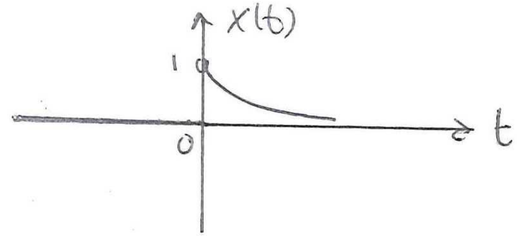
Answers to Selected Problems Chapter 1

Problem 1.3

a. (i) $x(t) = e^{-t}u(t)$



- $x(t) = 0$ for $t<0$, $x(t)$ is causal
- $x(-t) = e^{t}u(-t)$

  $x(-t) \neq x(t)$ for all t, not even

  $x(-t) \neq -x(t)$ for all t, not odd

(ii) $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$= \dfrac{e^{-t}u(t) + e^{t}u(-t)}{2}$

- $\underline{t<0}$ $\quad x_e(t) = \frac{1}{2}e^{t}$
- $\underline{t>0}$ $\quad x_e(t) = \frac{1}{2}e^{-t}$

Combining these two results: $x_e(t) = \frac{1}{2}e^{-|t|}$

b. $x(t) = e^{jt}$

$x_e(t) = \dfrac{e^{jt} + e^{-jt}}{2} = \cos(t)$

$x_o(t) = \dfrac{e^{jt} + e^{-jt}}{2} = j\,\dfrac{e^{jt} - e^{-jt}}{2j} = j\sin(t)$

c. Let $f(t) = g(t)h(t)$ with $g(t)$ an even and $h(t)$ an odd function of t.

$f(-t) = g(-t)h(-t) = g(t)\cdot[-h(t)] = -g(t)h(t)$

$= -f(t)$ for all t.

In other words, $f(t)$ is odd.

For an odd function $f(t)$, we have

$$\int_{t=-\infty}^{\infty} f(t)\,dt = \int_{t=-\infty}^{0} f(t)\,dt + \int_{t=0}^{\infty} f(t)\,dt$$

$$= \int_{t=0}^{\infty} f(-t)\,dt + \int_{t=0}^{\infty} f(t)\,dt$$

$$= -\int_{t=0}^{\infty} f(t)\,dt + \int_{t=0}^{\infty} f(t)\,dt$$

$$= 0$$

- Take $g(t) = x(t)$ and $h(t) = \sin(\Omega_0 t)$

  $g(t)$ is even, $h(t)$ is odd, $f(t) = g(t)h(t)$ is odd

  $$\int_{t=-\infty}^{\infty} x(t)\sin(\Omega_0 t)\,dt = \int_{t=-\infty}^{\infty} f(t)\,dt = 0$$

- Take $g(t) = x(t) + x(-t)$ and $h(t) = \sin(\Omega_0 t)$

  $g(t)$ is even, $h(t)$ is odd, $f(t) = g(t)h(t)$ is odd

  $$\int_{t=-\infty}^{\infty} [x(t) + x(-t)]\sin(\Omega_0 t)\,dt = \int_{t=-\infty}^{\infty} f(t)\,dt = 0$$

Problem 1.11

a. $$\int_{t=0}^{k} e^{j2\pi t}\,dt = \frac{1}{2\pi j} e^{j2\pi t}\Big|_{t=0}^{k} = \frac{1}{2\pi j}\left(e^{j2\pi k} - 1\right)$$

$e^{j2\pi k} = 1$ for any positive integer k

$$\int_{t=0}^{k} e^{j2\pi t}\,dt = 0, \quad k \text{ positive integer}$$

b. Let $$f(t_0) = \int_{t=t_0}^{t_0+T_0} x(t)\,dt$$

If we can show that $\frac{df}{dt_0} = 0$ then f does not depend on t_0 and we are done.

Although it is not necessary, let's turn this problem into a Dirac distribution exercise.

