Performance Evaluation of a Wavelet Packet-based Spectrum Estimator for Cognitive Radio Applications

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***Abstract* — The accurate and fast sensing of the radio spectrum is one of the key requirements of any Cognitive Radio (CR) architecture. Wavelet packet-based spectrum estimator (WPSE) is a newly developed method for the detection of active users or spectrum holes in the spectral domain. Various wavelet families with different wavelet parameters exhibit different detection performances. In this article, the receiver operating characteristics (ROC) is considered for the performance evaluation of wavelet packet-based spectrum estimator for a wide range of wavelet families, both orthogonal and non-orthogonal, with different wavelet parameters. Exploiting the underutilized radio spectrum, an efficient wavelet packet-based spectrum estimation model with reduced number of sensing measurements is also proposed. The developed model exhibits a significant reduction of number of sensing measurements with a good detection performance.**

***Index Terms*—Cognitive Radio, Wavelet packet transform, Receiver operating characteristics.**

I. Introduction

Rapid growth of wireless communications and its applications creates scarcity of the radio spectrum, the most valuable resource in wireless technology [1]. But significant underutilization of the radio spectrum by licensed wireless users at any particular time and geographic locations is also recognized [2],[3]. Cognitive radio (CR) [4],[5] is a revolutionary technology that enables the efficient spectrum utilization by allowing the unlicensed users to transmit in licensed user bands when they are vacant. Accurate scanning of wide frequency range to correctly detect the presence of licensed user or the spectrum hole is one of the main functionalities of CR system.

From the survey of all the conventional and modern spectrum estimation methodologies as reported in [6], a low complexity spectrum estimator with the property of good time-frequency resolution is desirable. One of the most recent trends in wireless digital communication is to apply the time-frequency localization property of wavelets for figuring out spectral singularities and edges [7]. An effective implementation of wavelet transform for sensing and management of dynamic spectrum is proposed in [8]. In [9] wavelet packets and filter bank based spectrum estimator is proposed. In [10], a method for best wavelet packet basis design for spectrum estimation is described and the method of calculating detection and false alarm probabilities to evaluate the receiver operating characteristics (ROC) is also illustrated. Mathematical theory of wavelets [11] gives a dynamic tunability of the time and frequency resolution but there is no pre-defined rule for wavelet selection for a specified function.

In this work the receiver operating characteristics is mainly considered as one of the major performance metrics of any spectral estimator including the wavelet packet-based spectrum estimator. In section II a brief introduction to the theoretical false alarm and detection probabilities of a conventional energy detector is presented. Section III illustrates the principle of WPSE. Along with that a model for wavelet packet based spectrum estimator with reduced number of wavelet packet coefficients is also discussed. Section IV illustrates the variation of ROC for different wavelet families by simulation results. In this section, the variation of ROC with different parameters of wavelet filters like length of the filter, orthogonality, level of decomposition etc. as well as the SNR of the input signal (with randomly activated and deactivated set of sub-carriers) is analyzed. The proposed model for WPSE with reduced number of sensing measurements is also simulated and its detection performance is evaluated. In section V the paper is summarized and future research areas are highlighted.

II. Analytical Formulation of Detection and False Alarm Probability in Conventional Energy Detector

The detection of any signal in an Additive White Gaussian noise environment is analyzed by the well-known binary hypothesis test problem based on Neyman-Pearson criterion [12].

This can be formulated by,

$$\left.\begin{aligned} r[n] &= w[n], && n = 1,2,\dots,N && : H_0 \\ r[n] &= x[n] + w[n], && n = 1,2,\dots\dots,N && : H_1 \end{aligned}\right\} \quad (1)$$

Where $r[n]$, $x[n]$ and $w[n]$ are respectively received signal at cognitive user nodes, transmitted original signal and noise signal whose samples are from a zero mean Gaussian random process with constant power spectral density σ_n^2. Hypotheses H_0 And H_1 correspond to the absence and presence of the target signal (licensed user in case of cognitive radio), respectively. N is the total number of samples of the input signal. Assuming that there is no prior knowledge about the transmitted signal, in the energy detector the average power is calculated and compared with a pre-defined threshold to decide the target signal's presence or absence. The energy detector is constructed as [13],

$$X(r)=\frac{1}{N}\sum_{n=1}^{N}r^{2}[n] \tag{2}$$

If λ is the threshold for detection, the criteria is,

$$\begin{aligned} X[r]>\lambda \quad &: H_1 \\ X[r]<\lambda \quad &: H_0 \end{aligned} \tag{3}$$

