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PERFORMANCE ANALYSIS OF THE STFC FOR COOPERATIVE ZP-OFDM: DIVERSITY, CAPACITY, AND COMPLEXITY

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ABSTRACT

In this paper, we consider a cooperative Zero-padding orthogonal frequency division multiplexing (ZP-ODFM) communication with multiple carrier frequency offsets (CFOs) and multipath channel, and propose a space time frequency coding (STFC) scheme to exploit the linear convolutional structure of the ZP-OFDM. Theoretical analysis of the proposed STFC is provided based on the analytical upper bound of the channel orthogonality deficiency. To reduce the system complexity, low-complexity linear equalizers, such as zero-forcing (ZF) and minimum mean square error (MMSE) equalizers are often adopted. We also show that with only linear receivers, the proposed code achieves the full cooperative diversity and improves the system capacity.

I. INTRODUCTION

Recently, cooperative diversity in wireless networks has received great interest and is regarded as a promising technique to mitigate multi-path fading, which causes fluctuations in the amplitude of the received signal. The basic idea behind cooperation is that several users in a network pool their resources in order to form a virtual antenna array which creates spatial diversity [1]. This cooperative diversity leads to an increased exponential decay rate in the error probability with increasing signal-to-noise ratio (SNR) [2].

A multiband (MB) ZP-OFDM based approach to design Ultra Wide Band (UWB) transceivers has been recently proposed in [3] for the IEEE Standard. In Dec. 2008, the European Computer Manufacturers Association (ECMA) adopted ZP-OFDM for the latest version of High Rate UWB Standard [4]. Because of its advantage in the low power transmission, ZP-OFDM has potentials to be used in other low power wireless communications systems [5].

To quantify the performance of different communication systems, two important criteria are the average bit-error rate (BER) and capacity. The BER performance of wireless transmissions over fading channels is usually quantified by two parameters: diversity order and coding gain. The diversity order is defined as the asymptotic slope of the BER versus signal-to-noise ratio (SNR) curve. It describes how fast the error probability decays with SNR, while the coding gain measures the performance gap among different schemes when they have the same diversity. The higher the diversity, the smaller the error probability at high-SNR regimes, to cope with the deleterious effects of fading on the system performance, diversity-enriched transmitters and receivers have well-appreciated merits. Most of the existing diversity-enabled schemes adopt maximum-likelihood equalizers (MLEs) or near-MLEs at the receiver to collect full diversity [6]. Although MLE enjoys the maximum diversity, its exponential decoding complexity makes it infeasible for certain practical systems. In order to reduce the system complexity, one may apply linear equalizers (LEs), when the system model is linear. The capacity is another important criterion to quantify the performance of a certain transmission strategy, and describes the maximum information rate for a transmission system with a certain equalizer employed at the receiver. Generally, besides diversity loss, LEs also lose capacity relative to systems with MLEs.

The channel orthogonality deficiency (*od*) [6] determines the fundamental condition when LEs collect the same diversity as the MLE. In other words, LEs usually have inferior performance relative to MLEs due to loss of diversity. According to [6], to collect the same diversity as MLE does and to improve the capacity, the equivalent channel matrix needs some “modification” to upper bound of *od* by a constant less than 1. In this paper, based on the *od* and some new results proposed in [6], we illustrate how to design a space time frequency code (STFC) to achieve full diversity, maximal capacity and with LEs to enable low system complexity.

The rest of the paper is organized as follows. Section II presents the system model of the cooperative ZP-OFDM system, and proposes a STFC for CFOs and multipath channel. Linear equalizer, ML equalizer and channel orthogonality deficiency are discussed in Section III. In Section IV, we present the cooperative diversity, capacity and complexity of the proposed STFC. Simulation results are illustrated in Section V to corroborate the theoretical claims, and finally Section VI concludes the paper.

Notations: Superscripts $(\cdot)^{-1}, (\cdot)^{T}, (\cdot)^{H}$, $(\cdot)^{\dagger}$ represent inverse, transpose, Hermitian and pseudo inverse, respectively, $\mathrm{diag}(\cdot)$ is diagonal matrix with main diagonal $(\cdot)$ .The notation $|\cdot|$ denotes the absolute value of a scalar or cardinality of a set, and $\|\cdot\|$ denotes the 2-norm of a vector/matrix.