We write

$$f(t_0) = \int_{t=-\infty}^{\infty} x(t)\,[u(t-t_0) - u(t-t_0-T_0)]\,dt$$

Taking the derivative with respect to t_0, we obtain

$$\frac{df}{dt_0} = \int_{t=-\infty}^{\infty} x(t)\,[-\delta(t-t_0) + \delta(t-t_0-T_0)]\,dt$$

$$= x(t_0+T_0) - x(t_0) = x(t_0) - x(t_0) = 0$$

c. $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$

$f(t) = \cos(2\pi t)$ and $t_0 = 1$

$\cos(2\pi t)\,\delta(t-1) = \delta(t-1)$

d. $x(t) = \cos(t)\,u(t)$

$$\frac{dx}{dt} = -\sin(t)\,u(t) + \cos(t)\,\delta(t)$$

$$= -\sin(t)\,u(t) + \delta(t)$$

e. $\int_{t=-\infty}^{\infty} e^{-t} u(t)\,\delta(t-2)\,dt = e^{-2}$

↑ typo in book

f. $x(t) = \cosh(t)\,u(t)$

$$\frac{dx}{dt} = \sinh(t)\,u(t) + \cosh(t)\,\delta(t)$$

$$= \sinh(t)\,u(t) + \delta(t)$$

g. $x(t)$ is not periodic

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{t=-T}^{T} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{t=0}^{T} \sin^2(t)\,dt$$

$$= \lim_{N\to\infty} \frac{1}{2NT_0} \cdot N \int_{t=0}^{T_0} \sin^2(t)\,dt$$

$$= \frac{1}{2T_0} \int_{t=0}^{T_0} \sin^2(t)\,dt \quad \text{with } T_0 = \pi \text{ s.}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{|\sin(t)|\,u(t) + |\sin(-t)|\,u(-t)}{2}$$

$$= \tfrac{1}{2}\,|\sin(t)|\,[u(t) + u(-t)]$$

$$= \tfrac{1}{2}\,|\sin(t)|$$

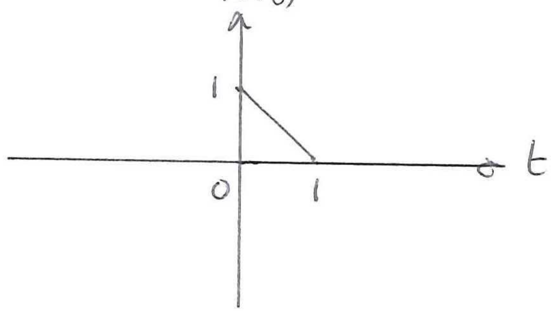
$x_e(t)$ is periodic with fundamental period $T_0 = \pi$ s.

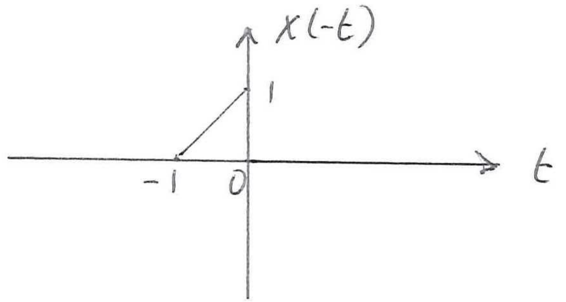
$$P_{x_e} = \frac{1}{T_0} \int_{t=0}^{T_0} x_e^2(t)\,dt = \frac{1}{4T_0} \int_{t=0}^{T_0} \sin^2(t)\,dt$$

$$= \frac{1}{2} P_x.$$

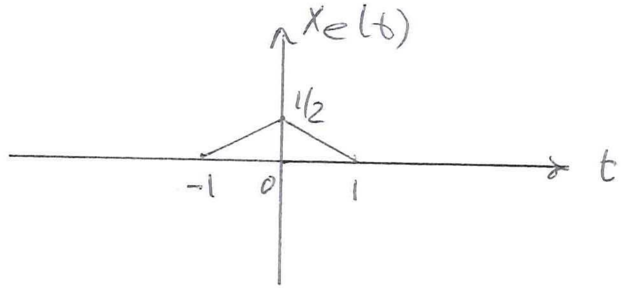
Problem 1.16

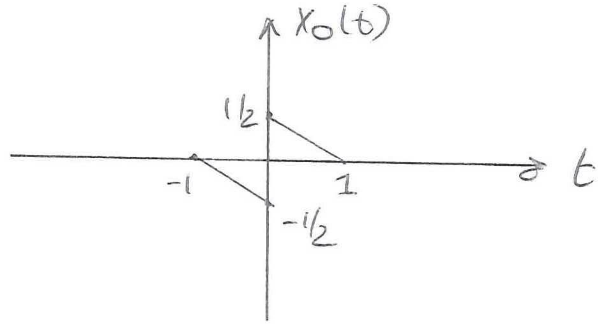
a.