The conditional probability of detecting the target signal subject to the condition that actually it is present is known as the probability of correct detection (P_d). The conditional probability of detection of the target signal subject to the condition that actually it is absent is known as the probability of false alarm (P_{fa}). If S_{av} is the average signal power and σ_n^2 is the noise variance then the false alarm and detection probabilities can be formulated as [13],

$$P_{fa}=Q\left(\frac{\lambda-\sigma_n^2}{\sqrt{\frac{2\sigma_n^4}{N}}}\right) \tag{4}$$

$$P_d=Q\left(\frac{\lambda-(S_{av}+\sigma_n^2)}{\sqrt{\frac{2(S_{av}+\sigma_n^2)^2}{N}}}\right) \tag{5}$$

Where, $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{\frac{-u^2}{2}}du$ is the Gaussian complementary cumulative distribution function. The (4) and (5) are the relations that exhibit the theoretical false alarm and detection probabilities in terms of signal and noise power. The variation of this detection probability with the false alarm probability for a wide range of threshold gives the theoretical receiver operating characteristics (ROC), which is shown in the simulation results section of this paper. In cognitive radio paradigm a high detection probability with very low false alarm is desirable.

III. Wavelet Packet-based Spectrum Estimator (WPSE) and Energy Detector

A. Principles of WPSE

From theory of wavelets it is seen that wavelets of compact support can be realized from perfect reconstruction filter banks [11]. In [9] an approach of application of wavelet packet transform for spectrum sensing in cognitive radio is illustrated. Wavelet packet transform (WPT) iteratively decomposes the received input signal into high and low frequency components [9]. The transform produces a binary tree based decomposition of the input signal. So, for an L level of decomposition the transform will generate a binary tree with 2^L number of terminal nodes as the leaves of the complete tree. Each terminal node contains a set of wavelet packet coefficients of finite length. An L level wavelet packet decomposition divides the frequency band ranging from 0 to $0.5f_s$ (where f_s denotes the sampling frequency) in 2^L frequency sub-bands [9]. The frequency ordering of the coefficients is on the basis of the sequential binary grey code values of the terminal node numbers [14]. The decomposition of the signal into different frequency bands with different resolutions is possible. Due to filtering and down-sampling of the input signal, at every level in wavelet packet based decomposition time resolution is halved and frequency resolution is doubled. Thus, the frequency resolution of the estimation relates to the level of decomposition. From the analysis of [9], as Parseval's relationship holds good for WPT, the power spectral density (PSD) at the k^{th} frequency band of the input signal can be written as,

$$\text{PSD}(f_k)=\frac{E(f_k)}{N_s f_k} \tag{6}$$

Where, N_s is total number of samples in each wavelet packet coefficient associated with each terminal node at L^{th} decomposition level, f_k is the frequency range spanned by k^{th} wavelet packet node and $E(f_k)$ is the energy (inner product of the wavelet coefficient vector of the corresponding node with itself) of the k^{th} wavelet packet node. The PSD estimation of any discrete signal $x[n]$, using WPSE is shown in Fig.1. In this figure $h[n]$ and $g[n]$ are low and high pass quadrature mirror filters (QMF), respectively.

The major advantage of wavelet packet transform for spectrum estimation is the orthogonality of the filters that makes the filtered output uncorrelated, minimizing the bias in the estimate.

