Improved Blind Separation Algorithm for Overlapping Secondary Surveillance Radar Replies

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***Abstract*—The secondary surveillance radar (SSR) is a transponder system used in air-traffic control. With the increase in air-traffic, replies from airplanes may overlap in time at ground station receivers, which results in loss of all replies for classic receivers. Blind source separation algorithms were proposed to separate such a mixture by the properties of SSR replies. Two known algebraic algorithms, the Manchester decoding algorithm (MDA) and the multishift zero-constant modulus algorithm, have the best performance but they still have performance degradation in cases with either small overlapping ratios or equal residual carrier frequencies. In this paper, we propose a modified subspace intersection method based on signed URV decompositions to preprocess the received data matrix for MDA. The proposed algorithm works on three successive time slots with the target time slot locating in the center and significantly improves the performance of MDA in all cases. Especially, in cases with small overlapping ratios, it touches the upper bound of the performance it can achieve. It shows the most stable performance compared with previous algorithms.**

I. Introduction

The secondary surveillance radar (SSR) is currently used in air-traffic control. In SSR ground stations, a rotating scanning beam is used to interrogate airplanes. The airplanes respond by transmitting SSR replies, burst pulse trains, modulated on carriers at 1090MHz. With the increase in air-traffic, the received replies may overlap in time at ground station receivers, which results in loss of all replies if simple receivers are used. By using antenna arrays in the ground stations, these replies can possibly be separated by methods such as blind beamforming.

To separate the overlapping replies, several algorithms were proposed, such as [1], [2], [3], [4], [5], [6]. Two algebraic blind source separation algorithms in [1] have the best performance at present, which are the Manchester decoding algorithm (MDA) and the multishift zero-constant modulus algorithm (MS-ZCMA). However, both algorithms have limitations. M-DA successfully separates replies when the replies are fully overlapping, but it fails when the replies trend to be non-overlapping. MS-ZCMA works well when the replies have unequal residual carrier frequencies (RCF) but it always fails with equal RCFs.

In this paper, we propose a new algorithm using subspace intersection (SI) based on signed URV decompositions (SURV) [7], called SI+SURV, to improve the performance of MDA. In this algorithm, we aim at recovering the target reply in the target time slot overlapped by interference replies from adjacent time slots. The principle is to convert the nonstationary data matrix for MDA into a stationary one. This algorithm initially works on the received data matrix across three successive time slots. By properly dividing this data matrix into submatrices, common signals and interferences are categorized. SI+SURV is then applied on these submatrices to find the interference-free subspace for the common signals. After projection onto this subspace, the replies with small overlapping ratios are filtered out in the data matrices. Next, the submatrices corresponding to the three time slots are added together to form a stationary data matrix for MDA, which finally finds the beamformer for the target reply.

II. Data Model and Preliminary

SSR replies [8]: SSR communication has two different protocols: mode A/C and mode S. Mode A/C is an old protocol and will be replaced by mode S. A mode S reply frame contains either 56 or 112 binary symbols. These bits are encoded by Manchester coding, 0 to $[0, 1]$ and 1 to $[1, 0]$. A preamble $[1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0]$ is added to the head of the encoded bits to form the source binary sequence $\bar{\mathbf{b}}$. Then $\bar{\mathbf{b}}$ is modulated by a kind of pulse-amplitude modulation (PAM) (pulses are absent for 0s) [8] into the transmitted signal

$$z(t) = \sum_{n=1}^{N_p} \bar{b}_n p(t - nT), \tag{2.1}$$

where $\bar{b}_n$ is the n-th bit of $\bar{\mathbf{b}}$, $p(t)$ is a PAM pulse, $N_p \in \{128, 240\}$, and $T = 0.5\mu s$ is the symbol period. In the following part of this paper, we call $\mathbf{z} = \{z(nT)\}_{1 \le n \le N_p} \in \mathbb{R}^{1 \times N_p}$ as a "SSR reply". We use denotation $z_k(t)$ and $\mathbf{z}_k$ for the k-th reply.

