Adaptive Precoding for Downstream Crosstalk Precancelation in DSL Systems Using Sign-Error Feedback

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***Abstract*—The performance of many very high bit rate digital subscriber line (VDSL) systems is limited by the effects of crosstalk among the wires in a bundle. For the downstream, a precoder can compensate for this, provided that some crosstalk channel information is available at the transmitter. In this paper, we investigate the issue of precoder design based on a particular type of feedback: it is assumed that each receiver sends back the (complex) sign of the noise on the symbol detection as a way to provide the needed information on crosstalk channels. We derive an algorithm that makes use of this limited information in an adaptive way to iteratively compute the precoder. The performance of the proposed algorithm is extensively analyzed, both with theoretical evaluations and simulation results. The algorithm is also compared to other solutions, showing the efficient performance for the limited amount of overhead it requires.**

***Index Terms*—Adaptive algorithm, channel estimation, crosstalk, precoding, vectored DSL.**

I. Introduction

DUE to the use of higher bandwidths and shorter loops, the FEXT (far end crosstalk) is becoming the main degradation in some DSL systems such as VDSL2 (very high bit rate digital subscriber line). For this reason, a number of precancelation techniques have been designed to decrease the effect of FEXT [1]–[4] in downstream, using the coordination at the central office (CO) or optical network unit (ONU). A suitable precoder is introduced on the vector of transmitted symbols. These methods assume that a good estimate of the crosstalk channel matrix is available. Essentially, the precoder is the inverse of this matrix.

We focus on downstream in this paper, as the upstream issue is simpler to solve. Due to the nature of the DSL network, the transmitters in the downstream are collocated and can be "coordinated" at the central office, while the receivers are physically separated and thus are uncoordinated. To be able to compute the precoder, combined knowledge on the crosstalk channel characteristics of each transmitter towards each user is required. Thus, it is necessary to centralize the available information that is present only distributed at the independent receivers. This requires feedback from the receivers.

One straightforward way to solve the problem is to use a set of pilot symbols, sent periodically, to perform the tracking of the downstream channels at the CPE. An example of this solution, applied to the VDSL system is analyzed in [5]. In [6], it is proposed to simplify the precoder to its off-diagonal elements only, and an LMS tracking algorithm is proposed that converges to the optimal off-diagonal solution. This is also essentially a pilot-based solution. These methods use part of the useful bit rate as pilot symbols and, in addition, the information about the estimates needs to be sent back to the CO periodically to update the precoder. So this may lead to a large overhead. In order to try to limit this overhead, some methods have been proposed that only require to feedback the *sign* of the error samples (slicer errors) at the receiver [7], [8]. The entire estimation processing is transferred at the central office. Recently, it has also been proposed in [9] to use SNR measurements, introducing small perturbations on the transmitted signal, and observing their effect on the received SNR, seen from the CPE side.

In this paper, we revisit the method that was briefly described in [8], and that is based on the feedback of the sign of the error samples from the receiver. In this method, the transmitter uses the combination of the feedback and of the knowledge of the transmitted symbols to iteratively compute the precoder. We propose a modified version of the algorithm that considerably decrease the complexity with respect to what was presented in [8], and we analyze the performance of the proposed (simplified) method. Approximate performance expressions are derived to quantify the potential of the method, and they are compared to simulation results to be validated. It is shown how these expressions can also easily be used to help setting the parameters of the method. Finally, the performance is compared to other existing methods.

II. System Model

A. Channel Model and Precoder

We consider a DSL environment where DMT modulation is employed. It is also assumed that the cyclic prefix is long enough and that the individual user signals are transmitted synchronously from the CO (central office) so that after DMT demodulation the channels (including crosstalk) are free of intersymbol interference and intercarrier interference. The considered model

