Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Resit exam EE2S11 SIGNALS & SYSTEMS
July 19, 2021
Block 1 (13:30-15:00)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 14:55–15:10

This block consists of three questions (25 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (11 points)

Given a causal time-domain signal $i(t)$. On its ROC, the one-sided Laplace transform of $i(t)$ is given by

$$I(s) = \frac{s-2}{s^2+2s+2}\,.$$

(a) What is its ROC?

(b) Determine $i(0^+)$.

(c) Determine $i(t)$.

(d) Compute $\dfrac{\mathrm{d}i}{\mathrm{d}t}$.

(e) Determine the inverse Laplace transform of

$$U(s) = \frac{s^2-2s}{s^2+2s+2}\,,$$

which has the same ROC as $I(s)$.

Question 2 (5 points)

Given the signal $x(t) = te^{-\alpha t}u(t)$ with $\alpha > 0$.

(a) Determine the convolution $y(t) = x(t) * x(t)$ directly using the convolution integral.

(b) Determine the convolution $y(t) = x(t) * x(t)$ using the Laplace transform.

Question 3 (9 points)

A periodic signal $x(t)$ with a fundamental period T_0 has a Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\alpha}{\beta + (k\pi)^2} e^{\mathrm{j}k\pi t} \quad \text{with } \alpha > 0 \text{ and } \beta > 0.$$

(a) What is the fundamental period T_0?

(b) What is the average value of $x(t)$?

(d) Is $x(t)$ even, odd, or neither? Motivate your answer.

One of the harmonics of $x(t)$ is expressed as $a\cos(4\pi t)$.

(d) What is a?

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Block 2 (15:25-16:55)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 16:50–17:05

This block consists of four questions (25 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 4 (9 points)

(a) Given the signals $x[n] = [\cdots, 0, 1, 2, \boxed{3}, 0, 0, \cdots]$ and $h[n] = [\cdots, \boxed{0}, 2, 1, 0, \cdots]$, where the 'box' denotes the value for $n = 0$.

Determine $y[n] = h[n] * x[n]$ using the convolution sum.

(b) Given an input signal $x[n] = \left(\frac{1}{3}\right)^{|n|}$. Determine the z-transform $X(z)$, also specify the ROC.

(c) Let $h[n] = (\frac{1}{2})^n u[n]$ be the impulse response of a filter, let

$$Y(z) = \frac{2z^2}{2z^2 + z - 1}\,, \qquad \text{ROC: } |z| > 1\,,$$

and let $y[n]$ be the corresponding signal.

Compute the input signal $x[n]$ for which this $y[n]$ is the output of the filter.

(d) A filter $H(z)$ is called *allpass* if its magnitude response (amplitude spectrum) is constant over frequency.

Consider

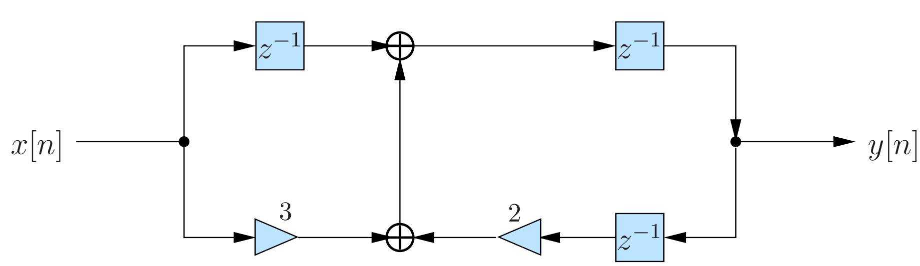
$$H(z) = z^{-1}\frac{1 + 3z^{-2}}{3 + z^{-2}}\,, \qquad \text{ROC: } |z| > \frac{1}{\sqrt{3}}$$

Determine if $H(z)$ is an allpass filter.

(e) Compute all poles and zeros of $H(z)$ in (d) and draw a pole-zero plot.

Question 5 (5 points)

Consider the following system realization:



(a) Determine the transfer function $H(z)$.

(b) Is this a minimal realization? (Why?)

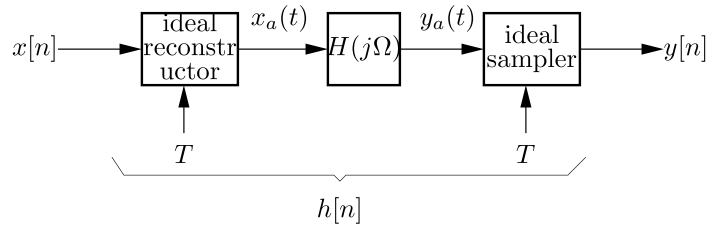
(c) Draw the corresponding Direct Form no. 2 realization.

(d) Determine $h[1]$, the impulse response at time $n = 1$.

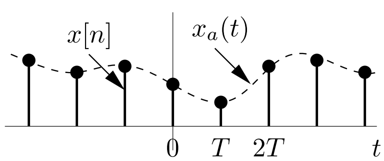
Question 6 (6 points)

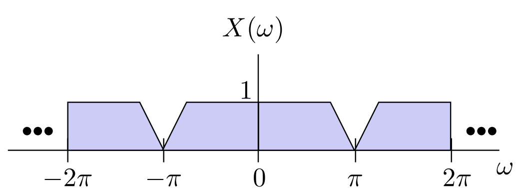
We have a discrete time sequence $x[n]$, and wish to implement a delay: $y[n] = x[n - \Delta]$. If Δ is not an integer, this has no formal meaning as we cannot shift the sequence $x[n]$ by anything but an integer.

To implement the effect of a non-integer delay, we consider the following setup:



The signal is first reconstructed (D/A conversion including an ideal reconstruction filter) assuming a certain sampling period of T, resulting in $x_a(t)$. Next, a suitable continuous-time filter $H(j\Omega)$ is applied, and the resulting signal $y_a(t)$ is sampled again with period T so that we obtain the series $y[n] = y_a(nT)$.





(a) The spectrum $X(\omega)$ corresponding to $x[n]$ is drawn schematically above. Sketch the spectrum $X_a(\Omega)$ after ideal reconstruction. (Also indicate values on the horizontal and vertical axes.)

(b) Express $x_a(t)$ in terms of $x[n]$.

(c) Relate $y_a(t)$ to $x_a(t)$ in an equation.

Based on this, specify $H(j\Omega)$ such that the desired delay is obtained.

(d) Express $y[n]$ in terms of $x[n]$ in an equation.

Based on this, specify the equivalent discrete-time filter $h[n]$ that implements the non-integer delay.

(e) How should T be selected?

Question 7 (5 points)

We use the bilinear transform to design a digital lowpass filter $H(z)$ with the following specifications:

– Passband: $0 \leq |\omega| \leq 0.25\pi$, maximal ripple 0.5 dB

– Stopband: $0.4\pi \leq |\omega| \leq \pi$, minimal damping 50 dB.

(a) Specify the passband and stopband frequencies for the design of the corresponding analog lowpass filter

(b) What is the required filter order if we use a Butterworth filter?

(c) Suppose $G(z) = H(-z)$. Give a plot of the magnitude response $|G(e^{j\omega})|$. Also specify values on both axes (derived from the specifications of $H(z)$).