Scenario: Consider d mode S replies of equal length received by an antenna array of M ($M \ge d$) elements in the three successive time slots (See Fig.2.1). Data matrices of the three time slots are given by $\mathbf{X}_k \in \mathbb{C}^{M \times N_s}, k = 1, 2, 3$. Two extension matrices for the first and the third slots are given by $\mathbf{X}_{ek} \in \mathbb{C}^{M \times N_e}, k = 1, 3$. We will call these time slots by their corresponding data matrices. $\mathbf{X}_2$ is the target slot, the start of which is determined from rank tracking algorithms [9] (A rank rising edge indicates the coming of one target reply). The target reply $\mathbf{z}_1$ is contained in $\mathbf{X}_2$. If more than one reply appear in $\mathbf{X}_2$, we select one of them to be the target reply. Two safe margins, each N_a bits long (although not equal in practice), are assumed to the both ends of $\mathbf{z}_1$ inside $\mathbf{X}_2$. Let

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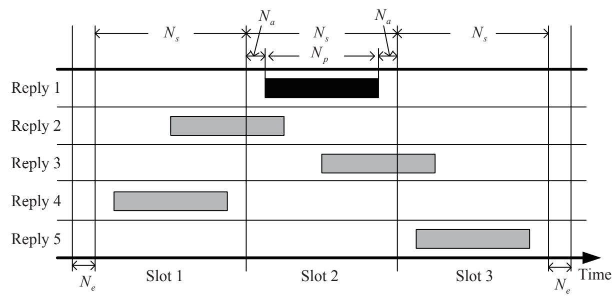

Fig. 2.1. The three successive time slots.

$\mathbf{X} = [\,\mathbf{X}_{e1}\ \mathbf{X}_1\ \mathbf{X}_2\ \mathbf{X}_3\ \mathbf{X}_{e3}\,]$. We have the data model

$$\mathbf{X} = \tilde{\mathbf{A}}\tilde{\mathbf{B}}(\mathbf{S} \odot \mathbf{\Phi}) + \mathbf{N}, \tag{2.2}$$

where $\odot$ is the Schur-Hadamard (pointwise multiplication) operator, $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N] \in \mathbb{C}^{M\times N}$, $N = 3N_s + 2N_e$, $\mathbf{x}_n = \mathbf{x}(nT_s)$, $1 \le n \le N$, is the received data matrix, $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}_1, \tilde{\mathbf{a}}_2, \ldots, \tilde{\mathbf{a}}_d] \in \mathbb{C}^{M\times d}$ is the array response matrix, $\tilde{\mathbf{B}} = \mathrm{diag}\{\tilde{b}_1, \tilde{b}_2, \cdots, \tilde{b}_d\} \in \mathbb{R}^{d\times d}$ contains the source power, $\mathbf{S} = [\mathbf{s}_1^H, \mathbf{s}_2^H, \ldots, \mathbf{s}_d^H]^H \in \mathbb{C}^{d\times N}$ ("H" is the conjugate transpose) is the source data matrix,

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \varphi_1^1 & \cdots & \varphi_1^{N-1} \\ 1 & \varphi_2^1 & \cdots & \varphi_2^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \varphi_d^1 & \cdots & \varphi_d^{N-1} \end{bmatrix}, \varphi_k = \mathrm{e}^{j2\pi\Delta f_k T_s} \tag{2.3}$$

contains the phase shifts caused by RCFs Δf_k (up to $\pm$3MHz) for sources, and $\mathbf{N} \in \mathbb{C}^{M\times N}$ is the white Gaussian noise matrix with covariance $\mathbf{R}_n = \mathrm{E}(\mathbf{n}\mathbf{n}^H) = \sigma_n^2\mathbf{I}$. T_s is the sample period. We set $T_s = T$.