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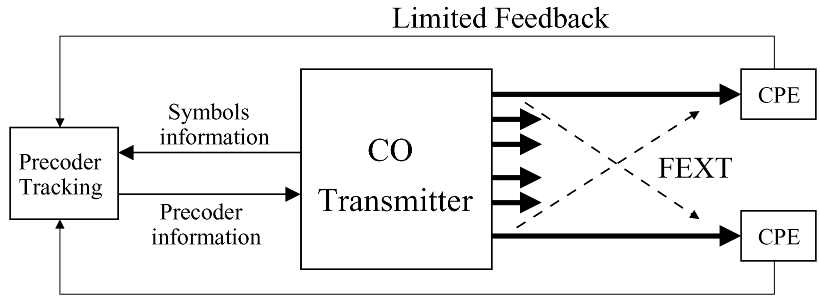

Fig. 1. Principle of the structure for the iterative precoder algorithm.

is depicted in Fig. 1. All operations can be applied tone-wise so we focus on one given tone only. There are N lines in the binder. The information symbols to be transmitted by the different users are denoted by u_i, $i = 1, \dots, N$ and are grouped into a vector $\mathbf{u} = [u_1 \dots u_N]^T$. The variance of the symbols on line i is denoted by $\sigma_{u_i}^2$. For simplicity, we assume some normalization of the symbols so that $\sigma_{u_i}^2 = 1$ on all lines. For the tone of interest, the channel model is written as

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{n}. \tag{1}$$

Here, $\mathbf{x}$ and $\mathbf{y}$ are N-dimensional vectors of transmitted and received samples with entries corresponding to the different users (or equivalently, to the different lines). $\mathbf{C}$ is the $N \times N$ channel matrix for this tone, and $\mathbf{n}$ is the vector of noise samples at the different receivers. The additive noise is assumed to be Gaussian with independent elements. The noise variance for user (receiver) i is denoted by $\sigma_{n,i}^2$. In the model (1), the diagonal entries of $\mathbf{C}$ correspond to the line transmission (also called direct channel later in this paper), the off-diagonal entries correspond to crosstalk channels. The off-diagonal entries are always much smaller than the diagonal entries in the row (row-wise diagonal dominance).

Because the receivers are not collocated, the ith receiver only has access to the ith entry of $\mathbf{y}$ for detection and/or estimation purposes. To mitigate the effect of crosstalk in advance, the CO uses a precoder. We assume a linear precoder as presented in [1] and [4], and later improved in [3]. To this end, the CO designs a precoding matrix $\mathbf{F}$ and sends

$$\mathbf{x} = \mathbf{F}\mathbf{u} \tag{2}$$

on the different lines. A normalization needs to be imposed on the precoder to ensure that each line gets a fixed transmitting power. Therefore, each line of the matrix $\mathbf{F}$ must have a power equal to 1. In [1], [3], and [4], the precoder design is such that the combined precoder-channel matrix $\mathbf{CF}$ is diagonal, or

$$\mathbf{F}_0 = \alpha \mathbf{C}^{-1} \mathbf{C}_d \tag{3}$$

where the notation $\mathbf{C}_d$ denotes the diagonal matrix formed by keeping only the diagonal entries of $\mathbf{C}$, and where α is a normalization factor that allows to fulfil the power condition described above. In most cases, the coefficient α can be approximated to 1. The computation of this precoder requires an estimate of the crosstalk channel matrix.

When the precoder is active, a new overall model can be obtained to take into account the presence of the precoder:

$$\mathbf{y} = \mathbf{H}\mathbf{u} + \mathbf{n} \tag{4}$$
$$\mathbf{H} = \mathbf{C}\mathbf{F}. \tag{5}$$

B. Sign Feedback

As mentioned above, the algorithm studied here is based on a limited feedback (sign of the noise on the decisions). Even though the information content of this coarsely quantized signal is low, it is shown in [7] that it is sufficient to provide a reasonable estimate of the channel and hence it should also be sufficient to compute the precoder directly. The feedback used can be seen as a replacement for the pilot tones, requiring less bit rate than what is consumed by the pilots, but using upstream instead of downstream. Note that in practice, it will not be necessary to feedback every symbol on every tone. The crosstalk channels are varying slowly and it is thus acceptable to feedback the symbols once in a while only. In this section, the model of the feedback is first described. The algorithm is derived in the next section.

