Chapter 1

Continuous–time Signals

1.1 Basic Problems

1.1 Notice that $0.5[x(t)+x(-t)]$, the even component of $x(t)$, is discontinuous at $t=0$, it is 1 at $t=0$ but 0.5 at $t \pm \epsilon$ for $\epsilon \to 0$. Likewise the odd component of $x(t)$, or $0.5[x(t)-x(-t)]$, must be zero at $t=0$ so that when added to the even component one gets $x(t)$.
$z(t)$ equals $x(t)$. See Fig. 1.

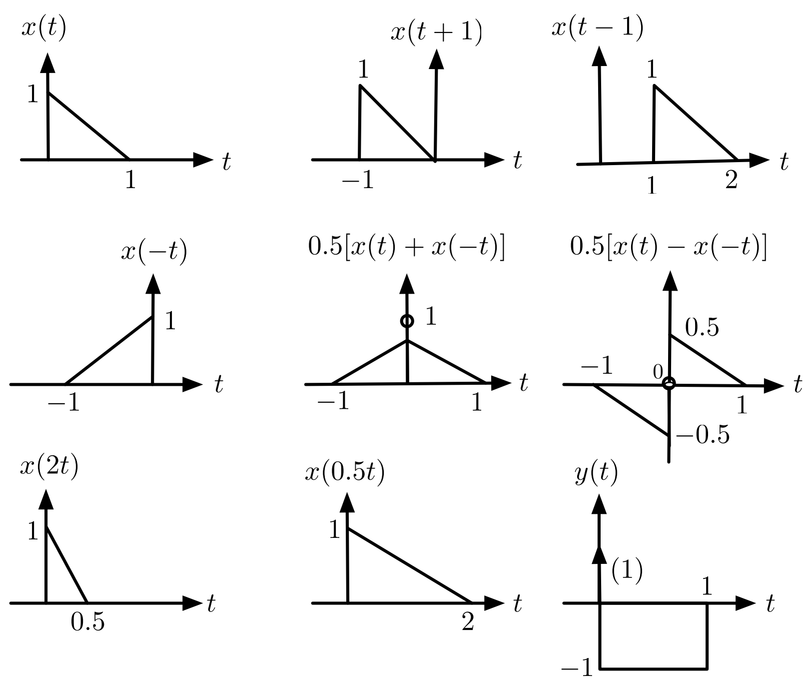

Figure 1.1: Problem 1

~~1.3~~ (3rd ed)1.2 (4th ed) 1.3

(a) We have that

i. $x(t)$ is causal because it is zero for $t < 0$. It is neither even nor odd.

ii. Yes, the even component of $x(t)$ is

$$\begin{aligned} x_e(t) &= 0.5[x(-t) + x(t)] \\ &= 0.5[e^t u(-t) + e^{-t}u(t)] = 0.5e^{-|t|} \end{aligned}$$

(b) $x(t) = \cos(t) + j\sin(t)$ is a complex signal, $x_e(t) = 0.5[e^{jt} + e^{-jt}] = \cos(t)$ so $x_o(t) = j\sin(t)$.

(c) The product of the even signal $x(t)$ with the sine, which is odd, gives an odd signal and because of this symmetry the integral is zero.

(d) Yes, because $x(t) + x(-t) = 2x_e(t)$, i.e., twice the even component of $x(t)$, and multiplied by the sine it is an odd function.

~~1.4~~ (3rd ed) 1.3 (4th ed) 1.4 The signal $x(t) = t[u(t) - u(t-1)]$ so that its reflection is

$$v(t) = x(-t) = -t[u(-t) - u(-t-1)]$$

and delaying $v(t)$ by 2 is

$$\begin{aligned} y(t) &= v(t-2) = -(t-2)[u(-(t-2)) - u(-(t-2)-1)] \\ &= (-t+2)[u(-t+2) - u(-t+1)] = (2-t)[u(t-1) - u(t-2)] \end{aligned}$$

On the other hand, the delaying of $x(t)$ by 2 gives

$$w(t) = x(t-2) = (t-2)[u(t-2) - u(t-3)]$$

which when reflected gives

$$z(t) = w(-t) = (-t-2)[u(-t-2) - u(-t-3)]$$

Comparing $y(t)$ and $z(t)$ we can see that these operations do not commute, that the order in which these operations are done cannot be changed, so that $y(t) \neq z(t)$ as shown in Fig. 1.4.