$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$





b. $x(t) = x_e(t) + x_o(t)$

$$x^2(t) = x_e^2(t) + 2x_e(t)x_o(t) + x_o^2(t)$$

$x_e^2(t)$ is even

$x_o^2(t)$ is even

$x_e(t)\,x_o(t)$ is odd

$$\int_{t=-\infty}^{\infty} x^2(t)\,dt = \int_{t=-\infty}^{\infty} x_e^2(t)\,dt + \int_{t=-\infty}^{\infty} x_o^2(t)\,dt,$$

Since

$$\int_{t=-\infty}^{\infty} x_e(t)\, x_o(t)\, dt = 0 \quad (\text{see } 1.3c)$$

Furthermore, since $x_e^2(t)$ and $x_o^2(t)$ are both even, we can also write

$$\int_{t=-\infty}^{\infty} x^2(t)\, dt = 2\int_{t=0}^{\infty} x_e^2(t)\, dt + 2\int_{t=0}^{\infty} x_o^2(t)\, dt$$

$$E_x = \int_{t=-\infty}^{\infty} x^2(t)\, dt = \int_{t=0}^{1} (1-t)^2\, dt$$

$$E_{x_e} = \int_{t=-\infty}^{\infty} x_e^2(t)\, dt = 2\int_{t=0}^{\infty} x_e^2(t)\, dt$$

$$= 2\int_{t=0}^{1} \frac{1}{4}(1-t)^2\, dt = \frac{1}{2}E_x$$

$$E_{x_o} = \int_{t=-\infty}^{\infty} x_o^2(t)\, dt = 2\int_{t=0}^{\infty} x_o^2(t)\, dt$$

$$= 2\int_{t=0}^{1} \frac{1}{4}(1-t)^2\, dt = \frac{1}{2}E_x.$$

Answers to Selected Problems Chapter 2

Problem 2.4

a. (i) $$z(t) = \int_{\tau=t-1}^{t} w(\tau)\,d\tau + 2$$

$$z(t) = \int_{\tau=-\infty}^{\infty} w(\tau)\left[u(t-\tau) - u(t-\tau-1)\right] d\tau + 2$$

$$\frac{dz}{dt} = \int_{\tau=-\infty}^{\infty} w(\tau)\left[\delta(t-\tau) - \delta(t-\tau-1)\right] d\tau$$

$$= w(t) - w(t-1)$$

$z(t)$ solves the differential equation

(ii) No. A scaled input signal $w(t)$ does not lead to an equally scaled output signal $z(t)$.

b. (i) $$v_c(t) = \int_{\tau=0}^{t} i(\tau)\,d\tau$$

We shift the input signal in time, time shift T.

$$\tilde{v}_c(t) = \int_{\tau=0}^{t} i(\tau - T)\,d\tau$$

$$= \int_{p=-T}^{0} i(p)\,dp + \int_{p=0}^{t-T} i(p)\,dp$$

$$\neq v_c(t-T)$$

In general, the capacitor is not time-invariant.

If $i(t) = 0$ for $t<0$ then the above becomes

$$\tilde{v}_c(t) = \int_{p=0}^{t-T} i(p)\,dp = v_c(t-T)$$

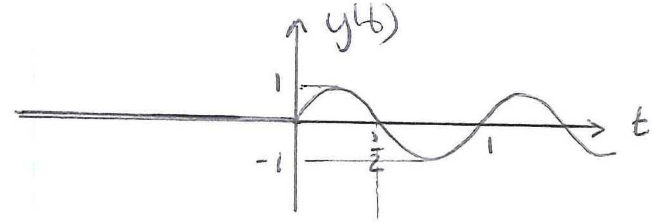
In this case, the capacitor is time-invariant.

