Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S1 (or EE2S11) SIGNALS AND SYSTEMS

Midterm exam, 30 September 2024, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has four questions (18 points).

Question 1 (3 points)

Consider the signals $x(t) = e^{\mathrm{j}2t}$ and $y(t) = e^{\mathrm{j}\pi t}$.

(a) Determine the fundamental period of $x(t)$.

(b) Determine the fundamental period of $y(t)$.

Next, consider the signal $z(t) = x(t) + y(t)$.

(c) Is the signal $z(t)$ periodic? Motivate your answer.

Finally, consider the signal $w(t) = x(t)\,y(t)$.

(d) Is the signal $w(t)$ periodic? Motivate your answer.

Answer

Note: only answers are provided with no or very little explanation. Your solutions at the exam should always be completely worked out and well motivated.

(a) $T_{0;x} = \pi$

(b) $T_{0;y} = 2$

(c) No, ratio of the fundamental periods of x and y is not a rational number.

(d) $w(t) = e^{\mathrm{j}(2+\pi)t}$. The signal w is periodic with fundamental period

$$T_{0;w} = \frac{2\pi}{2+\pi}$$

Question 2 (3 points)

Any signal $x(t)$ can be written as $x(t) = x_{\mathrm{e}}(t) + x_{\mathrm{o}}(t)$, where $x_{\mathrm{e}}(t)$ is the even part of $x(t)$ and $x_{\mathrm{o}}(t)$ is the odd part of $x(t)$.

Consider the signal $x(t) = e^{-t}u(t)$, where $u(t)$ is the Heaviside unit step function.

(a) Is it true that the odd part of $x(t)$ is given by

$$x_o(t) = \frac{1}{2}\,\mathrm{sign}(t)\,e^{-|t|},$$

where sign(t) is the sign-function? Motivate your answer.

(b) Is the signal $x(t)$ causal? Motivate your answer.

(c) Is its odd part $x_o(t)$ causal? Motivate your answer.

(d) Is its even part $x_e(t)$ causal? Motivate your answer.

Answer

(a)

$$x_o(t) = \frac{x(t) - x(-t)}{2} = \frac{1}{2}\left[e^{-t}u(t) - e^{t}u(-t)\right]$$

For $t < 0$ we have $x_o(t) = -\frac{1}{2}e^{t}$, while for $t > 0$ we have $x_o(t) = \frac{1}{2}e^{-t}$. Combining both results, we get

$$x_o(t) = \frac{1}{2}\mathrm{sign}(t)e^{-|t|}.$$

Yes, true.

(b) $x(t) = 0$ for $t < 0$. Yes, causal.

(c) $x_o(t) \neq 0$ for $t < 0$. No, not causal.

(d) $x_e(t) \neq 0$ for $t < 0$. No, not causal.

Question 3 (6 points)

Determine the inverse Laplace transforms of

(a)

$$X(s) = \frac{1}{10 - 5s}, \qquad \mathrm{Re}(s) > 2.$$

(b)

$$Y(s) = \frac{1}{3 - 4s} + \frac{3 - 2s}{s^2 + 49}, \qquad \mathrm{Re}(s) > \frac{3}{4}.$$

(c)

$$Z(s) = \frac{s + 3}{(s + 5)(s^2 + 4s + 5)}, \qquad \mathrm{Re}(s) > -2.$$

Answer

(a)

$$X(s) = \frac{1}{10 - 5s} = \frac{-1}{5s - 10} = -\frac{1}{5}\frac{1}{s - 2}, \qquad \mathrm{Re}(s) > 2$$

Using the table, we find

$$x(t) = -\frac{1}{5}e^{2t}u(t)$$

(b)

$$\begin{aligned}
Y(s) &= \frac{1}{3-4s} + \frac{3-2s}{s^2+49} \\
&= \frac{-1}{4s-3} + \frac{3}{s^2+49} - \frac{2s}{s^2+49} \\
&= -\frac{1}{4}\frac{1}{s-3/4} + \frac{3}{7}\frac{7}{s^2+7^2} - 2\frac{s}{s^2+7^2}, \qquad \mathrm{Re}(s) > 3/4
\end{aligned}$$

Using the table, we find

$$y(t) = -\frac{1}{4}e^{\frac{3}{4}t}u(t) + \frac{3}{7}\sin(7t)u(t) - 2\cos(7t)u(t)$$

(c)

$$Z(s) = \frac{s+3}{(s+5)(s^2+4s+5)} = -\frac{1}{5}\frac{1}{s+5} + \frac{1}{5}\frac{s+4}{s^2+4s+5}, \qquad \mathrm{Re}(s) > -2$$

Rewrite

$$Z(s) = -\frac{1}{5}\frac{1}{s+5} + \frac{1}{5}\frac{s+2}{(s+2)^2+1} + \frac{2}{5}\frac{1}{(s+2)^2+1} \qquad \mathrm{Re}(s) > -2.$$

Using the table, we find

$$z(t) = -\frac{1}{5}e^{-5t}u(t) + \frac{1}{5}e^{-2t}\cos(t)u(t) + \frac{2}{5}e^{-2t}\sin(t)u(t)$$

Question 4 (6 points)

The periodic signal $x(t)$ has a fundamental period $T_0 = 2\pi$, and one interval is given by

$$x(t) = |t|, \qquad -\pi \le t \le \pi.$$

The trigonometric Fourier series of this periodic signal is given by

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t).$$

(a) Explain why $d_k = 0$ for $k = 1, 2, \ldots$.

(b) Determine the dc component c_0 of $x(t)$.

(c) Explain why c_k decays as A/k^2 as $k \to \infty$ (A is a constant).

(d) Determine the Fourier coefficients c_k of $x(t)$.

Answer

(a) $x(t)$ is even.

(b) The dc component is $c_0 = X_0 = \pi/2$

(c) The signal is continuous, but its derivative is not.

(d)

$$c_k = \frac{1}{\pi k^2}[(-1)^k - 1], \qquad k \ge 1.$$