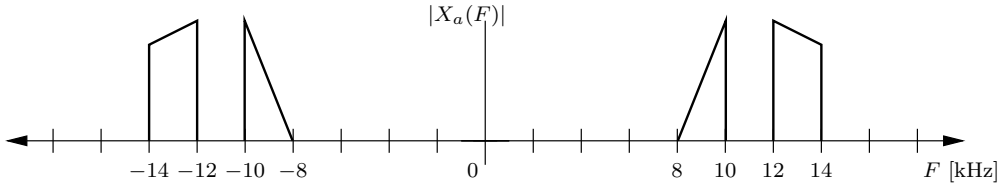
$$X(z) = \frac{1.2}{(1 - z^{-1})(1 + 0.2z^{-1})}, \qquad \text{ROC: } 0.2 < |z| < 1\,.$$

(d) Determine the frequency response $H(e^{j\omega})$ for the system defined by the difference equation:

$$y[n] = 0.5y[n-1] + x[n] + x[n-1], \qquad n \geq 0$$

Question 5 (3 points)

A continuous-time signal $x_a(t)$ has a Fourier transform $X_a(F)$ as shown in the figure:

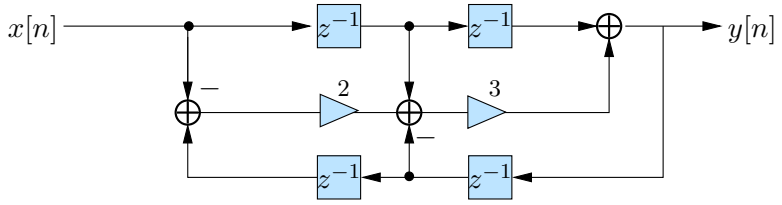


(a) What is the required sample frequency according to the Nyquist sample rate condition?

(b) The signal is sampled at a rate $F_s = 14$ kHz; no filtering is applied. Sketch the amplitude spectrum of the resulting discrete-time signal (carefully indicate the frequencies).

Question 6 (3 points)

Consider the following system realization:



(a) Determine the transfer function $H(z)$.

(b) Is this a stable system? (Why?)

(c) Is this a minimal realization? (Why?)

Question 7 (5 points)

A second-order analog lowpass filter (Butterworth filter) has transfer function

$$G_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1}\,.$$

The 3-dB cut-off frequency of this filter is $\Omega_c = 1$ rad/s.

Using the bilinear transform and the above filter as a template, we will now design a digital *high*-pass filter $H(z)$ with cut-off frequency $\omega_c' = \frac{2}{3}\pi$.

(a) What should be the corresponding cut-off frequency in the analog frequency domain?

(b) Which frequency transformation should be used?

(c) What is $H_a(s)$?

(d) What is $H(z)$?

(e) Verify that the design meets the specifications.