BLIND SOURCE SEPARATION BASED ON COMBINED DIRECTION FINDING AND CONSTANT MODULUS PROPERTIES

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In the blind separation of instantaneous mixtures of multiple cochannel signals using an antenna array, two types of techniques have been used: based on properties of the array manifold, or based on properties of the signals. A well-studied example of the first type is the ESPRIT algorithm, which exploits the parametric structure of the array manifold. A representative of the second type is ACMA, which gives algebraic expressions for the separation of sources based on their constant modulus property, valid for phase-modulated sources. In this paper, we show that the two properties can be combined into a single algorithm.

1. INTRODUCTION

Beamforming techniques try to separate superpositions of source signals from the outputs of a sensor array. The objective of blind beamforming is to do this without training information, relying instead on various structural properties of the problem. Direction finding can be considered as the earliest example of blind beamforming. The direction of each incoming wavefront is estimated, at the same time producing a beamformer to recover the signal from that direction while suppressing the others. This requires at least that the antenna array is calibrated and that the propagation scenario is simple, with a limited number of specular multipath rays per source.

More recently, new types of blind beamformers have been proposed that are not based on specific channel models, but instead exploit properties of the signals. A striking example is the constant modulus algorithm (CMA), which separates sources based on the fact that their baseband representation has a constant amplitude, such as is the case for FM or phase modulated signals. A prime advantage is that these beamformers are insensitive to multipath and not dependent on array calibration.

Mathematically, the problem can be described as follows. Let X be a data matrix containing the measurements at the antenna array (see section 2 for details). Under standard narrowband assumptions, we have the usual data model

$$X = ABS$$

where A is the array response matrix, B a diagonal scaling and S contains samples of the source signals. The blind source separation problem is, given X, to find the unknown factors A, B and S. Techniques of the first type (direction finding) assume that A satisfies a parametric model:

$$A = [\mathbf{a}_1 \cdots \mathbf{a}_d], \qquad \mathbf{a}_i = \mathbf{a}(\theta_i)$$

where $\mathbf{a}(\theta)$ is a known function of the direction-of-arrival θ. This model places constraints on the factor A, often sufficient to determine A and S uniquely up to scalings and permutations. A recent overview of parametric methods can be found in [1].

Techniques of the second type, on the other hand, place constraints on S and thus use structural information about the sources. In particular, constant modulus algorithms (CMAs) assume that sources are phase modulated and say that every entry of S should have a unit magnitude: $|S_{ij}| = 1$. Following the work of Treichler et al. [2], this has e.g., led to the "Constant-Modulus Array" [3, 4].

Most separation algorithms have used only properties of A or of S, but not both (except perhaps for initialization purposes). Nonetheless, a performance increase might be possible if information on both A and S is combined. For example, the work of Li et al. [5] shows this for direction estimation in the case of signals with known waveforms (i.e., known S and a parametrically specified A). Comparable benefits can be expected if we do not have complete knowledge of S but do know that sources have a constant modulus.

Traditional CMAs are iterative, need proper initialization, and have not been very reliable on short data sequences. These problems are not present in the "Analytic CMA" [6], which is an algebraic technique that can act on small data sets. Thus, it makes sense to extend the ACMA to also exploit the structure of the A-matrix. One case in which this can be done very nicely is for uniform linear arrays (ULAs), because for such arrays the direction finding problem admits algebraic solutions as well, as seen e.g., in the ESPRIT algorithm [7] and a specialization of MODE [8].

In this paper we show how ACMA can be combined with ESPRIT to produce an algebraic source separation algorithm that uses both the parametric structure of the ULA present in A and the CM property present in S. A companion paper [9] derives the relevant Cramer-Rao bounds for direction of arrival estimation.