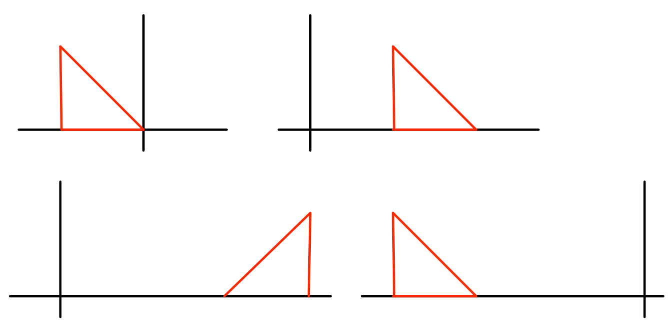

Figure 1.4: Problem 4: Reflection and delaying do not commute, $y(t) \neq z(t)$.

~~1.6~~ (3rd ed) 1.4 (4th ed) 1.5

(a) Using $\Omega_0 = 2\pi f_0 = 2\pi/T_0$ for

i. $\cos(2\pi t)$: $\Omega_0 = 2\pi$ rad/sec, $f_0 = 1$ Hz and $T_0 = 1$ sec.

ii. $\sin(t - \pi/4)$: $\Omega_0 = 1$ rad/sec, $f_0 = 1/(2\pi)$ Hz and $T_0 = 2\pi$ sec.

iii. $\tan(\pi t) = \sin(\pi t)/\cos(\pi t)$: $\Omega_0 = \pi$ rad/sec, $f_0 = 1/2$ Hz and $T_0 = 2$ sec.

(b) The fundamental period of $\sin(t)$ is $T_0 = 2\pi$, and $T_1 = 2\pi/3$ is the fundamental period of $\sin(3t)$, $T_1/T_0 = 1/3$ so $3T_1 = T_0 = 2\pi$ is the fundamental period of $z(t)$.

(c) i. $y(t)$ is periodic of fundamental period $T_0 = 1$.

ii. $w(t) = x(2t)$ is $x(t)$ compressed by a factor of 2 so its fundamental period is $T_0/2 = 1/2$, the fundamental period of $z(t)$.

iii. $v(t)$ has same fundamental period as $x(t)$, $T_0 = 1$, indeed $v(t + kT_0) = 1/x(t + kT_0) = 1/x(t)$.

(d) i. $x(t) = 2\cos(t)$, $\Omega_0 = 2\pi f_0 = 1$ so $f_0 = 1/(2\pi)$

ii. $y(t) = 3\cos(2\pi t + \pi/4)$, $\Omega_0 = 2\pi f_0 = 2\pi$ so $f_0 = 1$

iii. $c(t) = 1/\cos(t)$, of fundamental period $T_0 = 2\pi$, so $f_0 = 1/(2\pi)$.

(e) $z_e(t)$ is periodic of fundamental period T_0, indeed

$$\begin{aligned} z_e(t + T_0) &= 0.5[z(t + T_0) + z(-t - T_0))] \\ &= 0.5[z(t) + z(-t)] \end{aligned}$$

Same for $z_o(t)$ since $z_o(t) = z(t) - z_e(t)$.

~~1.8~~ (3rd ed) 1.6 (4th ed) 1.7

(a) $x(t)$ is a causal decaying exponential with energy

$$E_x = \int_0^\infty e^{-2t}dt = \frac{1}{2}$$

and zero power as

$$P_x = \lim_{T\to\infty} \frac{E_x}{2T} = 0$$

(b)

$$E_z = \int_{-\infty}^{\infty} e^{-2|t|}dt = 2\underbrace{\int_0^\infty e^{-2t}dt}_{E_{z_1}}$$

(c) i. If $y(t) = \text{sign}[x_1(t)]$, it has the same fundamental period as $x_1(t)$, i.e., $T_0 = 1$ and $y(t)$ is a train of pulses so its energy is infinite, while

$$P_y = \int_0^1 1\,dt = 1$$

ii. Since $x_2(t) = \cos(2\pi t - \pi/2) = \cos(2\pi(t-1/4)) = x_1(t-1/4)$, the energy and power of $x_2(t)$ coincide with those of $x_1(t)$.