Taking the start of $\mathbf{z}_1$ as the reference time point, replies have relative delays τ_k ($\tau_1 = 0$) so that $\mathbf{s}_k = \tilde{z}_k(nT_s - \tau_k)$, where $\tilde{z}_k(t)$ is the extension of $z_k(t)$ to the considered time interval. Although synchronization of all replies is impossible in overlapping cases, it does not affect the performance of MDA since it is based on the special property of SSR replies which holds for arbitrary delays [8]. Without loss of generality, we assume all the received replies have the same timing and integer delays $\tau_k = nT$, $n \in \mathbb{Z}$. Especially, for the replies overlapping $\mathbf{z}_1$, we define overlapping ratios (OVR) r_k by

$$r_k = l_k/N_p, -N_p \le \tau_k \le N_p, \tag{2.4}$$

where $l_k = N_p - |\tau_k|$ is the number of overlapping samples between $\mathbf{z}_k$ and $\mathbf{z}_1$.

$\tilde{\mathbf{A}}$ and $\mathbf{S}$ are unknown. Our objective is to find the beamformer $\mathbf{w}_1 \in \mathbb{C}^{M\times 1}$ for $\mathbf{s}_1$ (also for $\mathbf{z}_1$) such that

$$\hat{\mathbf{s}}_1 = \mathbf{w}_1^H\mathbf{X}, \tag{2.5}$$

where $\hat{\mathbf{s}}_1$ is the estimate of $\mathbf{s}_1$.

The additional assumptions are

1. Columns of $\tilde{\mathbf{A}}$ are linearly independent.
2. The replies are generated from random i.i.d. binary data.
3. The noise-free problem ($\mathbf{N} = \mathbf{O}$) is essentially identifiable [8], [10].

Manchester decoding algorithm [1]: This algorithm is based on the Manchester encoding property: A received mode S reply satisfies the following property independent of arbitrary delays

$$z(t-T)z(t)z(t+T) = 0, \forall t \in \mathbb{R}. \tag{2.6}$$

This property still holds for $\tilde{z}_k(t)$ and $\mathbf{s}_k$. MDA computes the third order tensors of the received data $\mathbf{X}$ based on the above property to form a tall matrix $\mathbf{P}$. The beamformers $\mathbf{W}$ are found by solving a joint diagonalization problem that finds the vectors best spanning $\ker(\mathbf{P})$.

Signed URV decomposition [9]: SURV implicitly computes

$$\mathbf{C}\mathbf{C}^H = \mathbf{Y}_1\mathbf{Y}_1^H - \mathbf{Y}_2\mathbf{Y}_2^H = \mathbf{A}\mathbf{A}^H - \mathbf{B}\mathbf{B}^H \tag{2.7}$$

and the column subspace bases of the indefinite matrix $\mathbf{C}\mathbf{C}^H$ from the following factorization

$$M\,[\overset{+}{\mathbf{Y}_1}\ \overset{-}{\mathbf{Y}_2}]\,\mathbf{\Theta} = [\overset{d_1}{\mathbf{Q}_\mathbf{A}}\ \overset{d_2}{\mathbf{Q}_\mathbf{B}}]\,[\overset{+}{\underset{d_1}{\mathbf{L}_\mathbf{A}}}\ \overset{+}{\mathbf{0}}\ \overset{-}{\underset{d_2}{\mathbf{L}_\mathbf{B}}}\ \overset{-}{\mathbf{0}}], \tag{2.8}$$

or in a compact form

$$[\mathbf{Y}_1\ \mathbf{Y}_2]\,\mathbf{\Theta}' = [\mathbf{Q}_\mathbf{A}\ \mathbf{Q}_\mathbf{B}]\,[\mathbf{L}_\mathbf{A}\ \mathbf{L}_\mathbf{B}], \tag{2.9}$$