(ii) $i(t) = u(t)$, $v_c(t) = t\,u(t) = r(t)$

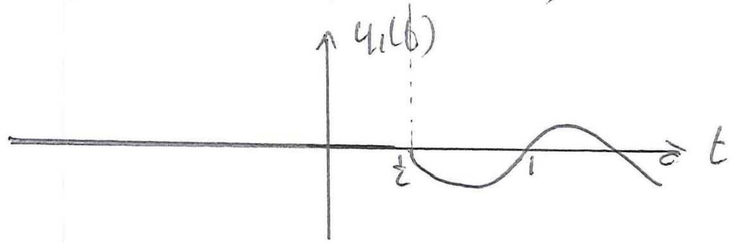
$i(t) = u(t-T)$, $\tilde{v}_c(t) = (t-T)\,u(t-T)$

$= r(t-T)$

c. $$y(t) = u(t)\sin(2\pi t)$$



$$y_1(t) = u(t-\tfrac{1}{2})\sin(2\pi t)$$



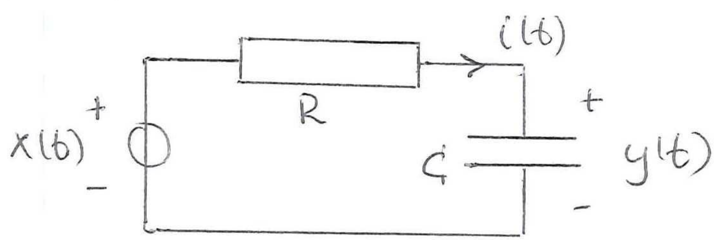
$$y(t-\tfrac{1}{2}) = u(t-\tfrac{1}{2})\sin[2\pi(t-\tfrac{1}{2})]$$

$$\neq y_1(t)$$

AM system is not time-invariant

Problem 25

a



$$i(t) = C\frac{dy}{dt} = \frac{x(t)-y(t)}{R}$$

$$\frac{dy}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$\frac{1}{RC} = 2, \quad C = 1F, \quad R = \tfrac{1}{2}\,\Omega.$$

$$\frac{dy}{dt} + 2y(t) = 2x(t)$$

b.

$$y(t) = e^{-2t}\int_{\tau=0}^{t} e^{2\tau}d\tau, \quad t>0$$

$$\frac{dy}{dt} = e^{-2t}\cdot e^{2t} - 2e^{-2t}\int_{\tau=0}^{t} e^{2\tau}d\tau$$

$$= 1 - 2y(t), \quad t>0$$

$$\frac{dy}{dt} + 2y(t) = 1, \quad t>0$$

Given output signal is incorrect.

The correct output signal is

$$y(t) = 2e^{-2t}\int_{\tau=0}^{t} e^{2\tau}d\tau, \quad t>0$$

$$\frac{dy}{dt} = 2e^{-2t}\cdot e^{2t} - 4e^{-2t}\int_{\tau=0}^{t} e^{2\tau}\,d\tau$$

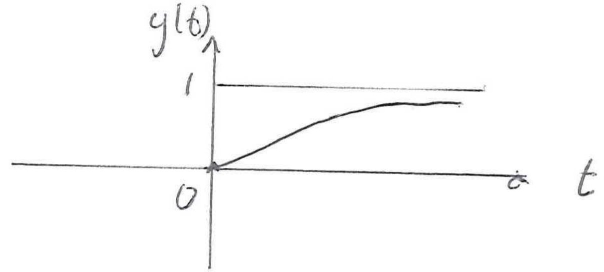
$$= 2 - 2y(t), \quad t>0$$

$$\frac{dy}{dt} + 2y(t) = 2, \quad t>0.$$

Evaluating the integral, we find

$$y(t) = 2e^{-2t}\cdot \frac{1}{2}e^{2\tau}\Big|_{\tau=0}^{t}$$

$$= e^{-2t}(e^{2t}-1) = 1-e^{-2t}, \quad t>0$$



Problem 2.7

a $q(t) = c(t)\,v(t)$

$$i(t) = \frac{dq}{dt} = c(t)\frac{dv}{dt} + v(t)\frac{dc}{dt}$$

b. $c(t) = 1+\cos(2\pi t)$

$v(t) = \cos(2\pi t)$

$$\frac{dc}{dt} = -2\pi\sin(2\pi t) \quad \frac{dv}{dt} = -2\pi\sin(2\pi t)$$

Substitution gives

$$i_1(t) = -2\pi\sin(2\pi t)\,[\,1 + 2\cos(2\pi t)\,]$$

c. Given the voltage-current relation, the system is clearly not TI.