(d) $v(t) = x_1(t) + x_2(t)$ is periodic of fundamental period $T_0 = 2\pi$, and its power is

$$P_v = \frac{1}{2\pi}\int_0^{2\pi}(\cos(t)+\cos(2t))^2dt = \frac{1}{2\pi}\int_0^{2\pi}(\cos^2(t)+\cos^2(2t)+2\cos(t)\cos(2t))dt$$

Using

$$\cos^2(\theta) = \frac{1}{2}+\frac{1}{2}\cos(2\theta)$$
$$\cos(\theta)\cos(\phi) = \frac{1}{2}(\cos(\theta+\phi)+\cos(\theta-\phi))$$

we have

$$\begin{aligned} P_v &= \underbrace{\frac{1}{2\pi}\int_0^{2\pi}\cos^2(t)dt}_{P_{x_1}} + \underbrace{\frac{1}{2\pi}\int_0^{2\pi}\cos^2(2t)dt}_{P_{x_2}} + \underbrace{\frac{1}{2\pi}\int_0^{2\pi}2\cos(t)\cos(2t))dt}_{0} \\ &= \frac{1}{2}+\frac{1}{2}+0 = 1 \end{aligned}$$

(e) Power of $x(t)$

$$\begin{aligned} P_x &= \frac{1}{T_0}\int_0^{T_0} x^2(t)dt \\ &= \int_0^1 \cos^2(2\pi t)dt \\ &= \int_0^1 (1/2+\cos^2(4\pi t)dt = 0.5+0 = 0.5 \end{aligned}$$

Power of $f(t)$

$$
\begin{aligned}
P_f &= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} y^2(t)dt \\
&= \lim_{N\to\infty} \frac{1}{2(NT_0)}\int_{0}^{NT_0} y^2(t)dt \\
&= \frac{1}{2T_0}\int_{0}^{T_0} y^2(t)dt = 0.5P_s
\end{aligned}
$$

~~1.10~~ (3rd ed) 1.8 (4th ed) 1.9 (a) Let $x(t) = x_1(t) + x_2(t) = \cos(2\pi t) + 2\cos(\pi t)$, so that $x_1(t)$ is a cosine of frequency $\Omega_1 = 2\pi$ or period $T_1 = 1$, and $x_2(t)$ is a cosine of frequency $\Omega_2 = \pi$ or period $T_2 = 2$. The ratio of these periods $T_2/T_1 = 2/1$ is a rational number so $x(t)$ is periodic of fundamental period $T_0 = 2T_1 = T_2 = 2$.
The average power of $x(t)$ is given by

$$P_x = \frac{1}{T_0}\int_0^{T_0} x^2(t)dt = \frac{1}{2}\int_0^2 [x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t)]dt$$

Using the trigonometric identity $\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta)$ we have that the integral

$$\begin{aligned}\frac{1}{2}\int_0^2 2x_1(t)x_2(t)dt &= \frac{1}{2}\int_0^2 4\cos(2\pi t)\cos(\pi t)dt\\ &= \int_0^2 [\cos(\pi t) + \cos(3\pi t)]dt = 0\end{aligned}$$

since $\cos(\pi t) + \cos(3\pi t)$ is periodic of period 2 and so its area under a period is zero. Thus,

$$\begin{aligned}P_x &= \frac{1}{2}\int_0^2 [x_1^2(t) + x_2^2(t)]dt\\ &= \frac{1}{2}\int_0^2 x_1^2(t)dt + \frac{1}{2}2\int_0^1 x_2^2(t)]dt\\ &= P_{x_1} + P_{x_2}\end{aligned}$$

so that the power of $x(t)$ equals the sum of the powers of $x_1(t)$ and $x_2(t)$ which are sinusoids of different frequencies, and thus orthogonal as we will see later.
Finally,