The principle of the feedback used in this scheme is the following. The received symbols on line i at the tone of interest is given by

$$y_i = H_{ii} u_i + \sum_{j \neq i} H_{ij} u_j + n_i \tag{6}$$

where H_{ij} is the corresponding entry in the matrix $\mathbf{H}$. After frequency equalization, assuming this equalization is accurate, the following decision variable is obtained

$$\hat{u}_i = u_i + \sum_{j \neq i} \frac{H_{ij}}{H_{ii}} u_j + \frac{n_i}{H_{ii}}. \tag{7}$$

The decision $\tilde{u}_i$ is taken as the closest symbol in the constellation. It is assumed that the decision is correct with very high probability. At the receiver, the decision noise (that is the noise that was present on the decision variable) is computed as

$$v_i = \hat{u}_i - \tilde{u}_i. \tag{8}$$

Finally, the principle used in this method is to feedback the sign of the decision noise (both real and imaginary parts):

$$v_i^{\mathrm{sgn}} = \mathrm{sign}(v_i). \tag{9}$$

Regrouping all the users simultaneously, it can be written as

$$\mathbf{y} = \mathbf{Cx} + \mathbf{n} = \mathbf{CFu} + \mathbf{n} \tag{10}$$
$$= \mathbf{Hu} + \mathbf{n} \tag{11}$$
$$\hat{\mathbf{u}} = \mathbf{H}_d^{-1}\mathbf{y} \tag{12}$$
$$\mathbf{v} = \hat{\mathbf{u}} - \mathbf{u} = \left(\mathbf{H}_d^{-1}\mathbf{CF} - \mathbf{I}\right)\mathbf{u} + \mathbf{H}_d^{-1}\mathbf{n} \tag{13}$$

where $\mathbf{I}$ denotes the identity matrix. As stated earlier, the notation $\mathbf{H}_d$ indicates the diagonal matrix constructed from the diagonal entries of $\mathbf{H}$.

III. Algorithm Derivation

We consider an adaptive update algorithm for the precoder, without resorting to explicit crosstalk channel estimation and inversion. To this end, we first have to select an objective function. In [8], a derivation was presented using the minimization of the mean square of the residual crosstalk interference at the receiver, under a fixed transmitted power constraint. The derivation based on this objective function however requires a few strong assumptions and leads to a complex algorithm. In this paper, we present another derivation, based on a slightly different criterion, which only relies on the diagonal dominance and uses standard methods. The obtained algorithm can be seen as a slightly modified version of the one in [8], but with a considerably lower complexity. The similarities and differences will be discussed in more detail below. It is however important to note that in practice, both versions exhibit identical performance.

The criterion used here aims at perfectly removing the crosstalk after precompensation (or equivalently diagonalizing the channel). Mathematically, this is also equivalent to zeroing out the correlation between the noise after detection $\mathbf{v}$ and the transmitted symbols $\mathbf{u}$. Hence, the following objective function can be used

$$J(\mathbf{F}) = \mathrm{E}[\mathbf{vu}^H] \tag{14}$$

where "$\mathrm{E}[\cdot]$" denotes the expectation operator, and superscript "^{H}" the complex conjugate transpose. In this expression, $\mathbf{v}$ is dependent on $\mathbf{F}$. As expected, if the diagonalizing precoder $\mathbf{F}_0$ (3) is used, we obtain $J(\mathbf{F}_0) = 0$. The algorithm will iteratively try to find the roots of the criterion (14), thereby zeroing the remaining crosstalk. Note that it is very similar to the orthogonality principle used for the minimum mean square error criterion.

In order to find these roots, we first investigate the expression of the criterion, and apply a few simplifications based on the property of (row-wise) diagonal dominance:

$$J(\mathbf{F}) = \mathrm{E}\left[\left(\mathbf{H}_d^{-1}\mathbf{CF} - \mathbf{I}\right)\mathbf{uu}^H + \mathbf{H}_d^{-1}\mathbf{nu}^H\right] \tag{15}$$
$$= \left(\mathbf{H}_d^{-1}\mathbf{CF} - \mathbf{I}\right) \tag{16}$$