where the sign $+$ and $-$ above matrices denote the positive and negative signatures of the corresponding columns, $[\mathbf{A}\ \mathbf{B}] = [\mathbf{Q}_\mathbf{A}\ \mathbf{Q}_\mathbf{B}]\,[\mathbf{L}_\mathbf{A}\ \mathbf{L}_\mathbf{B}]$, $\mathbf{\Theta}'$ is part of the $\mathbf{J}$-unitary matrix $\mathbf{\Theta}$ [11] for corresponding nonzero columns, $[\mathbf{L}_\mathbf{A}\ \mathbf{L}_\mathbf{B}] \in \mathbb{C}^{M\times M}$ is a lower triangular matrix, and $[\mathbf{Q}_\mathbf{A}\ \mathbf{Q}_\mathbf{B}] \in \mathbb{C}^{M\times M}$ is a unitary matrix. We call the subspace spanned by the columns of $\mathbf{Q}_\mathbf{B}$ (orthonormal basis) or $\mathbf{B}$ "the negative subspace" and the subspace spanned by the columns of $\mathbf{A}$ "the positive subspace". $\mathbf{Q}_\mathbf{A}$ is the orthogonal complement of the negative subspace. d_1 is the dimensionality of the positive subspace and d_2 is the dimensionality of the negative subspace. $M = d_1 + d_2$. SURV provides subspace estimates with good properties as

$$\mathrm{ran}\{\mathbf{Q}_\mathbf{B}\} \subset \mathrm{ran}\{\mathbf{Y}_2\}. \tag{2.10}$$

Subspace intersection based on SURV [7]: SI+SURV finds the interference-free subspace for the common signals in two observations with different interferences. Steps are listed as follows.

1. Collect two received noisy data matrices
$$\mathbf{Y}_k = \tilde{\mathbf{A}}_s\mathbf{S}_k + \tilde{\mathbf{A}}_{f_k}\mathbf{F}_k + \mathbf{N}_k \in \mathbb{C}^{M\times N_y}, k = 1, 2, \tag{2.11}$$
where $\tilde{\mathbf{A}}_s$ and $\tilde{\mathbf{A}}_{f_k}$ are the array response matrices of common signals and interferences, respectively.
2. Let $\mathbf{Y} = [\mathbf{Y}_1\ \mathbf{Y}_2]$ and project $\mathbf{Y}$ onto its principal subspace $\underline{\mathbf{Y}} = \mathbf{U}_p^H\mathbf{Y} = [\underline{\mathbf{Y}}_1\ \underline{\mathbf{Y}}_2]$.
3. Form two SURVs in compact forms
$$[\alpha\overset{+}{\underline{\mathbf{Y}}_1}\ \overset{-}{\underline{\mathbf{Y}}_2}\ \overset{-}{\gamma\mathbf{I}}]\,\mathbf{\Theta}'_1 = [\mathbf{Q}_{\mathbf{A}_1}\ \mathbf{Q}_{\mathbf{B}_1}]\,[\overset{+}{\mathbf{L}_{\mathbf{A}_1}}\ \overset{-}{\mathbf{L}_{\mathbf{B}_1}}], \tag{2.12}$$
$$[\alpha\overset{+}{\underline{\mathbf{Y}}_2}\ \overset{-}{\underline{\mathbf{Y}}_1}\ \overset{-}{\gamma\mathbf{I}}]\,\mathbf{\Theta}'_2 = [\mathbf{Q}_{\mathbf{A}_2}\ \mathbf{Q}_{\mathbf{B}_2}]\,[\overset{+}{\mathbf{L}_{\mathbf{A}_2}}\ \overset{-}{\mathbf{L}_{\mathbf{B}_2}}], \tag{2.13}$$
where $|\alpha| \ge \frac{\sqrt{N_y}+\sqrt{M}}{\sqrt{N_y}-\sqrt{M}}$, $\gamma = \sqrt{\alpha^2-1}\gamma_n$ is a threshold for noise power compensation in the negative subspace for the low signal-to-noise ratio (SNR) case with large α, and

$\gamma_n = \beta\sigma_n(\sqrt{N_y} + \sqrt{M})$ is a threshold used in subspace tracking algorithms [9], [12], [13].