$$\begin{aligned}P_x &= \frac{1}{2}\int_0^2 \cos^2(2\pi t)dt + \int_0^1 4\cos^2(\pi t)dt\\ &= \frac{1}{2}\int_0^2 [0.5 + 0.5\cos(4\pi t)]dt + \int_0^1 4[0.5 + 0.5\cos(2\pi t)]dt\\ &= 0.5 + 2 = 2.5\end{aligned}$$

remembering that the integrals of the cosines are zero (they are periodic of period 0.5 and 1 and the integrals compute their areas under one or more periods, so they are zero).
(b) The components of $y(t)$ have as periods $T_1 = 2\pi$ and $T_2 = 2$ so that $T_1/T_2 = \pi$ which is not rational so $y(t)$ is not periodic. In this case we need to find the power of $y(t)$ by finding the integral over an infinite support of $y^2(t)$ which will as before give

$$P_y = P_{y_1} + P_{y_2}$$

In the case of harmonically related signals we can use the periodicity and compute one integral. However, in either case the power superposition holds.

~~1.13~~ (3rd ed) 1.9 (4th ed) 1.12

(a) The signal $x(t)$ is

$$x(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \leq t \leq 0 \\ -1 & 0 < t \leq 1 \\ 0 & t > 1 \end{cases}$$

there are discontinuities at $t = 0$ and at $t = 1$. The derivative

$$\begin{aligned} y(t) &= \frac{dx(t)}{dt} \\ &= u(t+1) - u(t) - 2\delta(t) + \delta(t-1) \end{aligned}$$

indicating the discontinuities at $t = 0$, a decrease from 1 to -1, and at $t = 1$ an increase from -1 to 0.

(b) The integral

$$\begin{aligned} \int_{-\infty}^{t} y(\tau)d\tau &= \int_{-\infty}^{t} [u(\tau+1) - u(\tau) \\ &\quad -2\delta(\tau) + \delta(\tau - 1)]d\tau = x(t) \end{aligned}$$

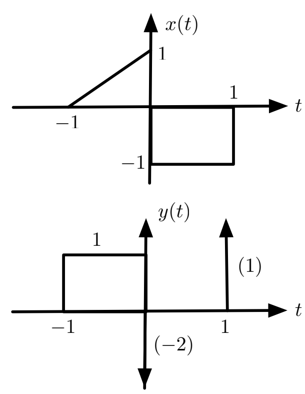

Figure 1.8: Problem 13

~~1.15~~ (3rd ed) 1.11 (4th ed) 1.14 (a) The signal $x(t) = t$ for $0 \leq t \leq 1$, zero otherwise. Then

$$x(2t) = \begin{cases} 2t & 0 \leq 2t \leq 1 \text{ or } 0 \leq t \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

that is, the signal has been compressed — instead of being between 0 and 1, it is now between 0 and 0.5.

(b) Likewise, the signal

$$x(t/2) = \begin{cases} t/2 & 0 \leq t/2 \leq 1 \text{ or } 0 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

i.e., the signal has been expanded, its support has doubled.
The following figure illustrates the compressed and expanded signals $x(2t)$ and $x(t/2)$.

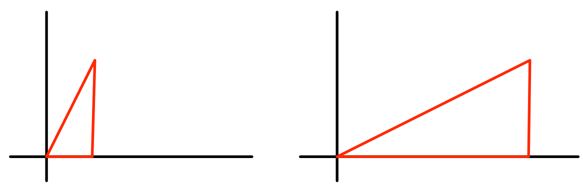

Figure 1.9: Problem 15: Compressed $x(2t)$, expanded $x(t/2)$ signals.

(c) If the acoustic signal is recorded in a tape, we can play it faster (contraction) or slower (expansion) than the speed at which it was recorded. Thus the signal can be made to last a desired amount of time, which might be helpful whenever an allocated time is reserved for broadcasting it.