II. COOPERATIVE ZP-OFDM AND STFC

A. System model of the Cooperative ZP-OFDM

We consider a cooperative ZP-OFDM system as shown in the Fig. 1. The Decode-and-forward (DF) protocol is adopted in the cooperative communication model. Relays can fully decode the information, and participate in the cooperation, and occupy different frequency bands to transmit data to the destination. Each relay-destination link undergoes multipath Rayleigh fading. For the relay r , $r \in [1, 2, \cdots, R]$, R is the number of relays, the received signal $\mathbf{y}_r$ can be formulated as

$$\mathbf{y}_r = \mathbf{F}_P \mathbf{D}_{P,r} \mathbf{H}_r \mathbf{T}_{ZP} \mathbf{F}_N^H \mathbf{x} + \mathbf{n} \qquad (1)$$

where $\mathbf{x} \triangleq [x_0, \cdots, x_{N-1}]^T$ is the vector of the so called frequency transmitted information signal, which is forwarded by R relays. N is the signal length. The subscript r in this paper indicates the variables or operators related to the r-th relay. To simplify the exposition, the noise term is denoted as $\mathbf{n}$, which stands for independently and identically distributed (i.i.d) complex white Gaussian noise with zero mean and variance σ_n^2. $\mathbf{F}_N$ and $\mathbf{F}_P$ stand for the N-point and P-point FFT matrix, respectively.

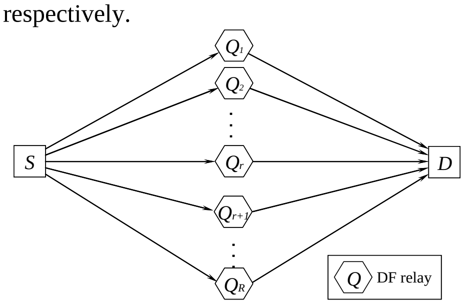

Figure 1: Cooperative ZP-OFDM system architecture, (S: Source, D: Destination, Q_r: r-th Relay).

The matrix

$$\mathbf{T}_{ZP} = \begin{bmatrix} \mathbf{I}_N \\ 0 \end{bmatrix}_{P\times N} \tag{2}$$

performs the zero-padding on the transmitted signal with V zeros, where $\mathbf{I}_N$ is $N \times N$ identity matrix, and $P = N + V$.

The matrix $\mathbf{H}_r$ is a $P \times P$ lower triangular matrix with its first column vector is $\left[h_{1,r},\cdots,h_{L,r},0\cdots 0\right]^T$, and its first row vector is $\left[h_{1,r},0\cdots 0\right]$, and $h_{L,r}$ denotes the L-th path gain over the r-th relay and destination link. Without loss of generality, we assume that the channel lengths of different relay-destination links are all L. To avoid inter-symbol interference (ISI), we should have $L \le V$, and we assume $L = V$ in this paper. $\mathbf{D}_{P,r}$ is a diagonal matrix representing the residual carrier frequency error over the r-th relay and destination link and is defined in terms of its diagonal elements as $\mathbf{D}_{P,r} = \mathrm{diag}\left(1,\alpha_r,\cdots,\alpha_r^{P-1}\right)$, with $\alpha_r = \exp\left(j2\pi\Delta q_r / N\right)$, and Δq_r is the normalized carrier frequency offset of r-th relay with the symbol duration of ZP-OFDM. Here, we notice that $\mathbf{H}_{T,r} = \mathbf{H}_r\mathbf{T}_{ZP}$ is a full column rank tall Toeplitz matrix, and its correlation matrix always guaranteed to be invertible. Consequently, (1) can be rewritten as

$$\mathbf{y}_r = \mathbf{F}_P\mathbf{D}_{P,r}\mathbf{H}_{T,r}\mathbf{F}_N^H\mathbf{x} + \mathbf{n} \tag{3}$$

In this paper, we consider a frequency division system for each relay, i.e., arranging transmitted symbols in different frequency bands according to the corresponding relay, as shown in the Fig. 2. By doing so, we can exploit the linear structure of ZP-OFDM to achieve the full cooperative diversity and improve the system capacity with linear receiver regardless of the existence of CFOs.

