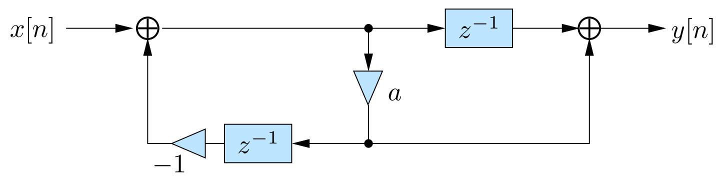
ee2s11 Signals and Systems

Exercises digital realizations

1. Question

Given the realization (wherein a is a real-valued parameter):



a Determine the transfer function $H(z)$ of this realization.

b Is this a minimal realization? (Why?)

c For which values of a is this a stable realization?

d Draw the corresponding “direct form no. 2” realization.

Answer

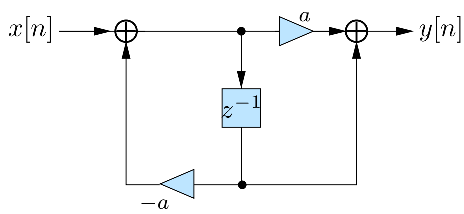
a

$$H(z) = \frac{a + z^{-1}}{1 + az^{-1}}$$

b Not minimal (order 1 and used 2 delay elements)

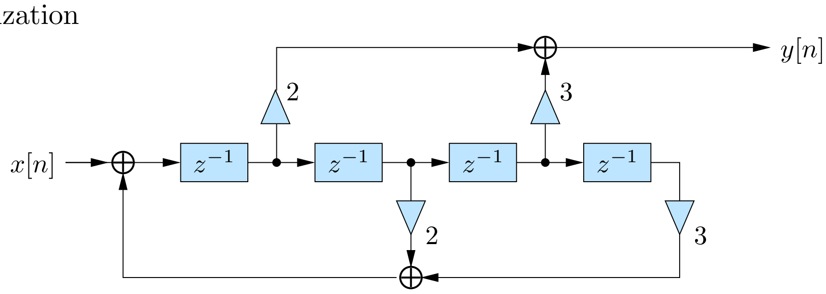
c Stable for $|a| < 1$.

d



2. Question

Given the realization



a Determine the transfer function $H(z)$ of this realization.

b Determine all poles and zeros of $H(z)$.

c Is this a stable realization?

d Is this a minimal realization?

e Draw the corresponding ”direct form no. 2 transposed”realization.

Answer

a This realization is already in direct form: you can immediately write down the transfer function (or easily derive it):

$$H(z) = \frac{2z^{-1} + 3z^{-3}}{1 - 2z^{-2} - 3z^{-4}}$$

b
$$H(z) = \frac{2z^{-1}(1 + \frac{3}{2}z^{-2})}{(1 - 3z^{-2})(1 + z^{-2})} = \frac{2z(z^2 + \frac{3}{2})}{(z^2 - 3)(z^2 + 1)}$$

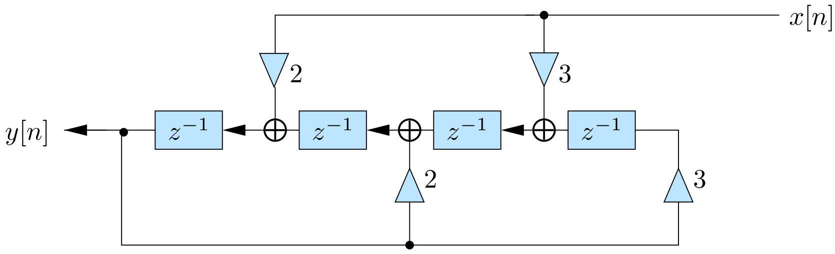
Poles: $z = \pm\sqrt{3}$, $z = \pm j$.

Zeros: $z = 0$, $z = \pm\sqrt{3/2}$, $z = \infty$.

c Poles outside the unit circle: not stable

d 4 delay elements used for a 4th order transfer: minimal realization

e The given realization is already in direct form.



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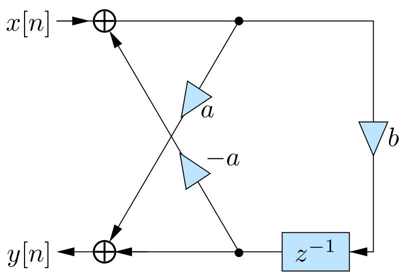
3. Question

Given the realization



a Determine the transfer function $H(z)$ of this realization.

b For which values of a, b is this a stable realization?

c Is this a minimal realization? (Why?)

d Is this a canonical realization for the class of first-order IIR systems? (Why?)