4. *Subspace intersection* [14]: Compute SVD $[\mathbf{Q}_{\mathbf{A}_1}\ \mathbf{Q}_{\mathbf{A}_2}] = \mathbf{U}\mathrm{diag}(\sigma_k)\mathbf{V}^H$. Let $\mathbf{U'}_c = \mathbf{U}(:, 1:\hat{d}_c)$ such that $\sigma_k \geq \sqrt{2},\ 1 \leq k \leq \hat{d}_c$, and $\sigma_k < \sqrt{2},\ k \geq \hat{d}_c{+}1$. Then $\mathrm{ran}\{\mathbf{U'}_c\}{=}\mathrm{ran}\{\mathbf{Q}_{\mathbf{A}_1}\}\bigcap\mathrm{ran}\{\mathbf{Q}_{\mathbf{A}_2}\}$. Let $\mathbf{U}_c = \mathbf{U}_p\mathbf{U'}_c$. We have $\mathrm{ran}\{\mathbf{U}_c\} = \mathrm{ran}\{\tilde{\mathbf{A}}_{f_1}, \tilde{\mathbf{A}}_{f_2}\}^{\perp}$. The columns of $\mathbf{U}_c$ are the orthonormal basis of the interference-free subspace for the common signals in $\mathbf{Y}$.

In the considered scenario, signals and interferences are categorized by dividing $\mathbf{X}$ into submatrices. In principle, we treat the target reply and other replies nearly fully overlapping it as the common signals.

III. Improved Separation Algorithm

In this section, we propose a new preprocessing algorithm for MDA. As discussed in [1], MDA has problems of convergence with small OVRs, where $\ker(\mathbf{P})$ becomes larger and the solution is no longer unique. To get rid of this imperfection, we propose to add the three matrices $\mathbf{X}_k, k = 1, 2, 3$ together to form a stationary data matrix. Previous to this addition, we perform SI+SURV on $\mathbf{X}$. This preprocessing provides two advantages:

1. No partially overlapping replies in the final data matrix.
2. Replies with small OVRs ($r_k < 0.5$) will be filtered out before MDA.

A. The Algorithm

Given the data model defined in Section II, we apply a modified SI+SURV on the following matrices.

$$\mathbf{Y}_1 = [\,\mathbf{X}_{e1}\ \mathbf{X}_1\ \mathbf{X}_2\,], \mathbf{Y}_2 = [\,\mathbf{X}_3\ \mathbf{X}_{e3}\,],\ \text{for (2.12).} \quad (3.1)$$

$$\mathbf{Y}_1 = [\,\mathbf{X}_{e1}\ \mathbf{X}_1\,], \mathbf{Y}_2 = [\,\mathbf{X}_2\ \mathbf{X}_3\ \mathbf{X}_{e3}\,],\ \text{for (2.13).} \quad (3.2)$$

Algorithm 1: Improved blind separation algorithm for SSR.

Step 1. Do SI+SURV to find $\mathbf{U}_c$ and $\hat{d}_c$.

Step 2. If $\hat{d}_c = 0$, $\mathbf{U}_c = \mathbf{I}$. Then do projection $\mathbf{X'}_k = \mathbf{U}_c^H\mathbf{X}_k, k = 1, 2, 3$.

Step 3. Do the addition $\mathbf{X'} = \sum_{k=1}^{3}\mathbf{X'}_k$.

Step 4. Do MDA on $\mathbf{X'}$ to find the beamformers $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots \mathbf{w}_{\hat{d}_c}] \in \mathbb{C}^{M\times\hat{d}_c}$.

For a finite number of samples (N_s, N_e), α must satisfy [7]

$$|\alpha| \geq \frac{N_s + N_e}{2N_s + N_e} \cdot \frac{\sqrt{N_s + N_e} + \sqrt{M}}{\sqrt{N_s + N_e} - \sqrt{M}}. \quad (3.3)$$

We set $\gamma = 0$ because in the real scenario the difference in power between replies is not big and SNR is normally high. Note that in Step 3, it brings three times more noise power into $\mathbf{X'}$ but it is worthy doing as mentioned before. The computational complexity of SI+SURV is of $O(M^2N)$.

