Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Resit exam EE2S11 SIGNAL PROCESSING
July 21, 2020
Block 1 (13:30-15:00)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 14:55–15:05

This block consists of three questions (30 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (10 points)

Given a SISO system with input signal $x(t)$ and output signal $y(t)$. For $T_1 \geq 0$ and $T_2 \geq 0$ and $T_1 + T_2 \neq 0$, the output signal $y(t)$ is related to the input signal $x(t)$ by

$$y(t) = \frac{1}{T_1 + T_2} \int_{\tau=t-T_1}^{t+T_2} x(\tau)\,\mathrm{d}\tau.$$

(a) The system is called a sliding window averager. Explain why.

(b) Is this system linear? Motivate your answer.

(c) Is this system time-invariant? Motivate your answer.

(d) Determine the transfer function of the system. What is its ROC?

(e) Determine the impulse response of the system.

(f) Is the system causal for $T_1 > 0$ and $T_2 > 0$? Motivate your answer.

(g) Is the system causal for $T_1 > 0$ and $T_2 = 0$? Motivate your answer.

Question 2 (10 points)

(a) Determine the Laplace transform $F(s)$ of the signal

$$f(t) = \sinh(t)u(t),$$

where $u(t)$ is the Heaviside unit step function.

(b) What is the ROC of $F(s)$?

For $t > 0$, the behavior of a system with input signal $x(t)$ and output signal $y(t)$ is governed by the differential equation

$$\frac{\mathrm{d}^4 y}{\mathrm{d}t^4} - y = x(t).$$

At $t = 0$, y and its first three derivatives vanish.

(c) Determine the impulse response $h(t)$ of the system.

(d) True or false: the output signal $y(t)$ of the system for a given input signal $x(t)$ and with vanishing initial conditions is given by

$$y(t) = \frac{1}{2} \int_{\tau=0}^{t} [\sinh(t-\tau) - \sin(t-\tau)]\, x(\tau)\, \mathrm{d}\tau, \quad t > 0.$$

Motivate your answer.

Question 3 (10 points)

Let $x(t)$ be a periodic signal with fundamental period $T_0 = 4$. On the interval $(-2, 2)$, $x(t)$ is given by

$$x(t) = t^2, \quad t \in (-2, 2).$$

(a) What can you say about the decay of the Fourier coefficients as $|k| \to \infty$ without computing these coefficients explicitly?

(b) Determine X_0, the dc-component of the signal $x(t)$.

(c) Determine the Fourier coefficients X_k for $k \neq 0$.

(d) Determine the power P_x of the signal.

(e) Use Parseval's power relation to show that

$$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}.$$

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Block 2 (15:00-16:30)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 16:30–16:45

This block consists of four questions (27 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 4 (10 points)

(a) Given the signals $x[n] = [\cdots, 0, 1, \boxed{2}, 3, 0, \cdots]$ and $h[n] = [\cdots, \boxed{0}, 1, 2, 0, \cdots]$. Determine $y[n] = h[n] * x[n]$ using the convolution sum.

(b) Given an input signal $x[n] = \left(\frac{1}{4}\right)^n u[n]$, and a system described by the difference equation

$$y[n] = 2x[n] - \frac{1}{2}y[n-1]\,.$$

Determine the output signal $y[n]$.

(c) Consider

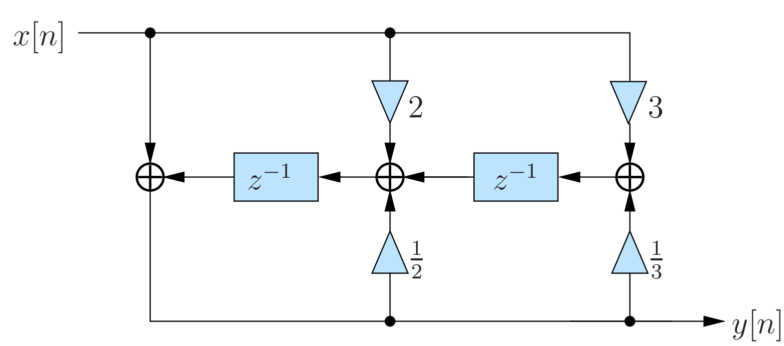
$$X(z) = \frac{z^2 - 1}{z^2 + 4}\,.$$

Make a pole-zero plot, and compute $x[n]$ for two cases: (i) ROC: $|z| < 2$, and (ii) ROC: $|z| > 2$.