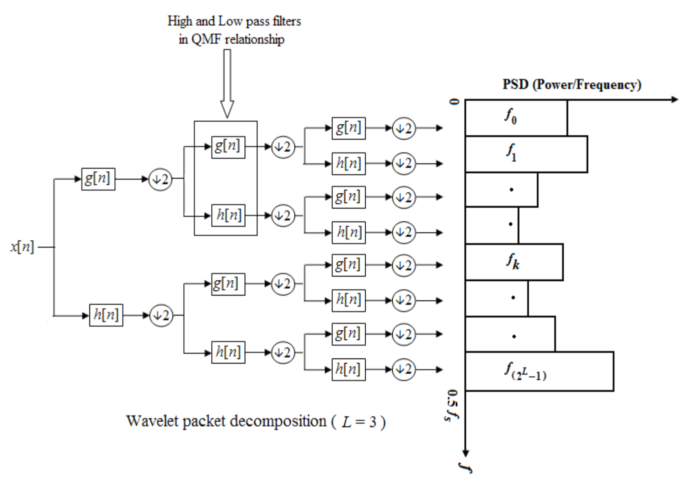

Fig. 1. 3-level Wavelet packet based spectrum estimator (WPSE); $h[n]$ and $g[n]$ are low and high pass quadrature mirror filters, respectively.

B. An efficient energy detection model by WPSE

The reduction of number of sensing measurements is a type of compression induced in the sensing and detection of the primary user. From the mathematical theory of compressed sensing [16], [17] we can see that it is possible to reconstruct a signal $x\in\mathbb{R}^N$ with N number of samples from $y\in\mathbb{R}^M$ of M number of random linear measurements where, $M << N$. Then the signal x is said to be compressible subject to the condition that it must be sufficiently sparse [18] i.e. a very small portion of the signal is carrying the essential information (very few non-zero elements in the signal vector). In cognitive radio paradigm the radio-spectrum also exhibits temporal and spatial sparse behavior [2], [3]. However, utilizing the sparsity of wideband spectrum a method for spectrum estimation and detection from sub-Nyquist samples with a wavelet based edge detector is already proposed in [19].

Here in Fig.2 we propose a model for detection of the primary user with reduced number of sensing measurements. In this scheme first the dynamic environment is sensed by a wavelet packet based spectrum estimator. It is seen that for an L level of decomposition there will be 2^L terminal coefficients. In the next step only R coefficients are selected out of 2^L. This selection of R coefficients is done based on the average energy content of the signal. The threshold (γ) for wavelet packet coefficient selection can be set dynamically based on the spectrum environment or any predefined statistical record regarding the power spectral density (PSD) of primary user. As it is already seen that significant part of the spectrum is underutilized for certain times or geographic locations, the number of coefficients with high energy content will be much smaller than total number of coefficients. So, these few coefficients, R ($R < 2^L$) can be used for the detection of the location of active users and informing the secondary/cognitive users for opportunistic spectrum access. The advantage of the above mentioned scheme is the reduced number of sensing measurements. Only, $M = R/2^L$, of the total coefficients are used to detect the spectral location of active/primary user for any level of decomposition L.

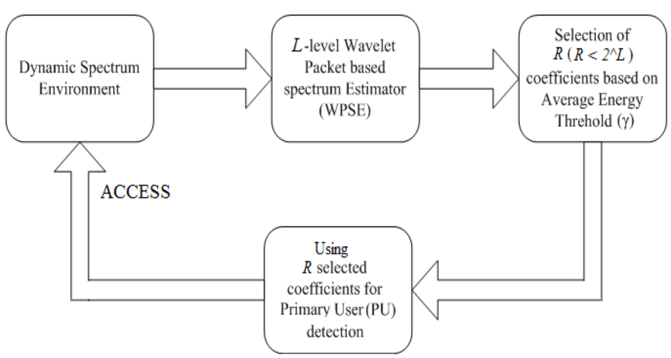

Fig. 2. Block diagram of the developed model for efficient detection of primary user (PU).

It is clear that if a small portion of the spectrum is occupied by primary user, which is the case for many real situations, R and consequently M will be small, reducing the total sensing load for primary user detection. Selection of threshold (γ) for wavelet packet coefficient selection is an important issue in this scheme. Here we have selected the threshold on the basis of the estimated (WPSE) active user power derived from wavelet packet transform of the signal with fixed SNR.
For this stage we have only considered the situation to be static. However, the information (coefficients indicating the presence of primary user) retrieved at one instant of time can be used for another instant for slowly varying dynamic environments.