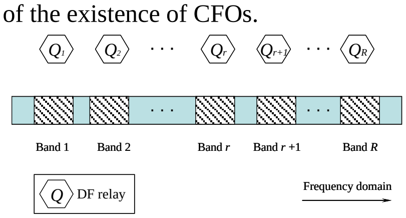

Figure 2: Frequency division cooperative ZP-OFDM system.

We take $\mathbf{x}$ as the information symbols correctly received at the r-th relay nodes involved in the DF-cooperative scheme. After full decoding, $\mathbf{x}$ is assigned to the corresponding r-th frequency band as shown in the Fig. 2, and forwarded to the destination.

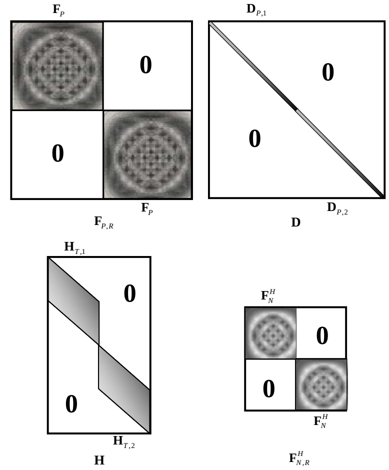

Figure 3: Structures of the FFT matrices, CFOs matrix and channel matrix for 2-relay cooperative system, left top: FFT matrix $\mathbf{F}_{P,R}$, right top: CFOs matrix $\mathbf{D}$, left bottom: channel matrix: $\mathbf{H}$, right bottom: FFT matrix $\mathbf{F}_{N,R}^H$. Blank parts are all 0's, others are non-zero values

Considering the frequency division system, the received signal at the destination of all R relay nodes yield

$$\mathbf{y} = \mathbf{F}_{P,R}\mathbf{D}\mathbf{H}\mathbf{F}_{N,R}^H\bar{\mathbf{x}} + \mathbf{n} \tag{4}$$

where $\mathbf{F}_{P,R} = \mathrm{diag}\left(\mathbf{F}_P,\mathbf{F}_P,\cdots,\mathbf{F}_P\right)$, $\mathbf{D} = \mathrm{diag}\left(\mathbf{D}_{P,1},\mathbf{D}_{P,2},\cdots,\mathbf{D}_{P,R}\right)$, $\mathbf{H} = \mathrm{diag}\left(\mathbf{H}_{T,1},\mathbf{H}_{T,2},\cdots,\mathbf{H}_{T,R}\right)$, $\mathbf{F}_{N,R}^H = \mathrm{diag}\left(\mathbf{F}_N^H,\mathbf{F}_N^H,\cdots,\mathbf{F}_N^H\right)$ are all diagonal matrices with R relay's components on their diagonals. For instance, we consider a 2-relay cooperation system, i.e., R = 2, the structures of $\mathbf{F}_{P,R}$, $\mathbf{D}$, $\mathbf{H}$ and $\mathbf{F}_{N,R}^H$ can be illustrated as Fig. 3. In (4) $\bar{\mathbf{x}} = \left[\mathbf{x}^T,\mathbf{x}^T,\cdots,\mathbf{x}^T\right]^T$ denotes the forwarded signal from R relays occupying R different frequency bands.

B. STFC for the Cooperative ZP-OFDM

We also design a linear structure STFC, which guarantees the full cooperative spatial diversity. By right multiplying a matrix $\mathbf{G} = \left[\mathbf{I}_1,\mathbf{I}_2,\cdots,\mathbf{I}_R\right]^T$ with $\mathbf{F}_N^H\mathbf{x}$, where $\mathbf{I}_r$ is an $N \times N$ identity matrix, $r \in \left[1,2,\cdots,R\right]$, the received signal at the destination from all R relay nodes can be rewritten in another form

$$\mathbf{y} = \mathbf{F}_{P,R}\mathbf{D}\mathbf{H}\mathbf{G}\mathbf{F}_N^H\mathbf{x} + \mathbf{n} \tag{5}$$