Problem 2.8

From the given input-output relation we know that $x(t)=0$ for $t<0$ ($x(t)$ is causal).

a Given relation is a convolution integral of a causal impulse response and a causal input signal:

$$y(t) = \int_{\tau=0}^{t} h(t-\tau)\,x(\tau)\,d\tau, \quad t\geq 0.$$

By inspection: $h(t) = e^{-t}u(t)$.

System is LTI

b. $h(t) = 0$ for $t<0$. System is causal.

c. $s(t) = \int_{\tau=0}^{t} e^{-(t-\tau)} d\tau = e^{-t} \int_{\tau=0}^{t} e^{\tau} d\tau$

$= 1 - e^{-t}, \quad t \geq 0$

$s(t) = 0, \quad t<0$

$s(t) = (1-e^{-t}) u(t)$

$h(t) = \frac{ds}{dt} = e^{-t} u(t) + (1-e^{-t}) \delta(t)$

$= e^{-t} u(t)$

$h(t)$ is absolutely integrable: system is BIBO stable.

d. $y(t) = s(t) - s(t-1)$

$= (1-e^{-t}) u(t) - [1-e^{-(t-1)}] u(t-1)$

Problem 2.9

a. $y(t) = \frac{1}{T} \int_{\tau=t-T/2}^{t+T/2} x(\tau) d\tau$

$= \frac{1}{T} \int_{\tau=-\infty}^{\infty} x(\tau) [u(t-\tau+\frac{T}{2}) - u(t-\tau-\frac{T}{2})] d\tau$

$= \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

with

$h(t) = \frac{1}{T} [u(t+T/2) - u(t-T/2)]$

Clearly, the system is noncausal.

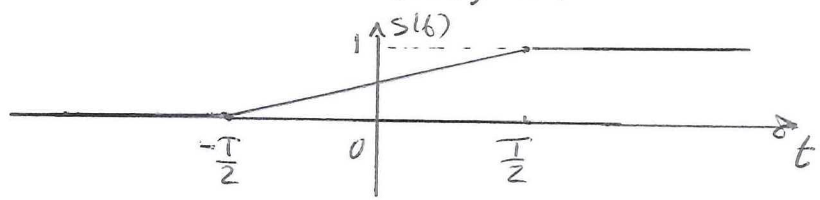
b. $s(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} u(\tau) d\tau$

$t < -T/2$: $s(t) = 0$

$|t| < T/2$: $s(t) = \frac{1}{T} \int_{\tau=0}^{t+T/2} d\tau$

$= \frac{t}{T} + \frac{1}{2}$

$t > T/2$: $s(t) = 1$



Observe that $\frac{ds}{dt} = h(t)$.

Problem 2.12

a. $y(0) = 0$: system is linear, linear combination of input signals produces the same linear combination of corresponding output signals

$y(0) \neq 0$: scale the input signal by a factor of two, for example. The output signal is not scaled by a factor of two. System is nonlinear.

b. $x(t) = 0$, no input signal. Response is called the zero-input response.