2. DATA MODEL

Consider d independent sources, transmitting signals $s_i(t)$ with constant modulus waveforms ($|s_i(t)| = 1$) in a wireless scenario. The signals are received by a uniform linear array of $M > d$ antennas spaced at Δ wavelengths. We stack the antenna outputs $x_i(t)$ into vectors $\mathbf{x}(t)$ and collect N complex samples into a matrix $X : M \times N$. Under standard simplifying assumptions (negligible multipath, sufficiently narrowband sources with discrete angles of incidence α_i), this leads to the well-known data model

$$X = ABS = \mathbf{a}_1 \beta_1 \mathbf{s}_1 + \cdots + \mathbf{a}_d \beta_d \mathbf{s}_d \,. \tag{1}$$

$A \in \mathbb{C}^{M\times d}$ is the array response matrix. $B \in \mathbb{C}^{d\times d}$ is a diagonal scaling matrix containing the complex gain parameters, and the rows $\mathbf{s}_i$ of $S \in \mathbb{C}^{d\times N}$ contain the samples of the source signals.

In the blind signal separation scenario, A, B and S are unknown and the objective is, given X, to find the factorization $X = ABS$. Alternatively, we try to find a beamforming matrix $W \in \mathbb{C}^{d\times M}$ of full row rank d such that $S = WX$.

Two types of properties are available to compute the factorization (1) from X. Firstly, under the present assumptions, we have a parametrized model for A:

$$A = [\mathbf{a}_1 \; \cdots \; \mathbf{a}_d] \,, \qquad \mathbf{a}_i = \mathbf{a}(\theta_i) \tag{2}$$

where each θ_i is related to the angle of arrival α_i and $\mathbf{a}(\theta)$ is the array manifold function, for a ULA given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ \theta \\ \vdots \\ \theta^{M-1} \end{bmatrix} , \qquad \theta = e^{j2\pi\Delta \sin(\alpha)} \,. \tag{3}$$

We can also try to find the factorization $X = ABS$ based on the constant-modulus property of S, i.e.,

$$|S_{ik}| = 1 \,. \tag{4}$$

3. SEPARATION ALGORITHM

For simplicity of exposition, we derive the algorithm from the noiseless case, but it will be clear how it can be extended. Given X, introduce its singular value decomposition (SVD)

$$X = \hat{U}\hat{\Sigma}\hat{V}$$

where $\hat{U}$ has d orthonormal columns, $\hat{V}$ has d orthonormal rows, and $\hat{\Sigma}$ is a $d \times d$ diagonal matrix containing the nonzero singular values of X. Any other rank-d factorization of X can be written as

$$X = \hat{U}\hat{\Sigma}T^{-1} \cdot T\hat{V}$$

where T is a $d \times d$ invertible matrix. This expression has to be matched with the model $X = ABS$, which leads to

$$\begin{cases} \hat{U}\hat{\Sigma} &= ABT \\ T\hat{V} &= S \end{cases}$$

(We can choose where to put the diagonal factors B and $\hat{\Sigma}$.) Thus, the problem of source separation is expressed in terms of finding the matrix T. This matrix has the interpretation of a beamformer on the whitened measurement data $\hat{V} = (\hat{\Sigma}^{-1}\hat{U}^*)X$. Once we know T, the beamformer W on the original data such that $WX = S$ is given by $W = T\hat{\Sigma}^{-1}\hat{U}^*$.

The parametric property of A in (2)–(3) is translated into an algebraic property by the ESPRIT algorithm [7]. Its solution can briefly be described as follows. The shift-invariance of $\mathbf{a}(\theta)$ in equation (3) leads to the observation that if we take two submatrices E_x and E_y of $\hat{U}\hat{\Sigma}$, consisting of rows 1 to $M-1$ and rows 2 to M of $\hat{U}\hat{\Sigma}$, respectively, then $E_y T^{-1} = E_x T^{-1}\Theta$, where $\Theta = \mathrm{diag}[\theta_i]$, so that

$$E_x^\dagger E_y = T^{-1}\Theta T \tag{5}$$

where $\dagger$ denotes the Moore-Penrose pseudo-inverse. This is an eigenvalue problem, and can directly be solved for T and Θ. (The columns of T^{-1} are the eigenvectors.)