where the symbol variance σ_u^2 has been assumed to be unity on all lines. Matrix $\mathbf{H}$ depends on the precoder $\mathbf{F}$. However, because of the diagonal dominance, we have $c_{ii} \gg c_{ij}$ for all $j \neq i$, and because we aim at diagonalizing $\mathbf{H} = \mathbf{CF}$, the precoder must also exhibit diagonal dominance: $f_{ii} \gg f_{ij}$ for all $j \neq i$. Due to the normalization of the precoder (in order to avoid any power increase), it means that $f_{ii} \approx 1$ for all lines i. Based on these considerations, the following approximation holds very accurately: $\mathbf{H}_d \approx \mathbf{C}_d$. It comes that the criterion

$$J(\mathbf{F}) = \mathbf{C}_d^{-1}\mathbf{CF} - \mathbf{I} \tag{17}$$

is linear in $\mathbf{F}$.

This is a matrix criterion, and as such, imposes N^2 conditions simultaneously. By using diagonal dominance again for both matrices $\mathbf{C}$ and $\mathbf{F}$, each individual entry J_{ij} of the criterion can be further simplified to the dominant terms. For instance, nondiagonal entries, $i \neq j$, become:

$$J_{ij} = \frac{1}{c_{ii}}\sum_{k=1}^{N} c_{ik}f_{kj} \tag{18}$$
$$\approx f_{ij} + \frac{c_{ij}}{c_{ii}}. \tag{19}$$

Clearly, in first order, all entries of the criterion function $J(\mathbf{F})$ are independent, and are linearly related to the corresponding entry of the precoder matrix $\mathbf{F}$. This means that the algorithm, whose aim is to find the zeros of the criterion, can work independently on all entries of the precoder matrix. In addition, since the criterion is (approximately) linear, the application of a simple stochastic gradient method leads to the following update equation:

$$\mathbf{F}(k+1) = \mathbf{F}(k) - \mu_{d,\mathrm{nosign}}\left[\mathbf{v}(k)\mathbf{u}^H(k)\right]_{nd} \tag{20}$$

where $\mathbf{F}(k)$ denotes the precoder at iteration k, where $\mu_{d,\mathrm{nosign}}$ is a *diagonal matrix* containing the stepsizes for the different lines. Each line has its own stepsize.

The final step to obtain an algorithm that uses the sign feedback, which is the only information available at the CO in our setup, is to simply replace the detection noise samples $\mathbf{v}(k)$ by their sign $\mathbf{v}^{\mathrm{sgn}}(k)$. The proposed algorithm is thus

$$\mathbf{F}(k+1) = \mathbf{F}(k) - \mu_d\left[\mathbf{v}^{\mathrm{sgn}}(k)\mathbf{u}^H(k)\right]_{nd} \tag{21}$$

with a different set of stepsizes. Making this replacement in (21) means that we assume that both expressions provide the same values, in expectation, up to a constant: $\mathrm{E}[\mathbf{vu}^H] = \mathbf{K}_d\mathrm{E}[\mathrm{sign}(\mathbf{v})\mathbf{u}^H]$ for some fixed diagonal matrix $\mathbf{K}_d$. This is not exactly true. However it can be shown, using the example of a Gaussian constellation, that it is closely followed when the residual crosstalk is low. More importantly however, it is easy to show that the two expectations have the same zeros. In other terms, a precoder that nulls out the interference and satisfies $\mathrm{E}[\mathbf{vu}^H] = 0$ will also satisfy $\mathrm{E}[\mathrm{sign}(\mathbf{v})\mathbf{u}^H] = 0$ and inversely. The algorithm based on sign feedback should therefore converge to the same precoder solution. The complexity of this update (21) is low since it only involves one multiplication with the stepsize per line, plus one multiplication with a bit of sign per line.

