(3rd ed) --
(4th ed) 6.2

8.2 (a) To find $X(\Omega)$, we use duality or find the inverse Fourier transform of a pulse of amplitude A and bandwidth Ω_0, that is

$$X(\Omega) = A[u(\Omega + \Omega_0) - u(\Omega - \Omega_0)]$$

so that

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\Omega_0}^{\Omega_0} Ae^{j\Omega t}d\Omega = \frac{A}{2\pi jt}e^{j\Omega t}\Big|_{-\Omega_0}^{\Omega_0} \\ &= \frac{A}{\pi t}\sin(\Omega_0 t) \end{aligned}$$

which when compared with the given $x(t) = \sin(t)/t$ gives that $A = \pi$ and $\Omega_0 = 1$ or

$$X(\Omega) = \pi[u(\Omega + 1) - u(\Omega - 1)]$$

indicating that $x(t)$ is band-limited with a maximum frequency $\Omega_{max} = 1$ (rad/sec).
(b) To sample without aliasing the sampling frequency should be chosen to be

$$f_s = \frac{1}{T_s} \geq 2\frac{\Omega_{max}}{2\pi}$$

which gives a sampling period

$$T_s \leq \frac{\pi}{\Omega_{max}} = \pi \;\text{ sec/sample}$$

(c) The spectrum of $y(t) = x^2(t)$ is the convolution in the frequency

$$Y(\Omega) = \frac{1}{2\pi}\left(X(\Omega) * X(\Omega)\right)$$

which would have a maximum frequency $\Omega_{max} = 2$, giving a sampling frequency which is double the one for $x(t)$. The sampling period for $y(t)$ should be

$$T_s \leq \frac{\pi}{2}.$$

(d) The signal $x(t) = \sin(t)/t$ is zero whenever $t = \pm k\pi$, for $k = 1, 2, \cdots$ so that choosing $T_s = \pi$ (the Nyquist sampling period) we obtain the desired signal $x_s(0) = 1$ and $x(nT_s) = 0$.

8.3 (a) The Fourier transform is

(3rd ed) 8.2
(4th ed) 6.3

$$X(\Omega) = 2\pi[u(\Omega + 0.5) - u(\Omega - 0.5)]$$

$x(t)$ is clearly band-limited with $\Omega_{max} = 0.5$ (rad/sec).
(b) According to the Nyquist sampling rate condition, we should have that

$$\Omega_s = \frac{2\pi}{T_s} \geq 2\Omega_{max}$$

or the sampling period

$$T_s \leq \frac{\pi}{\Omega_{max}} = 2\pi$$

The given value satisfies the Nyquist sampling rate condition so we can sample the signal with no aliasing. The given sampling period is the Nyquist sampling period.
Plotting the sinc function it can be seen that it is zero at values of $0.5t = \pm\pi k$ or $t = \pm 2\pi k$ for an integer k. The sampled signal using $T_s = 2\pi$ is

$$x(nT_s) = \frac{\sin(0.5\ 2\pi n)}{0.5n\ 2\pi} = \frac{\sin(\pi n)}{\pi n}$$

which is 1 for $n = 0$, and 0 for any other value of n.
(c) It seems the signal cannot be reconstructed from the samples, that frequency aliasing has occurred. Ideally, that is not the case. The spectrum of the sampled signal $x_s(t)$ for $T_s = 2\pi$ ($\Omega_s = 1$) is

$$X_s(\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\Omega - k\Omega_s) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(\Omega - k) = 1$$

Passing this signal through an ideal low-pass filter with amplitude 2π and cut-off frequency $\Omega_s/2 = 1/2$ the reconstructed signal, the output of this filter, is the inverse Fourier transform of a pulse in frequency, i.e., a sinc function, that coincides with the original signal.

8.5 (a) Inverse Fourier transform

(3rd ed) 8.3
(4th ed) 6.4

$$x(t) = \frac{1}{2\pi}\int_{-1}^{1} e^{j\Omega t}d\Omega = \frac{\sin(t)}{\pi t}$$

of infinite support in time but finite support in frequency.

(b) $y(t) = (x * x)(t)$ then $Y(\Omega) = X(\Omega)X(\Omega) = X(\Omega)$ so that $y(t) = x(t)$

(c) $2\pi f_{max} = 1$ then $f_s \geq 2f_{max} = 1/\pi$ so $T_s \leq \pi$.

(d) If $T_s = \pi$ we get samples

$$x(nT_s) = \begin{cases} 1/\pi & n = 0 \\ \sin(n\pi)/(n\pi^2) = 0 & n \neq 0 \end{cases}$$

$T_s = \pi$ barely satisfies Nyquist.

8.8 (3rd ed) 8.4 (4th ed) 6.6

(a) The maximum frequency of $s(t)$ is $f_{max} = 12 \times 10^3$ Hz thus according to Nyquist $f_s \geq 2f_{max} = 24 \times 10^3$ Hz.

(b) The different spectra are shown in Fig. 8.1. By the modulation property the sampler shifts the spectrum of $s(t)$ up and down to center frequencies $10m$ (in kHz) for $m = 0, \pm 1, \pm 2, \cdots$ giving the spectrum $S_s(f)$ in Fig. 8.1. If we filter the sampled signal with a low-pass filter of magnitude $T_s = 1/f_s = 10^{-4}$ and bandwidth 2 kHz we recover the original message.

