Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science

EE2S11 SIGNALS AND SYSTEMS

Part 1, 9 December 2019, 13:30 - 15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted.

This exam has three questions (28 points)

Question 1 (5 points)

Let $u(t)$ denote the Heaviside unit step function, and $\text{sign}(t)$ the sign-function.

Plot the signals

a) $f(t) = u[\sin(t)]$

b) $g(t) = 2u[\sin(t)] - 1$

c) $v(t) = \sin[\frac{\pi}{2}\text{sign}(t)]$.

Consider the signal

$$x(t) = u(2t + 1),$$

where $u(t)$ is the Heaviside unit step function. Its derivative $x'(t)$ can be written as

$$x'(t) = A\delta(t - \tau).$$

d) Determine the constants A and τ.

Question 2 (12 points)

a) Determine the Laplace transform of the signal

$$f(t) = t\cos(2t)u(t),$$

and provide its ROC.

Find the inverse Laplace transforms of

b) $F_1(s) = \dfrac{s+3}{s^2 + 7s + 10}$, $\qquad$ ROC $= \{s \in \mathbb{C}; \text{Re}(s) > -2\}$.

c) $F_2(s) = \dfrac{1}{(s^2 + 4)(s^2 + 9)}$, $\qquad$ ROC $= \{s \in \mathbb{C}; \text{Re}(s) > 0\}$.

d) $F_3(s) = \dfrac{1}{(s^2 + 4)^2}$, $\qquad$ ROC $= \{s \in \mathbb{C}; \text{Re}(s) > 0\}$.

Question 3 (11 points)

The Fourier series expansion of a periodic signal $x(t)$ is given by

$$X_k = \frac{1}{T_0}\int_{t=t_0}^{t_0+T_0} x(t)e^{-\text{j}k\Omega_0 t}\,\text{d}t, \qquad k \in \mathbb{Z}.$$

Let $x(t)$ be a periodic signal with fundamental period $T_0 = 2$. On the interval $(-1, 1)$, $x(t)$ is given by

$$x(t) = t, \quad t \in (-1, 1).$$

a) Determine the dc component X_0 of the signal $x(t)$.

b) Explain why the Fourier coefficients X_k of $x(t)$ are an odd function of k and purely imaginary.

c) Show that the Fourier components of $x(t)$ are given by

$$X_k = \mathrm{j}\frac{(-1)^k}{k\pi}, \quad k \neq 0.$$

The Fourier coefficients of a second real-valued periodic signal $y(t)$ with fundamental period $T_0 = 2$ are given by

$$Y_0 = 3/2 \quad \text{and} \quad Y_k = \frac{1-(-1)^k}{(k\pi)^2} + \mathrm{j}\frac{(-1)^k}{k\pi}, \quad k \neq 0.$$

d) Determine $y_\mathrm{o}(t)$, the odd part of the signal $y(t)$.

e) Is the even part of $y(t)$ continuous? Motivate your answer.

A third complex-valued signal $z(t)$ is also periodic with fundamental period $T_0 = 2$ and its Fourier coefficients are given by

$$Z_k = \begin{cases} 0 & k \in \mathbb{Z}, k \leq 0 \\ X_k & k \in \mathbb{Z}, k \geq 1. \end{cases}$$

f) Determine $\mathrm{Re}[z(t)]$, the real part of signal $z(t)$.