where $\mathbf{F}_{P,R}$, $\mathbf{D}$ and $\mathbf{H}$ are the same as (4). We denote $\mathbf{H}\mathbf{G}$=$\hat{\mathbf{H}}_T$ and $\mathbf{x}_t = \mathbf{F}_N^H\mathbf{x}$, here $\hat{\mathbf{H}}_T = \left[\mathbf{H}_{T,1}^T,\mathbf{H}_{T,2}^T,\cdots,\mathbf{H}_{T,R}^T\right]^T$ is a linear Toeplitz matrix, or tall Toeplitz matrix, and with $\hat{\mathbf{h}}_n = \left[h_{1,1},\cdots,h_{L,1},\mathbf{0},h_{1,2},\cdots,h_{L,2},\mathbf{0},\cdots,h_{1,R},\cdots,h_{L,R},\mathbf{0}\right]^T$ being $\hat{\mathbf{H}}_T$'s first column, $\mathbf{x}_t$ is the time domain signal. $\hat{\mathbf{H}}_T$ can also be regarded as a tall Toeplitz channel matrix, with the channel length $L_T = P \times \left(R-1\right) + L$.

We notice that matrix $\mathbf{G}$ spreads the R copies of the time domain signal $\mathbf{x}_t$ according to the corresponding R cooperative relays, and forms a cooperative frequency division system, since the relays perform the forwarding in the different bands, as shown in the Fig.2. Therefore, Matrix $\mathbf{G}$ can be regarded as a coding on the time domain signal, for different relays and difference bands, and so called space time frequency code. Then, (5) becomes

$$\mathbf{y} = \mathbf{F}_P \mathbf{D} \hat{\mathbf{H}}_T \mathbf{F}_N^H \mathbf{x} + \mathbf{n} \tag{6}$$

If we denote $\mathbb{H} = \mathbf{F}_P \mathbf{D} \hat{\mathbf{H}}_T \mathbf{F}_N^H$ as the equivalent channel matrix, we get

$$\mathbf{y} = \mathbb{H}\mathbf{x} + \mathbf{n} \tag{7}$$

$\mathbb{H}$ in (7) is regarded as the overall equivalent channel. In the section IV, we will exploit $\mathbb{H}$ to show that our STFC and tall Toeplitz channel design can achieve the full cooperative and multipath diversity and combat the CFOs, with only LEs.

III. Equalization and Orthogonality Deficiency

Given the equivalent channel model in (7), there are various ways to decode $\mathbf{x}$ from the observation $\mathbf{y}$. One often used and also optimal method (if there is no prior information on the symbols or symbols are treated as deterministic parameters) is the MLE, which is given as

$$\mathbf{x}_{ml} = \arg\min_{\tilde{\mathbf{x}} \in S^N} \left\| \mathbf{y} - \mathbb{H}\tilde{\mathbf{x}} \right\|^2 \tag{8}$$

where S is the finite alphabet of the transmitted symbols. On the other hand, LEs, such as ZF equalizer and MMSE equalizer are favored for their low decoding complexity.

The ZF equalizer is defined as

$$\mathbf{x}_{zf} = \mathbb{H}^{\dagger}\mathbf{y} \tag{9}$$

where $\mathbb{H}^{\dagger} = \left(\mathbb{H}^H \mathbb{H}\right)^{-1} \mathbb{H}^H$ denotes the Moore-Penrose pseudo-inverse of the channel matrix $\mathbb{H}$.

The MMSE equalizer is defined as

$$\mathbf{x}_{mmse} = \left(\mathbb{H}^H \mathbb{H} + \sigma_n^2 \mathbf{I}_N\right)^{-1} \mathbb{H}^H \mathbf{y} \tag{10}$$

Here, we notice that, with the definition of an extended system

$$\widehat{\mathbb{H}} = \begin{bmatrix} \mathbb{H} \\ \sigma_n \mathbf{I}_N \end{bmatrix} \text{ and } \widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N\times 1} \end{bmatrix} \tag{11}$$

The MMSE equalizer in (8) can be rewritten as $\mathbf{x}_{mmse} = \widehat{\mathbb{H}}^{\dagger}\widehat{\mathbf{y}}$, which indicates that ZF equalizer and MMSE equalizer are both LEs, and share the linear properties. To show the performance gap between LEs and MLEs, we adopt a parameter, orthogonality deficiency (*od*) of the channel matrix $\mathbb{H}$ as in [6].