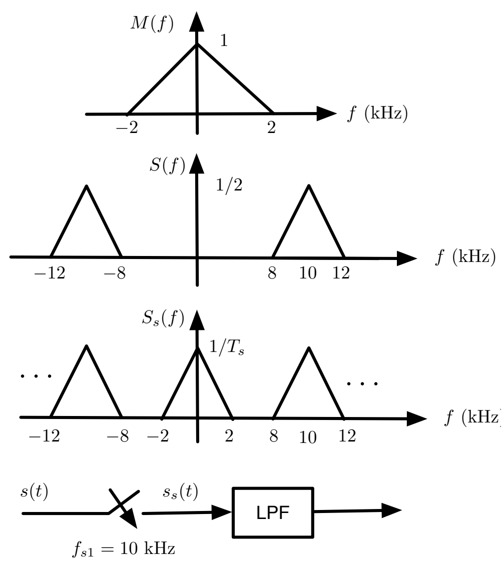

Figure 8.1: Problem 8

(3rd ed) --
(4th ed) 6.7

8.9 (a) If $\mathcal{F}(\cos(t)) = X(\Omega)$, then $\mathcal{F}(x^2(t)) = (X * X)(\Omega)/(2\pi)$, i.e., the convolution of $X(\Omega)$ with itself, having twice the bandwidth of $X(\Omega)$, so the maximum frequency of $x(t)$ is $\Omega_{max} = 2$, and $T_s \leq \pi/2$. Thus $x(t)$ is band-limited with twice the maximum frequency of $\cos(t)$.
Using $\cos^2(t) = 0.5(1 + \cos(2t))$ we see that its maximum frequency is 2 and it is band-limited and $T_s \leq \pi/2$.

(b) In general $\mathcal{F}(x^N(t))$ would be N convolutions of $X(\Omega)$ with itself, and the bandwidth being $N\times$bandwith of $\cos(t)$, i.e., N, so the maximum frequency is $\Omega_{max} = N$ and $T_s \leq \pi/N$.
If $N = 3$, $\cos^3(t) = 0.25(3\cos(t) + \cos(3t))$ with $\Omega_{max} = 3$, and $T_s \leq \pi/3$.

8.10 (a) If $T_s = 1$ then
(3rd ed) 8.5
(4th ed) 6.8

$$\Omega_s = \frac{2\pi}{T_s} \geq 2\Omega_{max}$$

or $\Omega_{max} \leq \pi$. To reconstruct the original signal we choose the cutoff frequency of the ideal low-pass filter to be

$$\Omega_{max} < \Omega_c < 2\pi - \Omega_{max}$$

and the magnitude $T_s = 1$.
(b) Since $T_s \leq \pi/\Omega_{max}$, and $T_s = 1$ then $\Omega_{max} \leq \pi$. If $\Omega_{max} = \pi$ for an ideal low-pass filter, then $\Omega_c = \pi$ to recover the original signal. Thus the maximum frequency has to be smaller than π to make it possible to use an ideal or a non-ideal low-pass filter to recover the original signal.

(3rd ed) --
(4th ed) --

8.13 (a) For $x(t)$ to be sampled without aliasing we need $\Omega_s > 2\Omega_{max} = 2$.

(b) If $y(t) = x^2(t)$ then $Y(\Omega) = [X * X](\Omega)/(2\pi)$ which gives a triangular spectrum in $[-2\ 2]$ frequency band, i.e., it is band-limited.

(c) The maximum frequency of $z(t)$ is $\Omega_{max} = 2$ (rad/sec) so $\Omega_s \geq 4$ (rad/sec) and $T_s \leq \pi/2$ sec.

8.15 (a) The sampling frequency is $\Omega_s = 4\pi$ then $T_s = 0.5$ sec/sample.

(3rd ed) 8.8
(4th ed) 6.11

(b) The spectrum of $x(t)$ is

$$X(\Omega) = r(\Omega+\pi) - 2r(\Omega) + r(\Omega-\pi)$$

where $r(.)$ is the ramp function. By duality, we have that if $z(t) = r(t+\pi) - 2r(t) + r(t-\pi)$ its Fourier transform is

$$\begin{aligned} Z(\Omega) &= Z(s)|_{s=j\Omega} = \frac{e^{s\pi} - 2 + e^{-s\pi}}{s^2}|_{s=j\Omega} \\ &= 2\frac{1-\cos(\pi\Omega)}{\Omega^2} = 4\frac{\sin^2(\pi\Omega/2)}{\Omega^2} \\ &= \left(\frac{\sin(\pi\Omega/2)}{\Omega/2}\right)^2 \end{aligned}$$

then

$$Z(t) \Leftrightarrow 2\pi z(-\Omega) = 2\pi z(\Omega)$$

so that

$$x(t) = \frac{Z(t)}{2\pi} = \frac{1}{2\pi}\left(\frac{\sin(\pi t/2)}{t/2}\right)^2$$

(c) An ideal low-pass filter of dc gain $T_s = 1/2$ and bandwidth $\pi < B < 3\pi$.