

# EE2S1 Signals and Systems

## Chapter 5.7: Basics of Filtering

Alle-Jan van der Veen

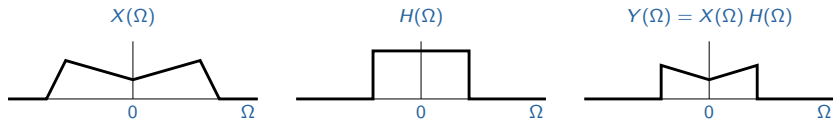
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# Basics of filtering

Recall

$$y(t) = x(t) * h(t) \quad \Leftrightarrow \quad Y(\Omega) = X(\Omega) H(\Omega)$$



In signal processing,  $x(t)$  is an input signal,  $y(t)$  is an output signal, and  $H(\Omega)$  the frequency response of a filter.

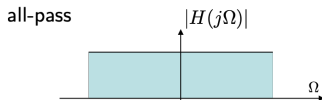
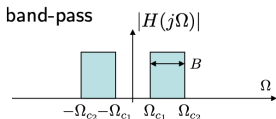
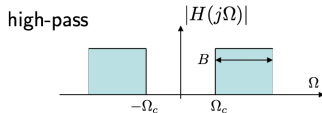
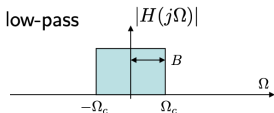
We consider rational filters, with transfer functions as

$$H(s) = \frac{B(s)}{A(s)} = \frac{\text{some polynomial}}{\text{another polynomial}}$$

# Frequency selective filters

Filter design: find  $H(s)$  to satisfy certain specifications, such as

- Flat frequency response in the pass band
- Sufficient suppression in the stop band

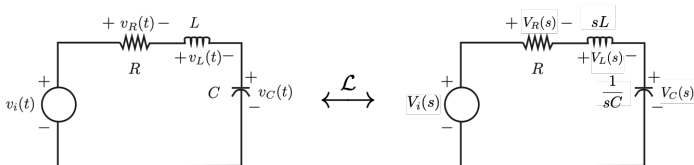


These ideal filters cannot be realized...

# Frequency selective filters

## Example: second-order RCL circuit

This (passive) circuit can generate many second-order rational filters:



(zero initial conditions)

$$V_i(s) = V_R(s) + V_L(s) + V_C(s)$$

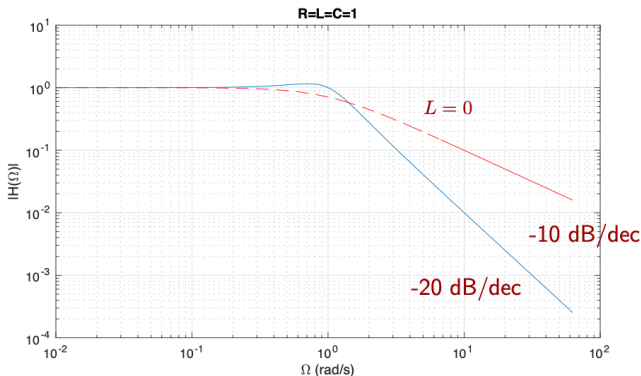
**Example** Voltage across the capacitor:

$$V_C(s) = \frac{V_i(s)}{s^2LC + sRC + 1} \quad \Rightarrow \quad H(s) = \frac{1}{s^2LC + sRC + 1}$$

# Frequency selective filters

The amplitude response is

$$|H(j\Omega)|^2 = \frac{1}{(1 - \Omega^2 LC)^2 + (\Omega RC)^2}$$



This is a second-order lowpass filter.

## Frequency selective filters

Similarly:

- Voltage across the inductor:

$$H(s) = \frac{s^2 LC}{s^2 LC + sRC + 1}$$

- Voltage across the resistor:

$$H(s) = \frac{sRC}{s^2 LC + sRC + 1}$$

- Voltage across the inductor and capacitor:

$$H(s) = \frac{s^2 LC + 1}{s^2 LC + sRC + 1}$$

What types of filters are these?

# Phasors

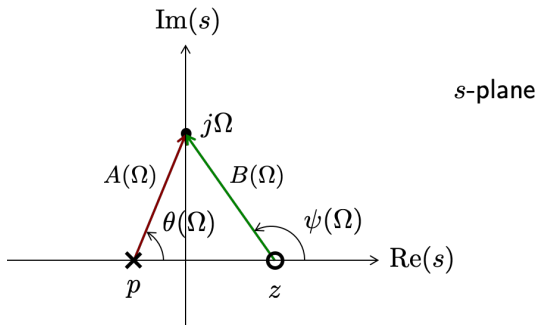
Consider a first-order rational transfer function

$$H(s) = \frac{s - z}{s - p}$$

Frequency response:

$$H(\Omega) = \frac{j\Omega - z}{j\Omega - p} = \underbrace{\frac{|j\Omega - z|}{|j\Omega - p|}}_{|H(j\Omega)|} \underbrace{e^{j(\angle(j\Omega - z) - \angle(j\Omega - p))}}_{e^{j\angle H(j\Omega)}}$$

# Phasors

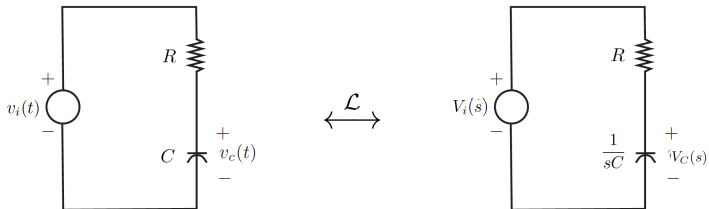


$$|H(j\Omega)| = \frac{B(\Omega)}{A(\Omega)} \quad \angle H(j\Omega) = \psi(\Omega) - \theta(\Omega)$$