(d) Given $x[n] = 2\,a^n \cos(\omega_0 n)$, with $|a| < 1$. Determine the DTFT $X(\omega)$.

Question 5 (4 points)

Consider the following system realization:



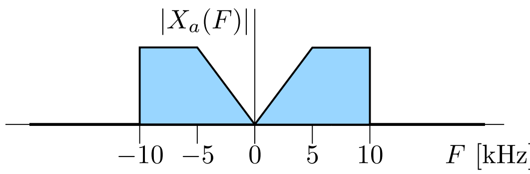
(a) Determine the transfer function $H(z)$.

(b) Is this a minimal realization? (Why?)

(c) Draw the corresponding Direct Form no. 2 realization.

Question 6 (5 points)

A continuous-time signal $x_a(t)$ has an amplitude spectrum $X_a(F)$ as shown below. The signal is sampled with period T so that we obtain a series $x[n] = x_a(nT)$.



For this question, draw the spectra at least for ω running from -2π until 2π.

(a) What is the Nyquist frequency at which we would have to sample to avoid any aliasing?

(b) We sample the signal at 30 kHz. Make a drawing of the resulting amplitude spectrum $|X(\omega)|$ of $x[n]$. Also mark the frequencies.

(c) After sampling, we apply an ideal digital highpass filter, with cutoff frequency $\omega_c = \frac{1}{3}\pi$. Make a drawing of the resulting amplitude spectrum $|Y(\omega)|$. Also mark the frequencies.

(d) After sampling, we invert every second sample of $x[n]$, resulting in $r[n] = (-1)^n x[n]$. Make a drawing of the resulting amplitude spectrum $|R(\omega)|$. Also mark the frequencies.

Question 7 (8 points)

In this question, we will design a Chebyshev type II lowpass filter $G(\Omega)$ with the following specifications:

Third order	
Passband:	$F_p = 3$ kHz
Stopband:	$F_s = 5$ kHz
Minimal stopband damping:	20 dB

Recall that a template Chebyshev (type I) filter has amplitude response

$$|H(\Omega)|^2 = \frac{1}{1 + \epsilon^2 T_n^2(\Omega)} .$$

A Chebyshev type II filter $G(\Omega)$ is derived from type I in two steps. First,

$$|F(\Omega)|^2 = 1 - |H(\Omega)|^2 = \frac{\epsilon^2 T_n^2(\Omega)}{1 + \epsilon^2 T_n^2(\Omega)} .$$

Next, apply a frequency transformation $\Omega \to \frac{\Omega_0}{\Omega}$:

$$|G(\Omega)|^2 = |F(\Omega_0/\Omega)|^2 = \frac{\epsilon^2 T_n^2(\Omega_0/\Omega)}{1 + \epsilon^2 T_n^2(\Omega_0/\Omega)} .$$

(a) Recall that the third order Chebyshev polynomial is given by

$$T_3(\Omega) = 4\Omega^3 - 3\Omega\,.$$

Give a plot of $T_3(\Omega)$. Determine Ω for which $T_3(\Omega)$ is 0, 1, ∞.

(b) Draw plots for $|H(\Omega)|^2$, $|F(\Omega)|^2$ and $|G(\Omega)|^2$ (for $n = 3$ and $\Omega_0 = 1$).

Indicate values on the horizontal and vertical axes. Pay attention to accurately draw the ripples.

(c) Determine Ω_0 and ϵ such that $G(\Omega)$ satisfies the specifications listed at the beginning of this question.

(d) How many dB is the maximal passband attenuation for this 3rd order Chebyshev II filter?