The constant-modulus property of S is used as in the ACMA [6], as follows. Let $\mathbf{t}^*$ be a row of T. The corresponding beamformer output is the row $\mathbf{s} = \mathbf{t}^*\hat{V}$. Our aim is to choose the beamformer such that $\mathbf{s}$ is a constant modulus signal, i.e., with entries $|s_k| = 1$ for $k = 1, \cdots, N$. For sufficiently large N, the uniqueness result for the CM factorization problem [6] claims that in that case $\mathbf{s}$ is almost surely one of the source signals, up to unimodular scaling. Substitution leads to the condition

$$\mathbf{t}^*(\mathbf{v}_k\mathbf{v}_k^*)\mathbf{t} = 1 \,, \qquad k = 1, \cdots, N \,,$$

where $\mathbf{v}_k$ is the k-th column of $\hat{V}$. This is an overdetermined system of quadratic equations, whose solution is more elaborate to derive. The ACMA technique in [6] transforms the conditions into d data matrices $Y_1, \cdots, Y_d$ of size $d \times d$, satisfying

$$\begin{array}{rcl} Y_1 &=& T^*\Lambda_1 T \\ Y_2 &=& T^*\Lambda_2 T \qquad \text{All } \Lambda_i \text{ diagonal} \\ &\vdots& \\ Y_d &=& T^*\Lambda_d T \end{array} \tag{6}$$

This is a simultaneous diagonalization problem (by congruence), and very much related to eigenvalue problems. It can be solved for T and the Λ_i.

The new observation in this paper is that (5) and (6) specify the same matrix T, and that it is possible to combine the two eigenvalue problems. To this end, introduce QR factorizations of T^{-1} and T^*,

$$\begin{array}{rclcrcl} T^{-1} &=:& ZR & \Leftrightarrow & T &=& R^{-1}Z^* \\ T^* &=:& QR' & & & & \end{array} \tag{7}$$

where Q and Z are unitary matrices, and R and R' are square and upper triangular. The combined diagonalization problem now becomes a generalized Schur decomposition problem of the form: find Q, Z (unitary) such that the following matrices become jointly upper triangular:

$$\left\{\begin{array}{rcl} Q^*Y_1Z &=& R_1 \\ &\vdots& \\ Q^*Y_dZ &=& R_d \end{array}\right. \qquad \begin{array}{l} Q,\, Z \text{ unitary} \\ \text{All } R_i \text{ and } R_\theta \text{ upper triang.} \end{array}$$
$$Z^*(E_x^\dagger E_y)Z = R_\theta$$

If we also introduce the QR factorization $Q_2R'' := E_yT^{-1} = E_xT^{-1}\Theta$ (where Q_2 is unitary and R'' is upper triangular with size $M-1\times d$) it is possible to put the problem into a nicer more symmetric form: find Q_1, Q_2, Z to make the following matrices upper triangular:

$$\left\{\begin{array}{rcl} Q_1^*Y_1Z &=& R_1 \\ &\vdots& \\ Q_1^*Y_dZ &=& R_d \end{array}\right. \qquad \begin{array}{l} Q_1,\, Q_2,\, Z \text{ unitary} \\ \text{All } R_i,\, R_x,\, R_y \text{ upper triang.} \end{array}$$
$$\left\{\begin{array}{rcl} Q_2^*E_xZ &=& R_x \\ Q_2^*E_yZ &=& R_y \end{array}\right. \tag{8}$$

(We have set $Q_1 := Q$. R_x and R_y are possibly nonsquare with size $M-1\times d$.) Each of the two subproblems looks like a Generalized Schur decomposition, and they are related as a Generalized SVD. With additive noise, these factorizations hold only approximately, and we can only expect to find Q_1, Q_2 and Z to make the data matrices approximately upper triangular.