$y(0) = 0$, the initial state of the system vanishes. Response is called the zero-state response.

c. Response to a Dirac distribution is called the impulse response

$$h(t) = \int_{\tau=0^-}^{t} e^{-(t-\tau)} \delta(\tau)\, d\tau = e^{-t},\ t > 0.$$

$$h(t) = e^{-t} u(t)$$

$s(t) = 0$ for $t < 0$. For $t > 0$, we find

$$s(t) = e^{-t} \int_{\tau=0}^{t} e^{\tau}\, d\tau = 1 - e^{-t}.$$

Conclusion: $s(t) = (1 - e^{-t})u(t)$

$$\frac{ds}{dt} = e^{-t} u(t) = h(t)$$

Problem 2.15

a. $x_1(t) = x(t) - x(t-2)$

$y_1(t) = y(t) - y(t-2)$

b. $x_2(t) = x(t+1) - x(t)$

$y_2(t) = y(t+1) - y(t)$

c. $x_3(t) = \frac{dx}{dt}$

$y_3(t) = \frac{dy}{dt}$

Answers Selected Problems Chapter 3

Problem 3.1

a (i) $X(s) = \int_{t=-\infty}^{\infty} \delta(t-1) e^{-st} dt = e^{-s}, \quad s \in \mathbb{C}$

(ii) $Y(s) = \int_{t=-\infty}^{\infty} [\delta(t+1) - \delta(t-1)] e^{-st} dt$

$= \int_{t=-\infty}^{\infty} \delta(t+1) e^{-st} dt - \int_{t=-\infty}^{\infty} \delta(t-1) e^{-st} dt$

$= e^{s} - e^{-s} = 2\frac{e^{s} - e^{-s}}{2} = 2\sinh(s), \quad s \in \mathbb{C}$

(iii) $Z(s) = \int_{t=-\infty}^{\infty} [u(t+1) - u(t-1)] e^{-st} dt$

$= \int_{t=-1}^{1} e^{-st} dt = \frac{2}{s}\sinh(s), \quad s \in \mathbb{C}$

Observe that $y(t) = \frac{dz}{dt}$

$Y(s) = s Z(s)$

(iv) $W(s) = \int_{t=-\infty}^{\infty} \cos(2\pi t) [u(t+1) - u(t-1)] e^{-st} dt$

$= \int_{t=-1}^{1} \cos(2\pi t) e^{-st} dt$

$= \int_{t=-1}^{1} \frac{e^{j2\pi t} + e^{-j2\pi t}}{2} e^{-st} dt$

$= \frac{1}{2} \frac{(e^{s} - e^{-s})(s + j2\pi) + (e^{s} - e^{-s})(s - j2\pi)}{s^2 + 4\pi^2}$

$= \frac{s(e^{s} - e^{-s})}{s^2 + 4\pi^2}$

$= \frac{2s\sinh(s)}{s^2 + 4\pi^2}$

b (ii) $Y_1(s) = \int_{t=1}^{\infty} e^{-t} e^{-st} dt$

$= \int_{t=1}^{\infty} e^{-(s+1)t} dt$

$= \frac{1}{s+1} e^{-(s+1)}, \quad Re(s) > -1$

Problem 3.4

a. $x(t) = e^{-\alpha t} u(t) - e^{\alpha t} u(-t)$

$$X(s) = \int_{t=-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_{t=0}^{\infty} e^{-\alpha t} e^{-st} dt - \int_{t=-\infty}^{0} e^{\alpha t} e^{-st} dt$$

$$= \int_{t=0}^{\infty} e^{-(s+\alpha)t} dt - \int_{t=-\infty}^{0} e^{-(s-\alpha)t} dt$$

convergent for $Re(s) > -\alpha$ | convergent for $Re(s) < \alpha$

$$= \frac{1}{s+\alpha} + \frac{1}{s-\alpha} \qquad |Re(s)| < \alpha$$

$$= \frac{2s}{s^2-\alpha^2}, \qquad |Re(s)| < \alpha$$

c. $s(t) = \sum_{n=0}^{\infty} u(t-n)$

$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}, \quad Re(s) > 0$

$u(t-1) \overset{\mathcal{L}}{\longleftrightarrow} e^{-s}\frac{1}{s}, \quad Re(s) > 0$

