12.11 (a) The dc gain is $|H(j0)| = 1$ and the gain at $\Omega = 1$ is

(3rd ed) 12.7
(4th ed) 10.11

$$|H(j1)| = \frac{1}{\sqrt{2}}|H(j0)|$$

so that $\Omega = 1$ is the half-power frequency.

(b) K matches the normalized half-power frequency $\Omega_{hp} = 1$ with the half-power frequency ω_{hp} according to

$$K = \frac{\Omega_{hp}}{\tan^{-1}(0.5\omega_{hp})} \quad \Rightarrow \quad \omega_{hp} = 2\tan(1) = \pi/2$$

(c) Replacing s by the bilinear transformation we get

$$\frac{1}{s^2 + \sqrt{2}s + 1}|_{s=(1-z^{-1})/(1+z^{-1})} = \frac{(1+z^{-1})^2}{(2+\sqrt{2}) + (2-\sqrt{2})z^{-2}}$$

(d) The analog $H(s)$ has no zeros, and its poles are

$$s_{1,2} = 1e^{\pm j3\pi/4}$$

or on the left-hand s-plane.
For the discrete filter,

$$H(z) = \frac{(z+1)^2}{(2+\sqrt{2})z^2 + (2-\sqrt{2})}$$

so zeros are $z = -1$, double, and poles

$$z_{1,2} = \left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)^{1/2} e^{\pm j\pi/2}$$

or inside the unit circle. Thus both filters are stable.

12.14 (a) From the realization

(3rd ed) 12.9
(4th ed) 10.14

$$E(z) = X(z) - E(z)(z^{-1} + Gz^{-2}) \;\; \Rightarrow E(z) = \frac{X(z)}{1 + z^{-1} + Gz^{-2}}$$
$$Y(z) = E(z)(1 + z^{-1} + z^{-2}) \;\; \Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + z^{-1} + z^{-2}}{1 + z^{-1} + Gz^{-2}}$$

or

$$Y(z)[1 + z^{-1} + Gz^{-2}] = X(z)[1 + z^{-1} + z^{-2}]$$

which gives the difference equation:

$$y[n] \;\; = \;\; -y[n-1] - Gy[n-2] + x[n] + x[n-1] + x[n-2]$$

(b) The transfer function of the filter is

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + z^{-1} + Gz^{-2}}$$

with poles

$$p_{1,2} = -0.5 \pm \frac{\sqrt{1 - 4G}}{2}$$

for these to be complex conjugates we need that $1 - 4G < 0$ or $G > 1/4$. The BIBO stability requires that $|p_{1,2}| < 1$ (inside the unit circle), or

$$\sqrt{\frac{1}{4} + \frac{4G - 1}{4}} < 1$$

or $G < 1$. Thus for the system to have complex conjugate poles and to be BIBO stable we need that $1/4 < G < 1$.

12.15 (a) Let

(3rd ed) --
(4th ed) 10.15

$$H(z) = \underbrace{\frac{2(1-z^{-1})}{1+0.5z^{-1}}}_{H_1(z)} \underbrace{\frac{1+\sqrt{2}z^{-1}+z^{-2}}{1-0.9z^{-1}+0.81z^{-2}}}_{H_2(z)}$$

The cascade realization of $H(z)$ is as shown in Fig. 12.2

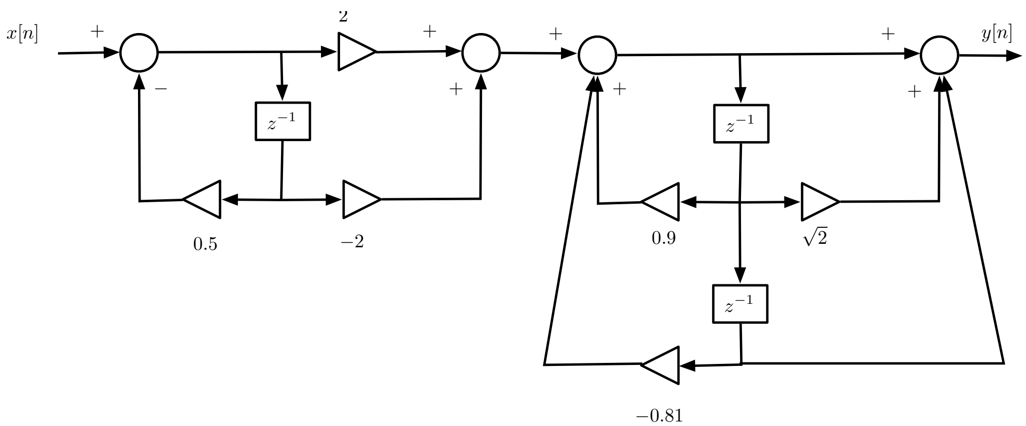

Figure 12.2: *Problem 15: Cascade realization of $H(z)$.*

(b) To obtain the parallel realization we do a partial fraction expansion of $H(z)$. Notice that $H(z)$ is not proper rational and so we have to obtain a constant term before performing the partial fraction expansion