B. Parameter Selection

The values of N_a and N_e need to be discussed. N_a shows the uncertainty of determining the start of the target reply due to the limitation of the practical rank tracking algorithm [9], where N_a is half the length of the sliding window N_w. N_a can not be set too large (it also puts limitation on N_w), otherwise the data in $\mathbf{X'}$ become nonstationary. From the results in [1], N_a must satisfy $N_a \leq 0.2N_p$. N_e is set to collect enough samples for replies partially overlapping the two ends of $\mathbf{X}$. Under the assumption $M \geq d$, we can set $N_e = N_p$ at most but in the following simulation we set $N_e = N_p/4$.

TABLE 4.1
Parameters of replies.

	DOA[1]	Delay	RCF (MHz)
Reply 1	$30°$	$\tau_1 = 0$	-1.0
Reply 2	$-20°$	$0 \leq \tau_2 \leq N_p$	2.0
Reply 3	$50°$	$-N_p \leq \tau_3 \leq 0$	1.2
Reply 4	$10°$	$\tau_4 > N_p$	0.3
Reply 5	$-30°$	$\tau_5 < -N_p$	0.0

[1] Direction of arrival.

IV. Simulation Results

In this section, we compare the performance of our proposed algorithm with original MDA and MS-ZCMA.

Consider 5 replies in the scenario defined in Section II. All replies have equal power. A linear antenna array with elements spaced at half wavelengths is used. $M = 10$, $f_s = 2$MHz ($T_s = 0.5\mu s$), $N_p = 100$, $N_s = 120$, $N_a = 10$ ($N_w = 20$), $N_e = 25$, $\beta = 1.3$. Other parameters are listed in Table 4.1. Reply 2 and 3 overlap the target reply, Reply 1, and will be shifted according to given OVRs (we set $r_2 = r_3$). Reply 4 and 5 do not overlap Reply 1 and stay fixed at given time positions.

Signal and interference powers are defined for every symbol. The signal-to-interference ratio SIR $:= 10\log_{10}(\sigma_1^2/\sigma_f^2)$, and SNR $:= 10\log_{10}(\sigma_1^2/\sigma_n^2)$, where σ_1^2 and σ_f^2 are the symbol power of the target reply and other replies, respectively. The performance measure is the residual signal-to-interference-plus-noise ratio (SINR), which is found as

$$\mathrm{sinr}(\mathbf{a}, \mathbf{w}) := \frac{\mathbf{w}^H(\mathbf{a}\mathbf{a}^H)\mathbf{w}}{\mathbf{w}^H(\bar{\mathbf{A}}\bar{\mathbf{A}}^H - \mathbf{a}\mathbf{a}^H + \sigma_n^2\mathbf{I})\mathbf{w}}, \quad (4.1)$$

$$\mathrm{SINR} := \max(\mathrm{sinr}(\bar{\mathbf{a}}_1, \mathbf{w}_1), \ldots, \mathrm{sinr}(\bar{\mathbf{a}}_1, \mathbf{w}_{\hat{d}_c})), \quad (4.2)$$

where $\bar{\mathbf{A}} = \tilde{\mathbf{A}}\tilde{\mathbf{B}}$, $\bar{\mathbf{a}}_1$ is the first column of $\bar{\mathbf{A}}$.

The algorithms in figures are named as our proposed algorithm ("SI+MDA"), the directly applied Manchester decoding algorithm ("MDA"), the multishift zero-constant modulus algorithm ("MS-ZCMA"), and the standard minimum mean square error algorithm ("SMMSE") on known $\mathbf{s}_1$. MDA, MS-ZCMA and SMMSE are directly applied on $\mathbf{X}$.

Fig. 4.1 shows the averaged SINR of algorithms at given OVRs with equal RCFs. SMMSE gives the upper bound. MDA and MS-ZCMA both show unstable performance in the entire region, while SI+MDA shows much more stable performance. SI+MDA touches the upper bound when OVR is small and turns into a "flat" performance when OVR goes over 0.5, where the gap between SI+MDA and SMMSE is due to the tripled noise power.