e Draw the corresponding "direct form no. 2 transposed" realization

Answer

a Denote by $P(z)$ the input of the delay element. Then

$$\left\{\begin{array}{lcl} P(z) & = & b(X(z) - az^{-1}P(z) \\ Y(z) & = & aX(z) + P(z)z^{-1}(1-a^2) \end{array}\right. \quad \Leftrightarrow \quad \left\{\begin{array}{lcl} P(z) & = & \frac{b}{1+abz^{-1}}X(z) \\ Y(z) & = & \left(a + \frac{bz^{-1}(1-a^2)}{1+abz^{-1}}\right) \end{array}\right.$$

$$\Rightarrow \quad H(z) = \frac{a+bz^{-1}}{1+abz^{-1}}$$

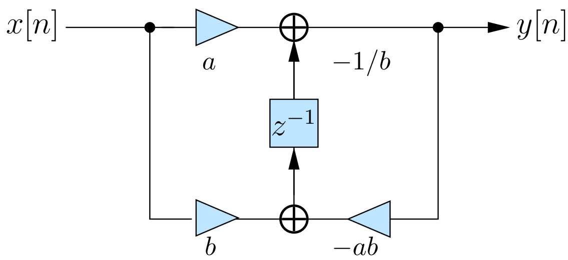
b Stable: pole within the unit circle: $|ab| < 1$.

c Yes: first-order system realized using 1 delay element.

d No.

Canonical for the class of first-order IIR system implies that I can realize all possible first-order rational systems (three degrees of freedom: 1 pole, 1 zero, gain, hence 3 coefficients) with a minimal number of multipliers (should be equal to 3). I do have 3 multipliers, nbut only 2 degrees of freedom (coefficients). I can make all pole/zero configurations but not all possible gains.

e



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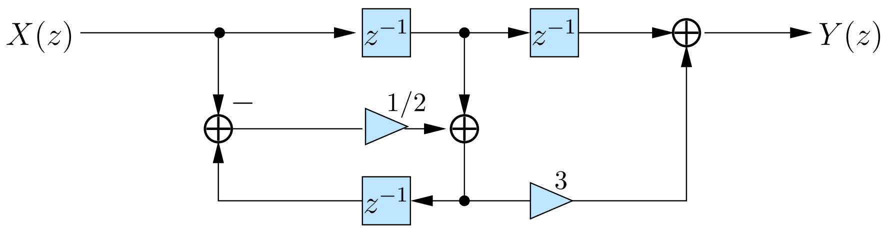
4. Question

Given the realization



a Determine the transfer function $H(z)$ of this realization.

b Is this a stable realization? (Why?)

c Is this a minimal realization? (Why?)

Answer

a Introduce an additional parameter $P(z)$ at the input of the multiplier ("3"). We obtain

$$\left\{\begin{array}{lcl} P(z) & = & z^{-1}X(z) - \frac{1}{2}X(z) + \frac{1}{2}z^{-1}P(z) \\ Y(z) & = & z^{-2}X(z) + 3P(z) \end{array}\right.$$

An expression for $P(z)$ is

$$P(z)(1 - \frac{1}{2}z^{-1}) = X(z)(z^{-1} - \frac{1}{2})$$

$$P(z) = X(z)\frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}}$$

Eliminate $P(z)$ from the expression for $Y(z)$, this yields

$$\begin{aligned} Y(z) &= X(z)\left(z^{-2} + \frac{3(z^{-1}-\frac{1}{2})}{1-\frac{1}{2}z^{-1}}\right) \\ &= X(z)\frac{-\frac{1}{2}z^{-3}+z^{-2}+3z^{-1}-\frac{3}{2}}{1-\frac{1}{2}z^{-1}} \end{aligned}$$

b De pole is located at $z = \frac{1}{2}$, within the unit circle, hence stable.

c The highest degree (filter order) is 3, the number of delay elements is also 3, hence minimal.

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5. Question

Given the transfer function $H(z) = \dfrac{z^{-1}(1 - z^{-1})}{1 + 2/3\, z^{-1}}$.

a Determine the poles and zeros (also at $z = 0$ and $z = \infty$).

b Is this a stable function? (Why?)

c Draw the "direct form no. II" realization, and also specify the coefficients.

d Determine the impulse response $h[n]$.

Answer

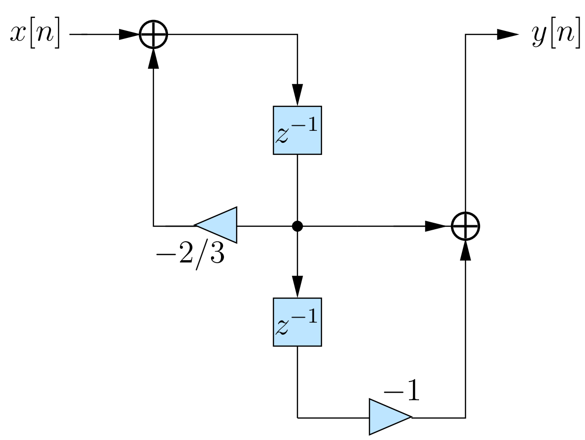
a

$$H(z) = \frac{z^{-1}(1 - z^{-1}}{1 + 2/3z^{-1}} = \frac{z - 1}{(z + 2/3)z}$$

The zeros are $z = 1$ and $z = \infty$, the poles are $z = 0$ and $z = -2/3$.

b yes (poles inside the unit circle)

c



d

$$h[n] = \left(-\frac{2}{3}\right)^{n-1} u(n-1) - \left(-\frac{2}{3}\right)^{n-2} u(n-2)$$