IV. Simulation Results And Analysis

A. Variation of ROC with different wavelet families and wavelet filter properties

The receiver operating characteristics (ROC) in WPSE can be evaluated by the technique mentioned in [10]. To evaluate the ROC of WPSE we have taken N_c no. of single tones as primary users in cognitive radio that constitutes the entire normalized frequency band of $[0, \pi]$ (radian/sample). For simplicity of the analysis we assumed that $N_c = 2^L$ for any level of decomposition L. To estimate the probability of false alarm and probability of correct detection, all N_c number of single tones/carriers are randomly activated and deactivated. This carrier activation and deactivation process is same as carrier activation and deactivation for orthogonal frequency division multiplexing (OFDM) [15] signals. ROC in WPSE is evaluated using different wavelet families and their performance is analyzed. The detection and false alarm probabilities are calculated out of 100 iterations. For every iteration, the detection threshold (λ) for active/primary user is varied in the range of -20 to 0 dB. For simplicity the channel is assumed to be additive white Gaussian noisy (AWGN) channel.

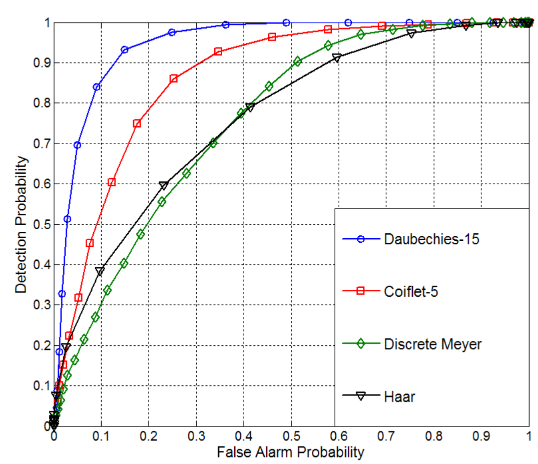

Fig. 3. ROC in WPSE for different standard wavelet families. N_c=128; Number of input samples=12800; Level of decomposition=7; SNR=0 dB.

Fig.3 shows the variation of ROC in WPSE for different standard wavelet families. It is seen that Daubechies wavelet is having the best performance because of its better frequency selectivity than the others. But performance of Haar wavelet is poor because of its small length of support. In the above analysis all the wavelets used were orthogonal.
In the next case both orthogonal and non-orthogonal wavelets are used and the result is shown in Fig.4.

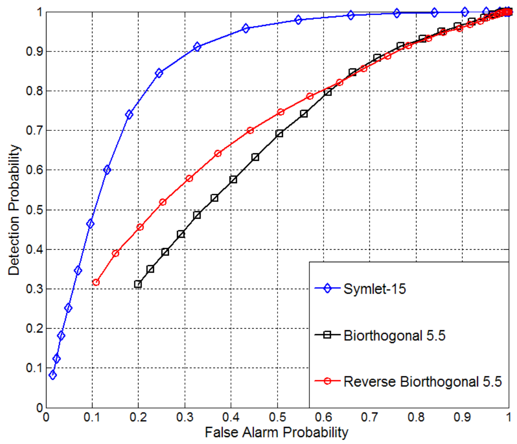

Fig. 4. ROC in WPSE for different standard wavelet families. N_c=128; Number of input samples=12800; Level of decomposition=7, SNR=0 dB.

It is clear that as biorthogonal and reverse biorthogonal wavelets are not orthogonal, the performance of ROC with Symlet-15 wavelet is much better than biorthogonal and reverse biorthogonal. This analysis shows that orthogonality of any wavelet filter is one of the main criteria for achieving a good frequency selectivity for WPSE.

In Fig.5 the variation of ROC performance is shown with the length of wavelet filters.

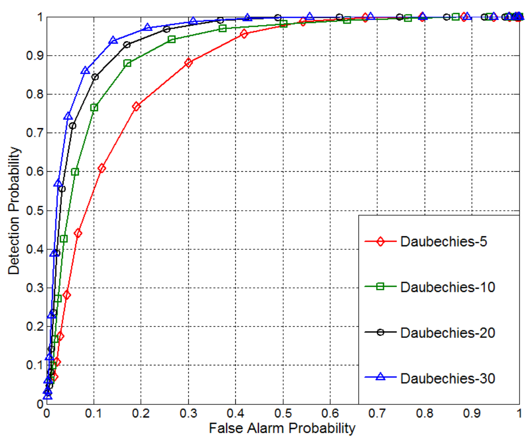

Fig. 5. ROC in WPSE for different length of wavelet filters. N_c=128; Number of input samples=12800; Level of decomposition=7, SNR=0 dB.

