Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

Resit exam EE2S1 (or EE2S11) SIGNALS AND SYSTEMS 17 December 2024, 09:00–12:00

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (35 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (7 points)

The behavior of a SISO system is governed by the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = x(t) + \frac{\mathrm{d}x}{\mathrm{d}t},$$

where $x(t)$ is the input signal and $y(t)$ the output signal.

(a) Determine the impulse response $h(t)$ of the system.

(b) Determine the output signal $y(t)$ in case the input signal is given by

$$x(t) = (1-t)e^{-t}u(t)$$

and the system is initially at rest.

(c) Determine $\lim_{t\to\infty} y(t)$ in case the input signal is given by

$$x(t) = 2\,u(t)$$

and the system is initially at rest.

Question 2 (5 points)

Determine the inverse Laplace transforms of

(a) $X(s) = \dfrac{s^2+2}{s^3-s}, \quad \mathrm{Re}(s) > 1.$

(b) $Y(s) = \dfrac{2}{s^2(s-1)}, \qquad \mathrm{Re}(s) > 1.$

Question 3 (4 points)

A periodic signal $x(t)$ with a fundamental period T_0 has a Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\alpha}{\beta + (k\pi)^2} e^{\mathrm{j}k\pi t} \quad \text{with } \alpha > 0 \text{ and } \beta > 0.$$

(a) What is the fundamental period T_0?

(b) What is the average value of $x(t)$?

(c) Is $x(t)$ even, odd, or neither? Motivate your answer.

One of the harmonics of $x(t)$ is expressed as $a\cos(4\pi t)$.

(d) What is a?

Question 4 (4 points)

(a) Determine the z-transform for the following discrete-time signal, also specify the ROC:

$$x[n] = \frac{1}{2}\left(1 + (-1)^n\right) u[n]\,.$$

(b) Assuming $x[n]$ is causal, determine the signal $x[n]$ corresponding to the z-transform:

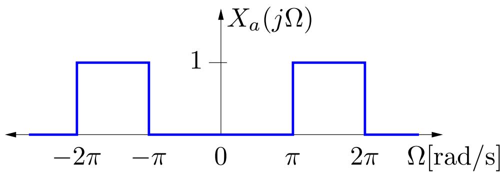
$$X(z) = \frac{z^{-2} + 10z^{-1}}{z^{-2} + 6z^{-1} + 8}\,.$$

(c) Determine the signal $x[n]$ corresponding to the DTFT

$$X(e^{j\omega}) = e^{j\pi/4}\delta(\omega - 1) + e^{-j\pi/4}\delta(\omega + 1)\,.$$

Question 5 (5 points)

The continuous-time signal $x_a(t)$ has a (real-valued) Fourier transform $X_a(j\Omega)$ as shown here:



(a) Is this a bandlimited signal? What is its maximal frequency (in Hz)?

(b) At what sampling period T_s should we sample this signal, avoiding aliasing?

We sample the signal at $T_s = \frac{1}{2}$ sec, and obtain the discrete-time signal $x[n]$. The DTFT spectrum of $x[n]$ is $X(e^{j\omega})$.

(c) How is ω related to Ω?

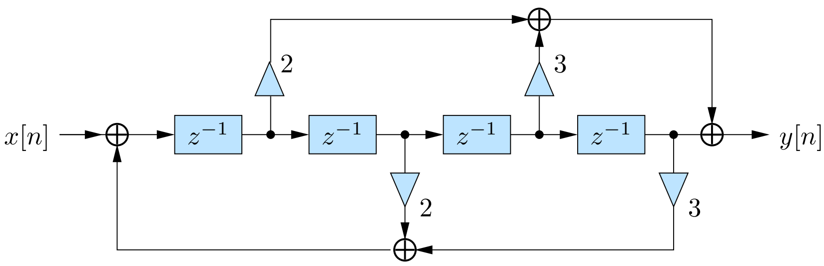
(d) Draw $X(e^{j\omega})$; also indicate the values on both axes.

Consider now the signal $y_a(t) = (x_a(t))^2$.

(e) Make a plot of $Y_a(j\Omega)$; also indicate the values on the horizontal axis.

Question 6 (4 points)

Consider the following system realization:



(a) Determine the transfer function $H(z)$.

(b) Determine all poles and zeros of $H(z)$, and draw a pole-zero diagram.

(c) Is this a stable system? (Why?)

(d) Is this a minimal realization? (Why?)

Question 7 (6 points)

We would like to design a digital *high*pass filter with the following specifications:

- Passband: starting at 3.0 kHz; ripple in the passband: ≤ 1 dB
- Stopband: below 2.0 kHz; stopband damping: ≥ 40 dB
- Sample rate: 12 kHz

The digital filter will be designed by applying the bilinear transform on an analog transfer function. We will use a Chebyshev filter. The expression for the frequency response of a prototype n-th order low-pass Chebyshev filter is

$$|H(\Omega)|^2 = \frac{1}{1 + \epsilon^2 \, T_n^2(\Omega)} \, .$$

(a) What are the passband and stopband frequencies (in rad) in the digital time domain?

(b) What are the filter specifications in the analog time domain?

(c) Which transformation will you use to transform a lowpass into a highpass filter? What are the specifications of the analog lowpass filter?

(d) Compute the required filter order n for a Chebyshev filter.

(Remark: $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$.)

(e) Suppose that the resulting Chebyshev lowpass filter has this form:

$$H(s) = \frac{B(s)}{A(s)} = \frac{1}{a_0 + a_1 s + \cdots + a_n s^n} \, .$$

How can you find the filter coefficients of the digital *high*pass filter?