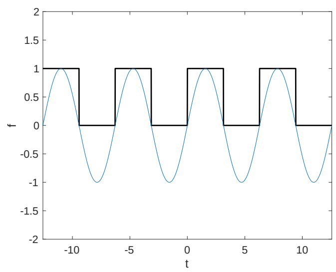
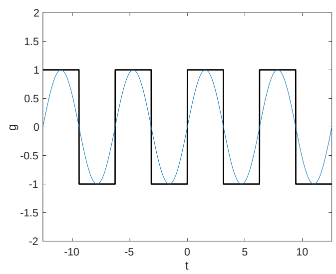
SOLUTIONS

Problem 1

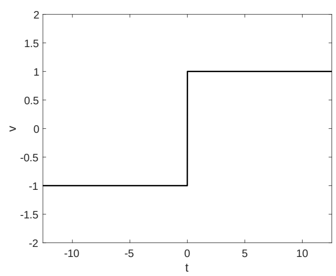
a) Bold line = graph of signal $f(t)$.



b) Bold line = graph of signal $g(t)$.



c) $v(t) = \text{sign}(t)$. Bold line = graph of signal $v(t)$.



d) $x(t)$ is a unit step function that starts at $t = -1/2$: $x'(t) = \delta(t + 1/2)$, $A = 1$, $\tau = -1/2$.

Problem 2

a) $f(t) = -(-t)x(t)$ with $x(t) = \cos(2t)u(t)$. Laplace transform

$$F(s) = -\frac{\mathrm{d}X}{\mathrm{d}s} \quad \text{with} \quad X(s) = \frac{s}{s^2+4} \quad \text{and} \quad \text{ROC} = \{s \in \mathbb{C}; \text{Re}(s) > 0\}.$$

Computing the derivative, we find

$$F(s) = \frac{s^2-4}{(s^2+4)^2}, \quad \text{ROC} = \{s \in \mathbb{C}; \text{Re}(s) > 0\}.$$

b)

$$F_1(s) = \frac{s+3}{(s+2)(s+5)} = \frac{1}{3}\frac{1}{s+2} + \frac{2}{3}\frac{1}{s+5}$$

We obtain: $f_1(t) = \frac{1}{3}e^{-2t}u(t) + \frac{2}{3}e^{-5t}u(t)$.

c)

$$F_2(s) = \frac{1}{(s^2+4)(s^2+9)} = \frac{1}{5}\frac{1}{s^2+4} - \frac{1}{5}\frac{1}{s^2+9} = \frac{1}{10}\frac{2}{s^2+2^2} - \frac{1}{15}\frac{3}{s^2+3^2}.$$

We obtain $f_2(t) = \frac{1}{10}\sin(2t)u(t) - \frac{1}{15}\sin(3t)u(t)$.

d)

$$F_3(s) = \frac{1}{(s^2+4)^2} = -\frac{1}{8}\frac{(s^2-4)}{(s^2+4)^2} + \frac{1}{8}\frac{1}{s^2+4} = \frac{1}{8}\frac{\mathrm{d}}{\mathrm{d}s}\frac{s}{s^2+4} + \frac{1}{16}\frac{2}{s^2+2^2}.$$

We obtain $f_3(t) = -\frac{1}{8}t\cos(2t)u(t) + \frac{1}{16}\sin(2t)u(t)$.

Problem 3

a) Since $x(t)$ is odd, we find

$$X_0 = \frac{1}{2}\int_{t=-1}^{1} x(t)\,\mathrm{d}t = 0.$$

b) Signal is odd and therefore $X_{-k} = -X_k$, that is, the Fourier coefficients are odd functions of k. Signal is also real-valued and therefore $X_k^* = X_{-k}$. Combining these two results, we find

$$X_k^* = -X_k$$

showing that X_k is purely imaginary.

c) For $k \neq 0$, evaluate the integral

$$X_k = \frac{1}{2} \int_{t=-1}^{1} t e^{-\mathrm{j}k\pi t} \, \mathrm{d}t$$

and the result follows.

d) Imaginary part of Y_k are the Fourier coefficients of $y_\mathrm{o}(t)$. Conclusion: $y_\mathrm{o}(t) = x(t)$.

e) Real part of Y_k are the Fourier coefficients of $y_\mathrm{e}(t)$. Even part is continuous (no $1/k$-term).

f) $\mathrm{Re}[z(t)] = \frac{1}{2}x(t)$.