Given $Y_1,\cdots,Y_d,E_x,E_y$, it is possible to set up a Jacobi rotation scheme to compute Q_1, Q_2 and Z iteratively, using 2×2 elementary rotations. Examples of such algorithms can be found in [6, 10, 11] and references therein. The general idea is to freeze Z and select Q_1 and Q_2 (independently) to make the first d, respectively last two matrices as much upper triangular as possible, then freeze Q_1, Q_2 and find Z to make the resulting matrices as much upper triangular as possible. A good initial point for the iteration is obtained from the Schur decomposition of $E_x^\dagger E_y$, so that convergence should be fast.

Finally, there are several ways to retrieve T from the decomposition. It can e.g., be shown that Q_1 and Z together parametrize T via (7), which can be found by solving a simple matrix equation. Perhaps better is to form a combined eigenvector estimate based on the solution of both subproblems, as follows. Define $H = T^{-1}$, with columns $\mathbf{h}_i$. We will estimate H. Firstly, note from (5) that

$$(\theta_iE_x - E_y)\mathbf{h}_i = 0 \tag{9}$$

where θ_i is the i-th eigenvalue of $E_x^\dagger E_y$, which will be estimated by the i-th entry of $\mathrm{diag}(R_x)^{-1}\mathrm{diag}(R_y)$. With slightly more effort, we can derive a similar property for the Y_i. If $\Lambda_1,\cdots,\Lambda_d$ in (6) would be known, then it is clear that we can compute linear combinations of $Y_1,\cdots,Y_d$ to arrive at rank-1 matrices $\mathbf{t}_i\mathbf{t}_i^*$. (Recall that $\mathbf{t}_i^*$ is the i-th row of T.) With only $R_1,\cdots,R_d$ known from (8), we can take the diagonal entries of these matrices, compute an inverse matrix $A = [\alpha_{ij}]$,

$$A = [\alpha_{ij}] = \begin{bmatrix} (R_1)_{11} & \cdots & (R_1)_{dd} \\ \vdots & & \vdots \\ (R_d)_{11} & \cdots & (R_d)_{dd} \end{bmatrix}^{-1},$$

and use these to construct linear combinations $\tilde{Y}_1,\cdots,\tilde{Y}_d$,

$$\tilde{Y}_i = \sum_{j=1}^{d} \alpha_{ij}Y_j\,, \qquad i = 1,\cdots,d\,.$$

As explained in [6], each matrix $\tilde{Y}_i$ is rank-1 and a scalar multiple of $\mathbf{t}_i\mathbf{t}_i^*$ (in the noise free case). Since $TH = I$, it follows that

$$\tilde{Y}_j\mathbf{h}_i = 0\,, \qquad \text{for } j \neq i. \tag{10}$$

Combining (9) with (10), we obtain the equations

$$\begin{bmatrix} \tilde{Y}_2 \\ \vdots \\ \tilde{Y}_d \\ \theta_1E_x - E_y \end{bmatrix}\mathbf{h}_1 = 0 \;,\cdots,\; \begin{bmatrix} \tilde{Y}_1 \\ \vdots \\ \tilde{Y}_{d-1} \\ \theta_dE_x - E_y \end{bmatrix}\mathbf{h}_d = 0$$

from which each of the $\mathbf{h}_i$ can be computed. With $H = [\mathbf{h}_i]$ in hand, we set $T = H^{-1}$ and $W = T\hat{\Sigma}^{-1}\hat{U}^*$.

Remarks

Several issues remain to be resolved. Most importantly, the relative scalings of the data matrices in (8) has not been optimized. Obviously, by scaling E_x and E_y, we can place more or less emphasis on the ESPRIT part or the ACMA part of the problem. The correct scaling should follow from an analysis of the signal-to-noise ratio on the data matrices $Y_1,\cdots,Y_d,E_x,E_y$, which is a significant effort not undertaken here. In the simulations, we have heuristically scaled the Y_i to have unit Frobenius norm, and E_x, E_y to have Frobenius norm 1 ('ACMA-ESPRIT(1)') and 2 ('ACMA-ESPRIT(2)').