$$S(s) = \frac{1}{s} + e^{-s}\frac{1}{s} + e^{-2s}\frac{1}{s} + \dots$$

$$= \frac{1}{s}(1 + e^{-s} + e^{-2s} + \dots)$$

$$= \frac{1}{s}\frac{1}{1-e^{-s}}, \qquad Re(s) > 0$$

Problem 3.9

a. $$Y_1(s) = \frac{e^{-2s}}{s^2+1} + \frac{(s+2)^2+2}{(s+2)^3}$$

$$= e^{-2s}\frac{1}{s^2+1} + \frac{1}{s+2} + \frac{2}{(s+2)^3},$$

$Re(s) > 0$

$$\frac{1}{s^2+1} \overset{\mathcal{L}}{\longleftrightarrow} \sin(t)\, u(t)$$

$$e^{-2s}\frac{1}{s^2+1} \overset{\mathcal{L}}{\longleftrightarrow} \sin(t-2)\, u(t-2)$$

$$\frac{1}{s+2} \overset{\mathcal{L}}{\longleftrightarrow} e^{-2t} u(t)$$

$$\frac{2}{(s+2)^3} \overset{\mathcal{L}}{\longleftrightarrow} 2 \cdot \frac{1}{2!} t^2 e^{-2t} u(t)$$

$$y_1(t) = \sin(t-2)u(t-2) + e^{-2t}u(t) + t^2 e^{-2t} u(t)$$

b. $Y_2(s) = \dfrac{-s^2 - s + 1}{s(s^2+3s+2)}$

<u>Bodi, p231</u>

$$Y(s) = \frac{B(s)}{A(s)} X(s) + \frac{I(s)}{A(s)}$$

n = degree of $B(s)$

N = degree of $A(s)$

$I(s)$ is due to the initial conditions

<u>Bodi, p. 231</u>

$N > n$ since then Dirac distributions and its derivatives do not appear in the solution

What is meant by this is that if the input signal does not contain any Dirac distributions or its derivatives and $N > n$ then the output signal also

does not contain any Dirac distributions or derivatives of this function/distribution.

With $N > n$ the system cannot "generate" Dirac distributions or derivatives of this distribution.

The point is that although this is true, it is also true for $N = n$.

To show this, first consider the case $N > n$.

We write $Y_2(s)$ accordingly as

$$Y_2(s) = -\frac{s^2+s-1}{s(s^2+3s+2)}$$

$$= -\frac{s(s+1)-1}{s(s^2+3s+2)}$$

$$= \frac{1}{s^2+3s+2} \cdot \frac{1}{s} - \frac{s+1}{s^2+3s+2}$$

$$= \frac{B(s)}{A(s)} X(s) + \frac{I(s)}{A(s)}$$

with $A(s) = s^2+3s+2$, $B(s) = 1$, $X(s) = \frac{1}{s}$, and $I(s) = -s-1$.

Note that $M = \text{degree}(B) = 0$

$N = \text{degree}(A) = 2$

so indeed $N > M$.

The differential equation follows from

$$A(s)\, Y_2(s) = B(s)\, X(s)$$

$$(s^2 + 3s + 2)\, Y_2(s) = X(s)$$

as

$$\frac{d^2 y_2}{dt^2} + 3\frac{dy_2}{dt} + 2y_2 = x(t) \qquad (*)$$

To find the initial conditions, we apply a one-sided Laplace transform to Eq. (*). Using the transformation rules

$$\frac{d^2 y_2}{dt^2} \overset{\mathcal{L}}{\longleftrightarrow} s^2 Y_2(s) - s y_2(0^-) - \left.\frac{dy_2}{dt}\right|_{t=0^-}$$

and

$$\frac{dy_2}{dt} \overset{\mathcal{L}}{\longleftrightarrow} s Y_2(s) - y_2(0^-),$$

we find

$$A(s)\, Y_2(s) = B(s)X(s) + s y_2(0^-) + 3 y_2(0^-) + \left.\frac{dy_2}{dt}\right|_{t=0^-}$$

$$= B(s)X(s) + I(s)$$

with

$$I(s) = s y_2(0^-) + 3 y_2(0^-) + \left.\frac{dy_2}{dt}\right|_{t=0^-}.$$

Now we know that

$$I(s) = -s - 1$$

and therefore

$$-s - 1 = s y_2(0^-) + 3 y_2(0^-) + \left.\frac{dy_2}{dt}\right|_{t=0^-}$$

Since this equality should hold for any s with $\text{Re}(s) > 0$, we conclude that

$$y_2(0^-) = -1 \quad \text{and} \quad 3 y_2(0^-) + \left.\frac{dy_2}{dt}\right|_{t=0^-} = -1$$

from which we obtain

$$y_2(0^-) = -1 \quad \text{and} \quad \left.\frac{dy_2}{dt}\right|_{t=0^-} = 2$$