$$H(z) = -4.94 + \frac{2.16}{1+0.5z^{-1}} + \frac{4.78-1.6z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

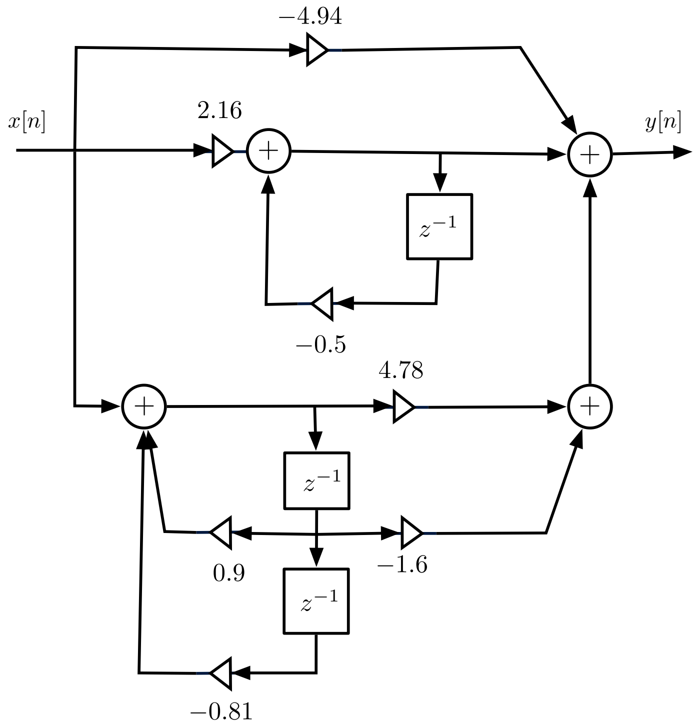

Figure 12.3: *Problem 15: Parallel realization of $H(z)$.*

(3rd ed) 12.10
(4th ed) 10.16

12.16 Using the variables $v[n]$ and $w[n]$ we obtain the following expressions

$$g[n] = 2v[n] + 1.8v[n-1] + 0.4v[n-2]$$
$$v[n] = x[n] + 1.3v[n-1] - 0.8v[n-2]$$

with Z-transforms

$$G(z) = (2 + 1.8z^{-1} + 0.4z^{-2})V(z)$$
$$V(z) = \frac{X(z)}{1 - 1.3z^{-1} + 0.8z^{-2}}$$

so that

$$\frac{G(z)}{X(z)} = \frac{2 + 1.8z^{-1} + 0.4z^{-2}}{1 - 1.3z^{-1} + 0.8z^{-2}}.$$

Thus, the difference equation relating $g[n]$ to $x[n]$ is

$$g[n] = 1.3g[n-1] - 0.8g[n-2] + 2x[n] + 1.8x[n-1] + 0.4x[n-2]$$

For the second cascade, we have

$$y[n] = 3w[n] + 4.5w[n-1] \;\Rightarrow\; Y(z) = (3 + 4.5z^{-1})W(z)$$
$$w[n] = g[n] + 0.3w[n-1] \;\Rightarrow\; W(z) = \frac{G(z)}{1 - 0.3z^{-1}}$$

so that

$$\frac{Y(z)}{G(z)} = \frac{3 + 4.5z^{-1}}{1 - 0.3z^{-1}} \;\Rightarrow\; y[n] = 0.3y[n-1] + 3g[n] + 4.5g[n-1]$$

where the term on the right is the difference equation relating $y[n]$ and $g[n]$.
(b) The overall transfer function is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{G(z)}\frac{G(z)}{X(z)} = \frac{(3 + 4.5z^{-1})(2 + 1.8z^{-1} + 0.4z^{-2})}{(1 - 0.3z^{-1})(1 - 1.3z^{-1} + 0.8z^{-2})}$$