Definition 1 (Orthogonality Deficiency): For an equivalent channel matrix $\mathbb{H} = \left[\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_N\right]$, with $\mathbf{h}_n$ being $\mathbb{H}$'s n-th column, its orthogonality deficiency $od\left(\mathbb{H}\right)$ is defined as

$$od\left(\mathbb{H}\right) = 1 - \frac{\det\left(\mathbb{H}^H \mathbb{H}\right)}{\prod_{n=1}^{N} \left\|\mathbf{h}_n\right\|^2} \tag{12}$$

If $\mathbb{H}$ is singular, $od\left(\mathbb{H}\right) = 1$. The smaller $od\left(\mathbb{H}\right)$ is, the more orthogonal $\mathbb{H}$ is. Given the model in (7), if $od\left(\mathbb{H}\right) = 0$, i.e., $\mathbb{H}^H \mathbb{H}$ is diagonal.

IV. Diversity, Capacity and Complexity of STFC

A. The Full Cooperative Diversity of STFC

In this section, we will show how LEs are only required to achieve full cooperative diversity order of *RL*. We first cite the following theorem from [6]:

Theorem 1: Consider a linear system in (7), The LEs collect the same diversity as MLE does, i.e., achieve the full diversity, if there exists a constant $\varepsilon \in (0,1)$ such that $\forall\ \mathbb{H}$, $od\left(\mathbb{H}\right) \le \varepsilon$.

In what follows, we will show that, the proposed STFC satisfies the condition in Theorem 1, and can achieve full diversity with LEs. Note that here the full diversity order is *RL*.

We notice that $\mathbf{F}_P$, $\mathbf{D}_P$ and $\mathbf{F}_N^H$ are all unitary matrices. Therefore, we have

$$\begin{aligned} \det\left(\mathbb{H}^H \mathbb{H}\right) &= \det\left(\mathbf{F}_N \hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T \mathbf{F}_N^H\right) \\ &= \det\left(\mathbf{F}_N\right)\det\left(\mathbf{F}_N^H\right)\det\left(\hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T\right) = \det\left(\hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T\right) \end{aligned} \tag{13}$$

where $\det\left(\mathbf{F}_N\right)\det\left(\mathbf{F}_N^H\right) = 1$. Since $\hat{\mathbf{H}}_T$ is a tall Toeplitz matrix, then $\det\left(\hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T\right) > 0$ for any nonzero channel response, i.e., $h_{l,r}$'s are not equal to zero simultaneously, where $l \in \left[1, 2, \cdots, L\right]$, $r \in \left[1, 2, \cdots, R\right]$, [7].

Consequently, we have $\det\left(\mathbb{H}^H \mathbb{H}\right) > 0$. For any practical channel, $\prod_{n=1}^{N} \left\|\mathbf{h}_n\right\|^2 > 0$ is always satisfied. Thus, we can verify that there exists a constant $\varepsilon \in (0,1)$ such that $\forall\ \mathbb{H}$, $od\left(\mathbb{H}\right) \le \varepsilon$, and the proposed STFC can achieve full cooperative and multipath diversity.

In order to provide the further insight into the channel factors that affect the cooperative transmission performance, we consider orthogonality deficiency of the pure channel effect, and denote $\overline{\mathbb{H}} = \mathbf{D}\hat{\mathbf{H}}_T$. The orthogonality deficiency of the pure channel effect can be represented as

$$od\left(\overline{\mathbb{H}}\right) = 1 - \frac{\det\left(\overline{\mathbb{H}}^H \overline{\mathbb{H}}\right)}{\prod_{n=1}^{N} \left\|\overline{\mathbf{h}}_n\right\|^2} = 1 - \frac{\det\left(\hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T\right)}{\prod_{n=1}^{N} \left\|\overline{\mathbf{h}}_n\right\|^2} \tag{14}$$

where $\overline{\mathbf{h}}_n$ is $\overline{\mathbb{H}}$'s n-th column. For the tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$, suppose $m = \arg\max_{l_t \in [1,\, L_T]} \left|h_{l_t}\right|^2$, and then $\left|h_m\right|^2 > 0$, the tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$ can be split into three submatrices as $\hat{\mathbf{H}}_T = \left[\hat{\mathbf{H}}_{T,o1}^T, \hat{\mathbf{H}}_{T,m}^T, \hat{\mathbf{H}}_{T,o2}^T\right]^T$, where matrix $\hat{\mathbf{H}}_{T,o1}^T$ consists of the first $(m-1)$ rows of $\hat{\mathbf{H}}_T$, $\hat{\mathbf{H}}_{T,o2}^T$ has the last $\left(L_T - m\right)$ rows, and $\hat{\mathbf{H}}_{T,m}^T$ is of size $N \times N$ with h_m on the diagonal entries. Therefore, we have $\hat{\mathbf{H}}_T^H \hat{\mathbf{H}}_T = \hat{\mathbf{H}}_{T,o1}^H \hat{\mathbf{H}}_{T,o1} + \hat{\mathbf{H}}_{T,m}^H \hat{\mathbf{H}}_{T,m}^T + \hat{\mathbf{H}}_{T,o2}^H \hat{\mathbf{H}}_{T,o2}^T$. It is easy to show that $\det\left(\hat{\mathbf{H}}_{T,m}^H \hat{\mathbf{H}}_{T,m}^T\right) = \left(\left|h_m\right|^2\right)^N$ when $N > L$. Thus, we bound $\det\left(\overline{\mathbb{H}}^H \overline{\mathbb{H}}\right)$ as

