Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science

Partial exam EE2S11 Signals and Systems
Part 2: 31 January 2020, 13:30–15:30

Closed book; two sides of handwritten notes permitted

This exam consists of five questions (36 points)

Question 1 (11 points)

a) Determine the Fourier transform of

$$x(t) = e^{-a|t|} \cos(\Omega_0 t)$$

for $a > 0$.

b) Determine the z-transform of

$$h[n] = (\delta[n] + \delta[n-1]) * a^n u[n]\,,$$

for $|a| < 1$, where '$*$' denotes convolution. Also specify the ROC.

c) The transfer function of an FIR filter is $H(z) = z^{-2}(0.5z + 1 - 0.5z^{-1})$. Find the frequency response of this filter. Is this a linear-phase filter? (Motivate)

d) Determine the inverse z-transform of

$$H(z) = \frac{z+1}{z^2 + 0.81}\,, \qquad \text{ROC: } |z| > 0.9\,.$$

Is this a stable filter (why)?

e) Determine the inverse z-transform of

$$H(z) = \frac{z+1}{z^2 + 0.81}\,, \qquad \text{ROC: } |z| < 0.9\,.$$

Is this a stable filter (why)?

Question 2 (8 points)

The output of a discrete-time causal filter with transfer function

$$H(z) = \frac{z+1}{z^2 + 0.81}$$

is a sequence $y[n] = \delta[n-1] + \delta[n-2]$.

a) Determine the input sequence $x[n]$ such that the output of the filter is the given $y[n]$.

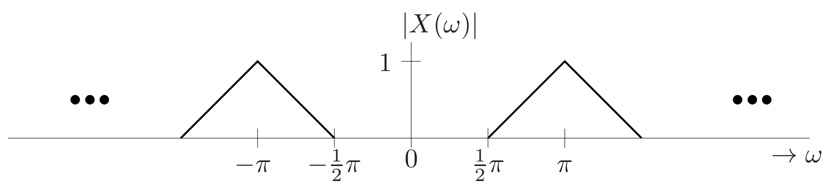
b) Determine all poles and zeros of the filter and draw a pole-zero plot.

c) Using b), sketch the amplitude spectrum $|H(e^{j\omega})|$, also indicate values on the axes.

d) Draw the "Direct form no. II" realization of the filter and also specify the coefficients.

Question 3 (6 points)

Consider an analog signal $x_a(t)$ with Fourier transform $X_a(F)$ (with F in Hz). Suppose that the signal is bandlimited with maximal frequency 5 kHz. The signal is sampled at $F_s = 10$ kHz. The resulting discrete-time signal $x[n]$ has spectrum $X(\omega)$ as shown below:



a) Using ideal components, is it possible to recover $x_a(t)$ from $x[n]$? (How? Or why not?)

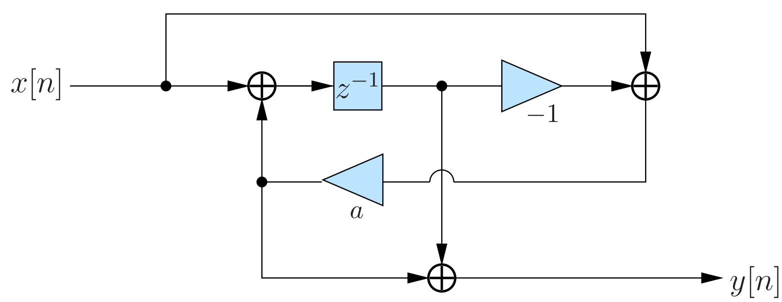
b) What is the relation between F and ω?

c) Plot $|X_a(F)|$.

d) Suppose that, instead, we sample $x_a(t)$ at 5 kHz. Draw the spectrum of the resulting digital signal (also clearly mark the frequencies).

Question 4 (4 points)

a) Determine the transfer function $H(z)$ of the following realization:



b) Is this a minimal realization? (Why?)

Question 5 (7 points)

A second-order analog lowpass filter is given by

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}.$$

a) Determine the squared magnitude response, $|H(j\Omega)|^2$, and give a sketch of it. In the plot, also specify the cut-off frequency.

b) What transformation is needed to obtain a high-pass filter with cut-off frequency Ω_c [rad/s]? Give an expression for the resulting high-pass filter $G_a(s)$.

We want to use $G_a(s)$ to design a second order *digital* high-pass filter with cut-off frequency $\omega_c = 0.2\pi$ [rad].

c) Is the bilinear transform suitable? (Motivate.)

d) Give an expression for the resulting digital high-pass filter $G(z)$.