From Fig.5 it is seen that as the length of wavelet filters increases the performance gets better. This is due to the reduction of variance of the estimated PSD in the unoccupied band and the improvement of frequency resolution of wavelet transform with increasing length of wavelet filters.

In Fig.6 the variation of ROC with the level of decomposition of wavelet packet transform is shown. It is seen that as the level of decomposition increases the ROC performance deteriorates. The total band of $[0, \pi]$, used for simulation consists of N_c single tones as primary users, which are randomly activated or deactivated. As the level of decomposition (L) increases the number of single tones or primary users is also increased as we have assumed $N_c = 2^L$. At the high level of decomposition, the frequency selectivity for wavelet packet-based estimation of any single tone increases but simultaneously the amount of side lobe power for individual single tone estimation is also increased. Thus, in case of higher decomposition level the over all false alarm rate for the entire band increases.

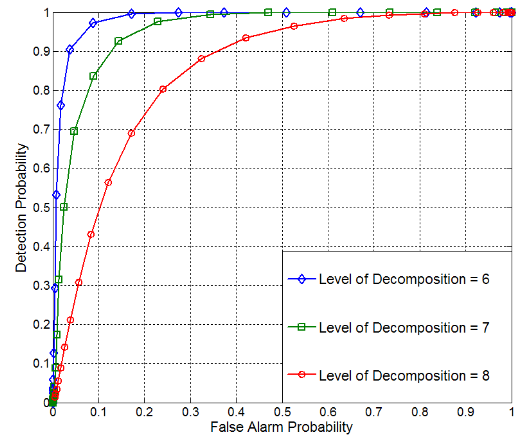

Fig. 6. ROC in WPSE for different level of decomposition; $N_c = 2^L$; Number of input samples=12800; Wavelet=Daubechies-15, SNR=0 dB.

B. Variation of ROC with different Signal-to-Noise ratios

In the previous section the signal-to-noise ratio (SNR) was kept constant for all the analysis. The theoretical variation of probability of correct detection and probability of false alarm with SNR can be achieved from equation (4) and (5). In this section, the variation of the detection and false alarm probabilities for WPSE is studied with different SNR and the results are compared with theoretical values. Fig.6 shows the nature of ROC for both theoretical and WPSE. Noise added is assumed to be additive white Gaussian (AWGN).

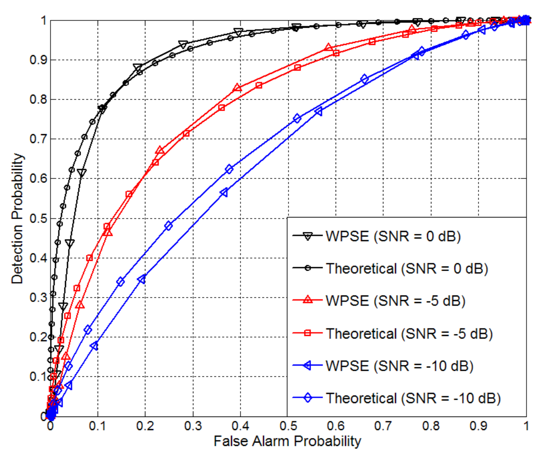

Fig. 7. Variation of ROC with SNR; N_c=128; Number of input samples=12800; Wavelet=Daubechies-10.

Fig.7 illustrates both theoretical ROC and ROC developed from WPSE. In case of WPSE it is seen that detection performance improves with increasing SNR which is analogous to the theoretical results of eq. (4) and (5).

C. Study of ROC with reduced number of samples

As only a small portion of any spectrum is generally occupied at a certain time, instead of all, only a small number of wavelet packet coefficients can be effectively used to detect the presence of the primary user. In Fig.2 a model for primary user detection using less number of coefficients is proposed. The amount of sensing measurements reduction is given by the coefficient selection factor i.e. compression rate ($M = R/2^L$). R is the number of selected coefficients for detection for any level of decomposition L. It is clear that as M reduces, lesser number of sensing measurements is required for the primary user detection.