$$\det\left(\overline{\mathbb{H}}^H\overline{\mathbb{H}}\right)\geq\det\left(\hat{\mathbf{H}}_{T,m}^H\hat{\mathbf{H}}_{T,m}^T\right)=\left(\max_{l_t\in[1,\,L_T]}\left(\left|h_{l_t}\right|^2\right)\right)^N \quad (15)$$

For the CFOs matrix, we notice that $\left|\alpha_r^p\right|^2=1$, $p\in[0,1,\cdots,P-1]$. In this case, we can show the upper bound of the (14) as

$$od\left(\overline{\mathbb{H}}\right)\leq 1-\frac{\left(\max_{l_t\in[1,\,L_T]}\left(\left|h_{l_t}\right|^2\right)\right)^N}{\left(\sum_{l_t=1}^{L_T}\left|h_{l_t}\right|^2\right)^N}\leq 1-\frac{\left(\max_{l_t\in[1,\,L_T]}\left(\left|h_{l_t}\right|^2\right)\right)^N}{\left(RL\left(\max_{l_t\in[1,\,L_T]}\left(\left|h_{l_t}\right|^2\right)\right)\right)^N}=1-\frac{1}{(RL)^N} \quad (16)$$

Note that RL is the full diversity order, If we keep RL as a constant, and reduce the upper bound of $od\left(\overline{\mathbb{H}}\right)$ by decreasing N, i.e., the channel becomes more orthogonal, the upper bound of BER also becomes smaller, which indicates that LEs may achieve better BER performance with the full diversity order. Later, we will verify this theoretical claim by simulations.

B. *Capacity Analysis of the STFC*

To depict the capacity, one not only needs the capacity, but also the outage capacity, i.e. C^{th}, a capacity threshold indicate the outage behaviour. In this section, we compare the outage capacity of ZF equalizer with that of MLE. The results can be easily extended to other LEs. Let us first consider the capacity when no channel state information is available at the transmitter, and the MLE is adopted at the receiver. Given the linear equivalent channel model in (7), the capacity achieved by MLE, i.e., C_{ml} is given as

$$C_{ml}(\mathbb{H})=\log_2\left[\det\left(I_N+\left(1/\sigma_n^2\right)\mathbb{H}^H\mathbb{H}\right)\right] \quad (17)$$

When ZF equalizer is adopted at the receiver, the capacity of ZF equalizer given $\mathbb{H}$ can be expressed as [8]

$$C_{zf}(\mathbb{H})=\log_2\left[\det\left(I_N+\left(1/\sigma_n^2\right)\mathbb{N}^{-1}\right)\right] \quad (18)$$

where $\sigma_n^2\mathbb{N}$ is called the covariance matrix of the equivalent noise vector with $\mathbb{N}=\mathrm{diag}\left[k_{1,1},k_{2,2},\cdots,k_{N,N}\right]$, and $k_{i,i}$ being the (i,i)-th entry of matrix $\mathbb{k}=\left(\mathbb{H}^H\mathbb{H}\right)^{-1}$. It is well known that $C_{zf}(\mathbb{H})\leq C_{ml}(\mathbb{H})$ is always satisfied, and the difference between $C_{zf}(\mathbb{H})$ and $C_{ml}(\mathbb{H})$ for each realization of $\mathbb{H}$ can be shown as

$$C_{ml}(\mathbb{H})-C_{zf}(\mathbb{H})\approx-\log_2\left(1-od\left(\left(\mathbb{H}^\dagger\right)^H\right)\right) \quad (19)$$

This expression shows that the capacity difference between ZF and MLEs is also related to the *od* of the channel matrix. Similar to the discussion in the previous Section, we also consider the pure channel effect $\overline{\mathbb{H}}$ here. We observe that as $od\left(\left(\overline{\mathbb{H}}^\dagger\right)^H\right)$ decreases, i.e., the inverse of the channel matrix is more orthogonal, the capacity gap between the ML and ZF equalizers decreases.