Fig. 4.2 shows a similar result to Fig. 4.1 but here RCFs are unequal. In Fig. 4.2, MS-ZCMA is expected to work better than in Fig. 4.1 but it still gives unstable performance because

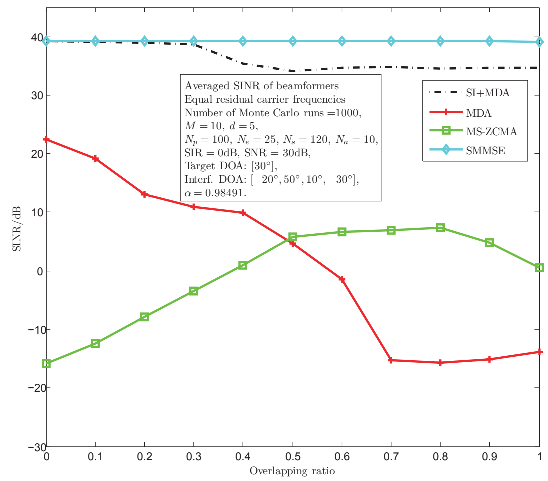

Fig. 4.1. Performance of algorithms at given OVRs with equal RCFs.

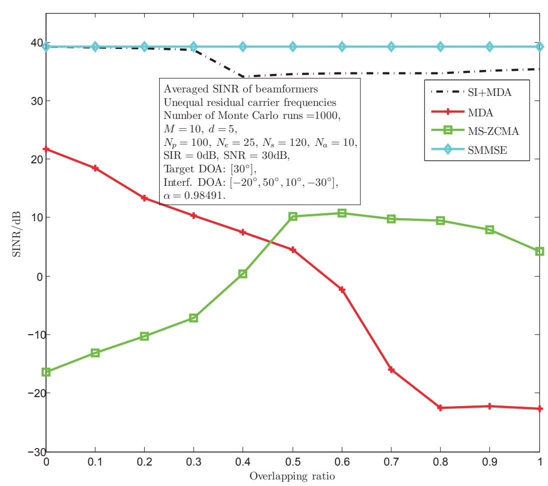

Fig. 4.2. Performance of algorithms at given OVRs with different RCFs.

it depends on optimized shifts, which is hard to explore, and moreover, the number of interferences are more than in [1]. Contrarily, SI+MDA sill gives the same performance as in Fig. 4.1.

Fig. 4.3 shows the averaged SINR of algorithms at given SNRs with equal RCFs. OVR is fixed to 0.1. It is seen from Fig. 4.3 that SI+MDA gives the same performance to SMMSE while MDA and MS-ZCMA give much worse performance.

V. Conclusion

We proposed an improved blind separation algorithm for overlapping secondary surveillance radar replies. This algorithm significantly improved the performance of the Manchester decoding algorithm in all cases, especially in cases with small overlapping ratios. This algorithm was shown robust to equal or unequal residual carrier frequencies. Contrary to the previous algorithms, this algorithm worked well on a larger number of replies with any overlapping patterns. The presented idea can be also applied to separate signals in other similar asynchronous systems, e.g. constant modulus signals.

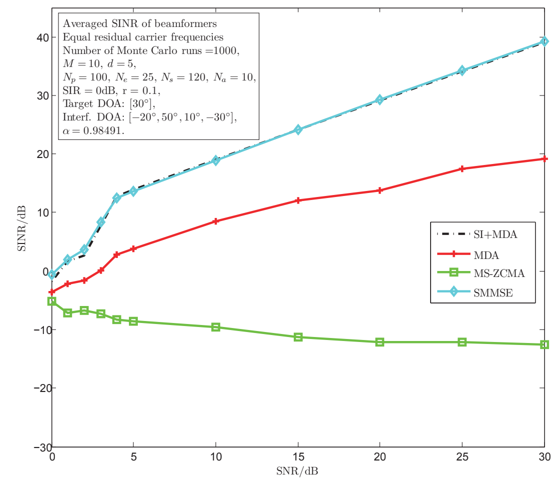

Fig. 4.3. Performance of algorithms at given SNRs with equal RCFs.

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