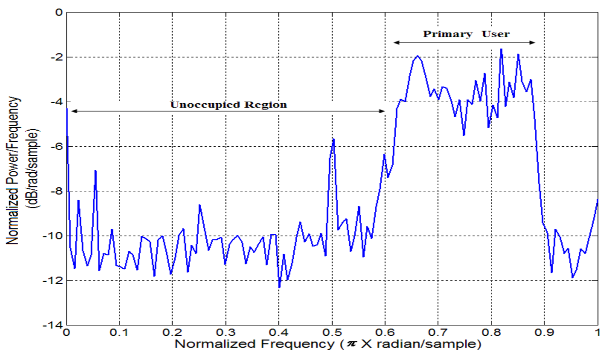

Fig. 8. Wavelet packet based estimate of $[0,\pi]$ spectrum whose 30% is occupied by primary user; SNR=0 dB; Wavelet= Daubechies-20

To study the detection performance of the developed model we considered that only a small portion of the entire spectrum with normalized frequency of 0-π (radian/sample), is occupied by the primary user as shown in Fig.8. The approximate width 0.3π is chosen arbitrarily. The width of the partial sub-band is kept intentionally low (30% of the entire spectrum) keeping the significant underutilization of radio spectrum in mind. Keeping the width of the occupied portion of the spectrum constant in the simulation, the location of this occupied partial sub-band is varied randomly through out the entire spectrum and detection and false alarm probabilities are calculated out of 1000 iterations of primary user spectral locations. The decision threshold for detection is kept constant to -10 dB. The threshold for wavelet packet coefficient selection (γ) is varied from -30 to -3 dB resulting different compression rates. Fig. 8 exhibits the variation of detection performance with the compression rate ($R/2^L$) with Daubechies-15 and Symlet-20 wavelet families.

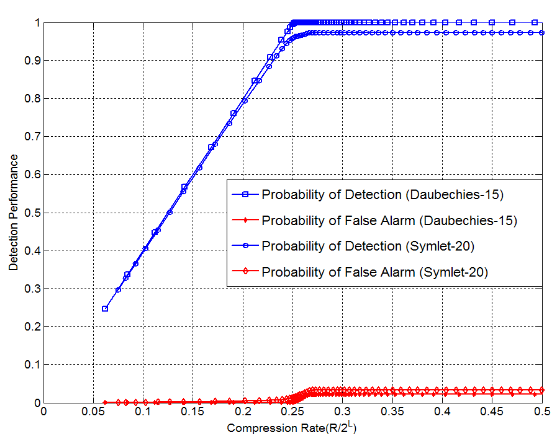

Fig. 9. Variation of detection performance with compression rate; SNR=0 dB.

From Fig.9 it is seen that only 25-30% of the total coefficient can achieve 100% probability of detection for Daudechies-15 wavelet. The false alarm probability is almost constant at 0.01-0.02, which is analogous to theoretical results because of the fixed value of the decision threshold (λ) in (4). The slight increase of false alarm probability near the compression rate of 0.25 is due to side lobes of wavelet packet based estimator particularly at lower threshold levels for coefficient selection (γ).

V. Conclusion and Future Research

Variation of the ROC for WPSE with various types of wavelet families and their properties are studied. It is observed that orthogonality of the wavelet filters is one of the major criteria for achieving a good detection performance. Secondly, as the length of the wavelet filters is increased, the frequency selectivity is also improved giving rise to better detection performances. However, as the level of decomposition is increased the performance deteriorates due to greater side lobes of the estimate though the frequency selectivity is improved. The improvement of detection performance with increasing signal-to-noise ratio of the input signal is studied for both WPSE (simulated) and the theoretical ROC. Finally, it has also been exhibited that the wavelet packet coefficient selection based on average energy threshold reduces the number of sensing measurements for detection of primary user.

Studying the dynamic behavior of the proposed model for the dynamic spectrum access (DSA) applications and study of ROC of present scenario in different propagation environment and channel conditions indicate the future scope of the work. In the ROC performance evaluation with reduced number of measurements, coefficients were selected based on average energy threshold. Future research may involve random selection of wavelet packet coefficients and appropriate wavelet basis selection to establish WPSE in the light of mathematical theory of compressed sensing [16],[17].

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