# Phasors

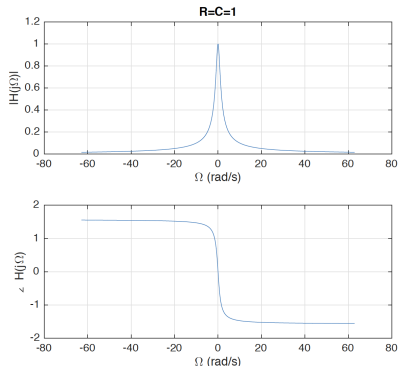
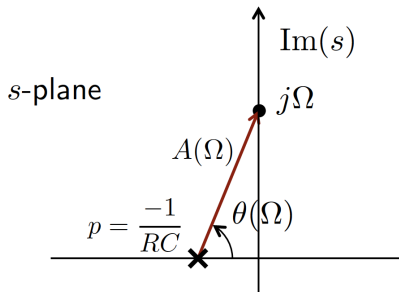
## Example: first-order RC circuit



$$H(s) = \frac{V_c(s)}{V_i(s)} = \frac{1}{1 + sRC} = \frac{(RC)^{-1}}{(RC)^{-1} + s}$$

- One zero at  $\infty$
- One pole at  $-\frac{1}{RC}$

# Phasors



This is a first-order lowpass filter. Close to the pole, the response will peak, and the phase changes quickly. Far away, the response slowly goes to zero.

# Phasors

More general:

$$H(s) = \frac{b_0 + b_1s + \dots + b_Ms^M}{a_0 + a_1s + \dots + a_Ns^N} = \frac{b_M \prod_{k=1}^M (s - z_k)}{a_N \prod_{k=1}^N (s - p_k)}$$

Write  $s = j\Omega$  and express the complex-valued factors in polar form as

$$j\Omega - z_k = B_k(\Omega)e^{j\psi_k(\Omega)}, \quad j\Omega - p_k = A_k(\Omega)e^{j\theta_k(\Omega)}$$

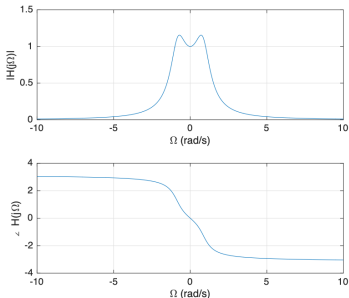
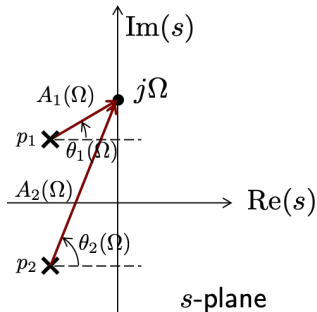
Frequency response:

$$\begin{aligned} |H(\Omega)| &= \frac{|b_M| \prod_{k=1}^M B_k(\Omega)}{|a_N| \prod_{k=1}^N A_k(\Omega)} \\ \angle H(\Omega) &= \angle \frac{b_M}{a_M} + \sum_{k=1}^M \psi_k(\Omega) - \sum_{k=1}^N \theta_k(\Omega) \end{aligned}$$

# Phasors

## Example 1

$$H(s) = \frac{1}{s^2 + s + 1} = \frac{1}{(s - p_1)(s - p_2)}, \quad p = \frac{1}{2}(-1 \pm j\sqrt{3})$$

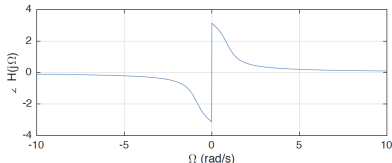
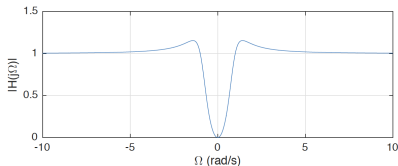
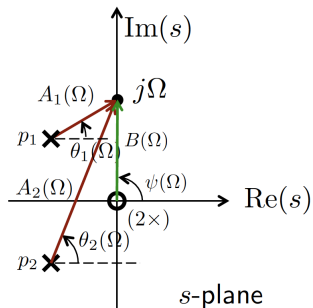


This is a second-order lowpass filter (with a ripple in the passband).

# Phasors

## Example 2

$$H(s) = \frac{s^2}{s^2 + s + 1}$$

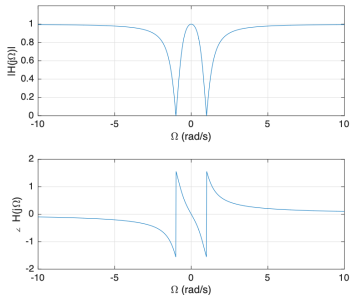
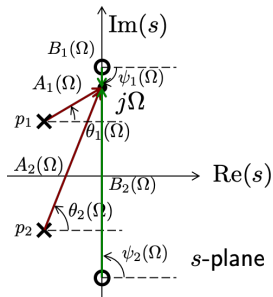


This is a second-order highpass filter. The zeros pull down the response.

# Phasors

## Example 3

$$H(s) = \frac{s^2 + 1}{s^2 + s + 1} = \frac{(s + j)(s - j)}{(s + p_1)(s + p_2)}, \quad p = \frac{1}{2}(1 \pm j\sqrt{3})$$



This is a second-order bandstop filter. Zeros on the imaginary axis.

# Summary

- Phasors are used to construct a magnitude/phase spectrum from a given pole-zero plot.
- Close to poles, the magnitude peaks; close to zeros the magnitude drops
- This allows to judge the type of filter (lowpass, highpass, bandpass, ...)

Filter design from given specifications comes later in the course (Chapter 7.3).