4. SIMULATION RESULTS

Some performance results are shown in figure 3. In this simulation, we took $N = 20$ samples, a ULA($\frac{\lambda}{2}$) consisting of $M = 4$ antennas, and $d = 3$ constant-modulus sources with directions $[-\alpha, 0, \alpha]$, for varying α. The signal to noise ratio (SNR) was set at 10 dB. The figure shows the signal to interference ratio (SIR) after beamforming, which measures how well the estimated W is a left inverse of A.

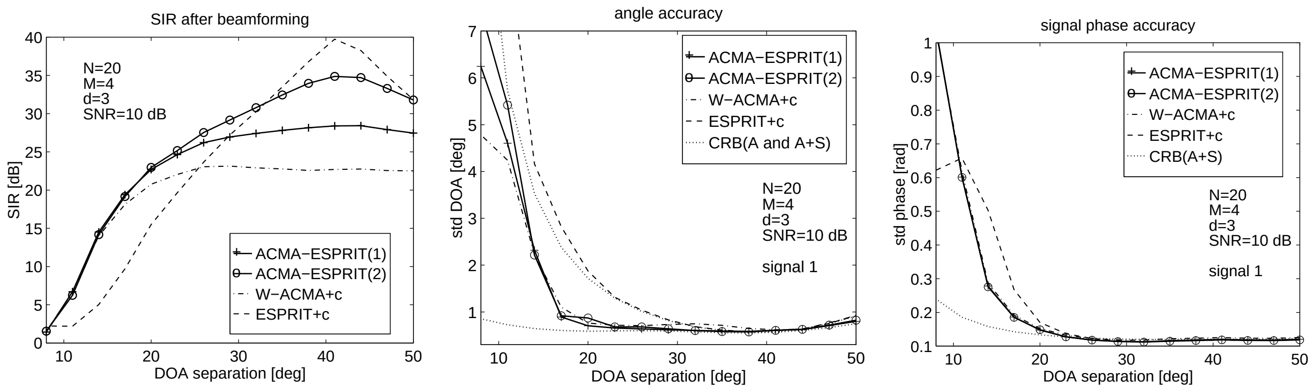

Fig. 1. (a) Signal-to-interference ratio after beamforming; (b) accuracy of DOA estimate and of (c) phase estimate of first signal.

The "ESPRIT+c" algorithm is the standard LS-ESPRIT algorithm, but acting on an extended data matrix $[X \;\; \Pi\bar{X}]$, which uses the centro-symmetry of the array. (Π is a permutation matrix which reverses the ordering of rows.) Similarly, "ACMA+c" is the ACMA algorithm [6], acting on this extended data matrix.[1] "ACMA-ESPRIT" is the new combined algorithm, also acting on the extended data matrix. We try two scalings of E_x, E_y: to have a Frobenius norm of 1 and 2. It is seen from the SIR plot that the matrix W computed by ACMA-ESPRIT is almost always more accurate than that of ACMA, although the difference is significant only when ESPRIT is more accurate than ACMA+c, which occurs for wide source separations. The impact on DOA estimates and signal phase estimates (or SINR) is negligible. In the DOA plot, we show the Cramer-Rao bounds "CRB(A)" for the model with structure only in A, and "CRB(A+S)" for the model with structure in both A and S [9], which is the lower dotted curve.

5. CONCLUSIONS

We have obtained an algorithm that separates sources based on both directional and constant modulus properties. The algorithm is a small extension of the ACMA, but raises interesting and unsolved issues, such as how to weight the matrices in a joint diagonalization. It was found that the benefits of the combined approach are only small, because ACMA is by itself already almost always more accurate than ESPRIT. As seen by the CRBs, there is still room for improvement at the small source separations.

6. REFERENCES

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[1] In fact, we have used the recently derived Weighted-ACMA, which removes the bias due to noise present in the original algorithm.