In what follows, we will show that, with the ZF equalizer, the proposed STFC collects the same outage diversity as that of MLEs. The outage diversity order G_o is defined as

$$G_o=\lim_{\mathrm{SNR}\to\infty}-\frac{\log\left(\mathrm{Prob}\left(C<C^{th}\right)\right)}{\log(\mathrm{SNR})}, \quad (20)$$

If two cumulative density functions (CDFs) of channel capacities are in parallel, it can be shown that they have the same outage diversity [6]. In order to prove the proposed STFC of this paper with ZF equalizer achieves the same outage diversity as MLEs, we cite the results from [6] in the following theorem:

Theorem 2: Given the system model in (7) with channel state information at the receiver but not at the transmitter, if $od\left(\overline{\mathbb{H}}\right)\leq\varepsilon$, $\forall\ \overline{\mathbb{H}}$, and $\varepsilon\in(0,1)$, then at high-SNR regime, ZF equalizers collect the same outage diversity as that of MLEs.

Note that the condition in the *Theorem 2* is the same as the condition in the *Theorem 1*. Similar to the verification for the full cooperative diversity, by taking advantage of the linear Toeplitz structure of the proposed STFC, it is ready to show that by utilizing the proposed STFC, ZF equalizers have the same outage diversity as that of MLEs.

C. *Complexity comparison between LEs and MLEs*

To quantify the complexity of different equalizers, we count the average number of arithmetic operations needed to estimate in (7). Using ZF equalizer in (9) as an example, the complexity results from computing $\mathbb{H}^\dagger=\left(\mathbb{H}^H\mathbb{H}\right)^{-1}\mathbb{H}^H$ using the QR decomposition and calculating $\mathbb{H}^\dagger\mathbf{y}$ .As shown in [9], if we consider $\mathbb{H}$ as a $M\times N$ matrix, $M=R\times(N+L)$, the number of real multiplications for ZF equalizer is $O\left(N^3\right)+O\left(N^2M\right)+O\left(NM^2\right)$ while the number of real additions is also $O\left(N^3\right)+O\left(N^2M\right)+O\left(NM^2\right)$, $O(\cdot)$ denotes the Landau notation. The optimum equalizer, MLE in (8), while enjoys the best performance, requires the highest complexity as well. As shown in [9], the number of arithmetic operations for the MLE in (8) is $O\left(|\mathbf{x}|^N\right)MN$.

V. SIMULATION

To show the effect of the proposed STFC and $od\left(\overline{\mathbb{H}}\right)$ on the performance, we consider the N subcarriers ZP-OFDM systems with ZP accounts for 25% of the OFDM symbol duration undergoes Rayleigh channel.

Fig. 4 shows the BER vs. E_b/N_0 performance with R = 1, 2, respectively, under the same CFO, $\Delta q_r=0.3$. The frequency-selective channel order L is fixed to be 2. MMSE equalizer is adopted at the receiver. We observe that, without the proposed STFC, i.e., direct combining the 2-relays signals in the same frequency band at the destination, the 2-relays system only yields 3 dB power gain. The 2-relays BER vs. E_b/N_0 curve without STFC is parallel to the 1-relay case, indicating no diversity is achieved. After adopting the proposed STFC, $od\left(\overline{\mathbb{H}}\right)\leq\varepsilon<1$, the full cooperative diversity is achieved. We also notice that when ε is smaller as the N decreases, the BER performance of the proposed STFC gets better. This is consistent with the analysis, as shown in (16),

$od(\overline{\mathbb{H}})$ decreases with the decrease of N. When ε is smaller, the channel is more orthogonal, the upper bound of the BER performance also becomes smaller. In general, for LEs, a smaller $od(\overline{\mathbb{H}})$ bound indicates higher coding gain while the diversity is the same.