From the differential equation we conclude that y_2 and its derivative must be continuous at $t=0$: We obtain

$$y_2(0^-) = y_2(0^+) = y_2(0) = 1$$

and

$$\left.\frac{dy_2}{dt}\right|_{t=0^-} = \left.\frac{dy_2}{dt}\right|_{t=0^+} = \left.\frac{dy_2}{dt}\right|_{t=0} = 2$$

Now consider the case $\underline{M=N=2}$.

$$Y_2(s) = \frac{B(s)}{A(s)} X(s) + \frac{I(s)}{A(s)}$$

with $A(s) = s^2+3s+2$, $B(s) = -s^2-s+1$, $X(s) = \frac{1}{s}$ and $I(s) = 0$

In this case the differential equation follows from

$$A(s)\, Y_2(s) = B(s) X(s)$$

as

$$\frac{d^2y_2}{dt^2} + 3\frac{dy_2}{dt} + 2y_2 = -\frac{d^2x}{dt^2} - \frac{dx}{dt} + x(t)$$

and the initial conditions follow from

$$I(s) = s y_2(0^-) + 3y_2(0^-) + \left.\frac{dy_2}{dt}\right|_{t=0^-} = 0$$

as

$$y_2(0^-) = 0 \quad \text{and} \quad \left.\frac{dy_2}{dt}\right|_{t=0^-} = 0$$

In this case $y_2(t)$ exhibits a jump discontinuity at $t=0$.

The point is that with $M=N=2$, we again have no Dirac distribution or a derivative of a Dirac distribution in the response.

Requiring no Dirac distribution generation makes physical sense, but then the requirement should be $N \geq M$. However, such a requirement is not sufficient for uniqueness.

$\underline{c.}$

$$Y(s) = \frac{1}{s\,[(s+1)^2+4]}$$

$$= \frac{1}{5}\frac{1}{s} - \frac{1}{5}\left[\frac{s+1}{(s+1)^2+4} + \frac{1}{2}\frac{2}{(s+1)^2+4}\right]$$

$\mathrm{Re}(s) > 0$

$$y(t) = \frac{1}{5}u(t) - \frac{1}{5}e^{-t}\cos(2t)\,u(t) - \frac{1}{10}e^{-t}\sin(2t)\,u(t)$$

$$= y_t(t) + y_{ss}(t)$$

$$y_t(t) = -\frac{1}{5}e^{-t}\cos(2t)\,u(t) - \frac{1}{10}e^{-t}\sin(2t)\,u(t)$$

$$y_{ss}(t) = \frac{1}{5}u(t)$$

Problem 3.10

a. $(s^2+3s+2)\,Y(s) = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)}$$

Poles at $s=-1$ and $s=-2$, system is BIBO stable

b. $$Y(s) = \frac{1}{s(s+1)(s+2)} = \frac{1}{2}\frac{1}{s} - \frac{1}{s+1} + \frac{1}{2}\frac{1}{s+2}$$

$$y(t) = \frac{1}{2}u(t) - e^{-t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

$$y_{ss}(t) = \frac{1}{2}u(t)$$

Nonzero initial conditions

$$Y(s) = H(s)X(s) + \frac{I(s)}{s^2+3s+2}$$

$I(s)$ is due to the initial conditions

$I(s)$ is a polynomial in s of degree ≤ 1

$$\frac{I(s)}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2}$$

Inverse Laplace transform leads to exponentially decaying signals.

Steady-state response is the same.

Problem 3.20

a. $y(t) = \alpha x(t-T) + \alpha^3 x(t-3T) \quad 0<\alpha<1$

$$h(t) = \alpha\,\delta(t-T) + \alpha^3\,\delta(t-3T)$$

b. $H(s) = \alpha e^{-sT} + \alpha^3 e^{-3sT}, \quad s \in \mathbb{C}$

no poles, system is BIBO stable