~~1.16~~ (3rd ed) 1.12 (4th ed) 1.15 (a) Because of the discontinuity of $x(t)$ at $t=0$ the even component of $x(t)$ is a triangle with $x_e(0)=1$, i.e.,

$$x_e(t)=\begin{cases}0.5(1-t) & 0<t\leq 1\\ 0.5(1+t) & -1\leq t<0\\ 1 & t=0\end{cases}$$

while the odd component is

$$x_o(t)=\begin{cases}0.5(1-t) & 0<t\leq 1\\ -0.5(1+t) & -1\leq t<0\\ 0 & t=0\end{cases}$$

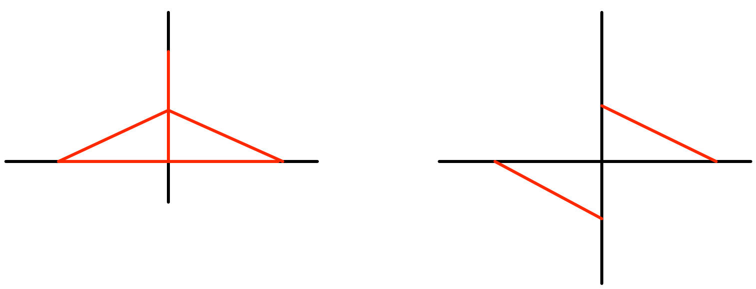

Figure 1.10: Problem 16: Even and odd decomposition of $x(t)$.

(b) The energy of $x(t)$ is

$$\begin{aligned}\int_{-\infty}^{\infty}x^2(t)dt &= \int_{-\infty}^{\infty}[x_e(t)+x_o(t)]^2dt\\ &= \int_{-\infty}^{\infty}x_e^2(t)dt+\int_{-\infty}^{\infty}x_o^2(t)dt+2\int_{-\infty}^{\infty}x_e(t)x_o(t)dt\end{aligned}$$

where the last equation on the right is zero, given that the integrand is odd.
(c) The energy of $x(t)=1-t$, $0\leq t\leq 1$ and zero otherwise, is given by

$$\int_{-\infty}^{\infty}x^2(t)dt=\int_0^1(1-t)^2dt=t-t^2+\frac{t^3}{3}\,|_0^1\,=\frac{1}{3}$$

The energy of the even component is

$$\int_{-\infty}^{\infty}x_e^2(t)dt=0.25\int_{-1}^{0}(1+t)^2dt+0.25\int_0^1(1-t)^2dt=0.5\int_0^1(1-t)^2dt$$

where the discontinuity at $t=0$ does not change the above result. The energy of the odd component is

$$\int_{-\infty}^{\infty}x_o^2(t)dt=0.25\int_{-1}^{0}(1+t)^2dt+0.25\int_0^1(1-t)^2dt=0.5\int_0^1(1-t)^2dt$$

so that

$$E_x=E_{x_e}+E_{x_o}$$

~~1.17~~ (a) The function $g(t)$ corresponding to the first period of $x(t)$ is given by

(3rd ed) 1.13
(4th ed) 1.16

$$g(t) = u(t) - 2u(t-1) + u(t-2)$$

(b) The periodic signal $x(t)$ is

$$\begin{aligned} x(t) = & \quad g(t) + g(t-2) + g(t-4) + \cdots \\ + & \quad g(t+2) + g(t+4) + \cdots = \sum_{k=-\infty}^{\infty} g(t+2k) \end{aligned}$$

(c) Yes, the signals $y(t)$, $z(t)$ and $v(t)$ are periodic of period $T_0 = 2$ as can be easily verified.
(d) The derivative of $x(t)$ is

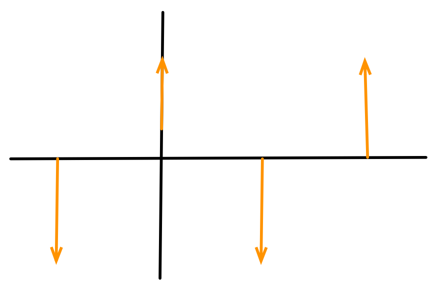

Figure 1.11: Problem 17: Derivative of $x(t)$.

$$\begin{aligned} w(t) = 2\delta(t) & \quad - \quad 2\delta(t-1) + 2\delta(t-2) + \cdots \\ & \quad - \quad 2\delta(t+1) + 2\delta(t+2) + \cdots \end{aligned}$$

which can be seen to be periodic of period $T_0 = 2$.