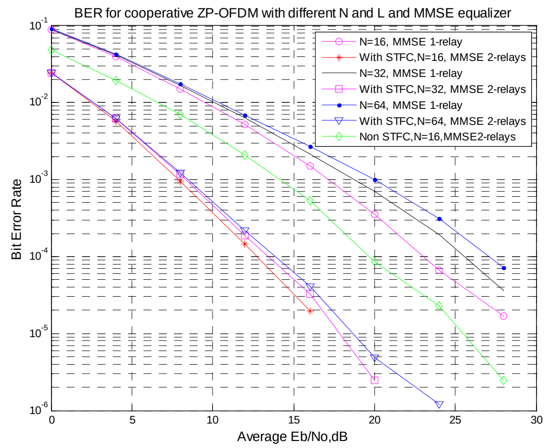

Figure 4. Performance of MMSE equalizer for STFC cooperative ZP-OFDM

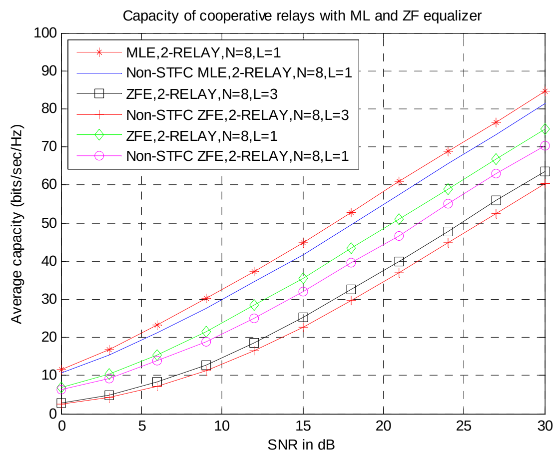

Figure 5: Average capacity for STFC cooperative ZP-OFDM

Fig. 5 shows the average capacity of Rayleigh channel with and without proposed STFC for two relays cooperation, and $N = 8$. As shown in the figure, the proposed STFC slightly improves the system capacity, because of exploiting the linear structure of the channel. We notice that the $od(\overline{\mathbb{H}})$ gets smaller as the channel length decreases, and thus the capacity gets higher. This confirms the observation in (19) that, the capacity not only depends on SNR but also the channel orthogonality. The CDFs of the capacity $\mathrm{Prob}(C < C^{th})$ with ZF and ML equalizer are depicted in Fig.6, with SNR = 25 dB. We notice that, for the ZF equalizer (ZFE) case without STFC, the curve is not in parallel with the one of the MLE case, which means loss of outage diversity. By adopting the proposed STFC, the curve of the ZFE becomes parallel with that of MLE, which indicates that the proposed STFC achieves the same outage diversity as MLE. This is consistent with the Theorem 2 and our analysis in Section IV. *B*.

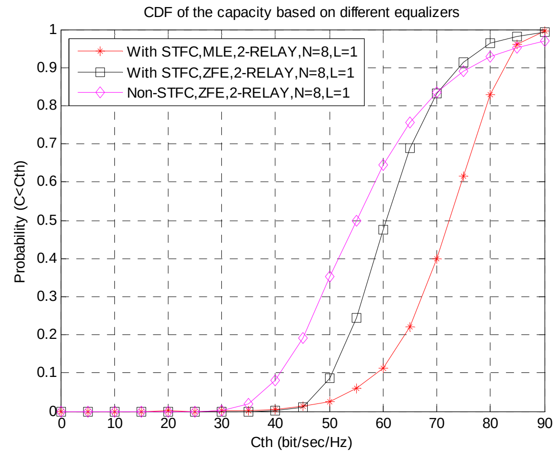

Figure 6: CDF of the capacity with and without the proposed STFC

VI. CONCLUSIONS

In this paper, we first designed an STFC for the cooperative ZF-OFDM system, with CFO and multipath channel, i.e., doubly time-frequency selective channel. Then, we showed the analytical upper bound of the channel orthogonality deficiency with the proposed STFC system, and illustrated how the change of channel factors affects the system performance in terms of the cooperative diversity and capacity. The proposed STFC improves the system capacity, by taking advantage of the linear structure of the cooperative ZP-OFDM system. According to the theoretical analysis and simulation results, only with linear equalizers, the proposed STFC achieved the same cooperative and outage diversity as those of MLEs, while significantly reduced the system complexity.

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