Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S1 (or EE2S11) SIGNALS AND SYSTEMS

Midterm exam, 30 September 2024, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has four questions (18 points).

Question 1 (3 points)

Consider the signals $x(t) = e^{\mathrm{j}2t}$ and $y(t) = e^{\mathrm{j}\pi t}$.

(a) Determine the fundamental period of $x(t)$.

(b) Determine the fundamental period of $y(t)$.

Next, consider the signal $z(t) = x(t) + y(t)$.

(c) Is the signal $z(t)$ periodic? Motivate your answer.

Finally, consider the signal $w(t) = x(t)\,y(t)$.

(d) Is the signal $w(t)$ periodic? Motivate your answer.

Question 2 (3 points)

Any signal $x(t)$ can be written as $x(t) = x_{\mathrm{e}}(t) + x_{\mathrm{o}}(t)$, where $x_{\mathrm{e}}(t)$ is the even part of $x(t)$ and $x_{\mathrm{o}}(t)$ is the odd part of $x(t)$.

Consider the signal $x(t) = e^{-t}u(t)$, where $u(t)$ is the Heaviside unit step function.

(a) Is it true that the odd part of $x(t)$ is given by

$$x_{\mathrm{o}}(t) = \frac{1}{2}\,\mathrm{sign}(t)\,e^{-|t|},$$

where $\mathrm{sign}(t)$ is the sign-function? Motivate your answer.

(b) Is the signal $x(t)$ causal? Motivate your answer.

(c) Is its odd part $x_{\mathrm{o}}(t)$ causal? Motivate your answer.

(d) Is its even part $x_{\mathrm{e}}(t)$ causal? Motivate your answer.

Question 3 (6 points)

Determine the inverse Laplace transforms of

(a)
$$X(s) = \frac{1}{10 - 5s}\,, \qquad \mathrm{Re}(s) > 2.$$

(b)
$$Y(s) = \frac{1}{3 - 4s} + \frac{3 - 2s}{s^2 + 49}\,, \qquad \mathrm{Re}(s) > \frac{3}{4}.$$

(c)
$$Z(s) = \frac{s + 3}{(s + 5)(s^2 + 4s + 5)}\,, \qquad \mathrm{Re}(s) > -2.$$

Question 4 (6 points)

The periodic signal $x(t)$ has a fundamental period $T_0 = 2\pi$, and one interval is given by

$$x(t) = |t|\,, \qquad -\pi \leq t \leq \pi\,.$$

The trigonometric Fourier series of this periodic signal is given by

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t)\,.$$

(a) Explain why $d_k = 0$ for $k = 1, 2, \ldots$.

(b) Determine the dc component c_0 of $x(t)$.

(c) Explain why c_k decays as A/k^2 as $k \to \infty$ (A is a constant).

(d) Determine the Fourier coefficients c_k of $x(t)$.