The similarity with the algorithm version proposed in [8] is obvious:

$$\mathbf{F}(k+1) = \mathbf{F}(k) - \mu\mathbf{F}^{-H}(k)\left[\mathbf{v}^{\mathrm{sgn}}(k)\mathbf{u}^H(k)\right]_{nd} \tag{22}$$

That algorithm however requires a matrix inverse operation of the current precoder $\mathbf{F}(k)$ at each step, which is clearly too complex to be used in practice. It has been shown above that the precoder itself exhibits diagonal dominance (and is in fact very close to the identity matrix). The new version of the algorithm simply replaces $\mu\mathbf{F}(k)$ by a fixed diagonal matrix. Doing so, it also gains in generality by allowing the different lines to have different stepsizes. As is shown below, this allows us to tune the performance more efficiently. Once again, it is important to note that, for equal stepsizes, the two versions of the algorithms have identical performance in all practical scenarios investigated.

IV. Performance Evaluation

In this section, we present theoretical analyses of the proposed algorithm (21). The basic tool used here for measuring the performance of the algorithm is the evolution of the *residual interference* during the updates of the precoder. The matrix of residual interference is given by

$$\mathbf{R} = \mathbf{H}_d^{-1}\mathbf{C}\mathbf{F} - \mathbf{I}. \quad (23)$$

It measures how well the crosstalk is suppressed by the precoder. If we focus on a given line i, we can observe the evolution of the residual interference vector

$$\mathbf{r}_i = [\mathbf{R}_{i,0} \quad \dots \quad \mathbf{R}_{i,i-1} \quad \mathbf{R}_{i,i+1} \quad \dots \quad \mathbf{R}_{i,N-1}] \quad (24)$$

which is line i of the residual interference matrix $\mathbf{R}$ from which the diagonal element has been removed.[1] Ideally, this vector should get close to zero as quickly as possible. Inserting the update (21) into the definition of the residual interference, it follows that

$$\mathbf{R}(k+1) = \mathbf{R}(k) - \mathbf{H}_d^{-1}\mathbf{C}\mu_d\left[\mathbf{v}^{\mathrm{sgn}}(k)\mathbf{u}^H(k)\right]_{nd}. \quad (25)$$

Thanks to the diagonal dominance, the behavior of each line can be approximated by

$$\mathbf{r}_i(k+1) \approx \mathbf{r}_i(k) - \mu_i H_{ii}^{-1} C_{ii} v_i^{\mathrm{sgn}}(k)\bar{\mathbf{u}}^H(k) \quad (26)$$
$$\approx \mathbf{r}_i(k) - \mu_i v_i^{\mathrm{sgn}}(k)\bar{\mathbf{u}}^H(k) \quad (27)$$

where μ_i is the stepsize for line i and where $\bar{\mathbf{u}}$ denotes the vector of symbols $\mathbf{u}$ from which the i^{th} entry was removed. The last line comes by assuming that $H_{ii} \approx C_{ii}$, as already shown above. Using this approximation, we essentially decouple the algorithm among the different lines. Each line can thus be analyzed separately. By taking the expectation of the second term in (27), it is possible to compute the average effect of the update equation on the residual interference for the line of interest. This expectation depends on the constellations used for the symbols u_i on the different lines. In order to compute this, we approximate the probability distribution of the symbols with Gaussian distributions. In that case, it can be shown that (time index k is dropped for simplicity)

$$\mathrm{E}\left[v_i^{\mathrm{sgn}} u_j\right] = \frac{2}{\sqrt{\pi}} \frac{r_{ij}}{\sqrt{|\mathbf{r}_i|^2 + \sigma_{n_i}^2/|C_{ii}|^2}} \quad (28)$$

for $i \neq j$. The evolution of the residual interference vector is then given by (on average, and for normalized symbol variance $\sigma_u^2 = 1$ on all lines)

$$\mathbf{r}_i(k+1) = \mathbf{r}_i(k) - \frac{2\mu_i}{\sqrt{\pi}} \frac{\mathbf{r}_i(k)}{\sqrt{|\mathbf{r}_i(k)|^2 + \sigma_{n_i}^2/|C_{ii}|^2}}. \quad (29)$$

[1]This element is always zero by definition of the matrix $\mathbf{R}$.

It is interesting to analyze this formula. It generates different behaviors whether the system is in acquisition or in tracking. In acquisition, the crosstalk is dominant $|\mathbf{r}_i(k)|^2 \gg \sigma_{n_i}^2/|C_{ii}|^2$, and the evolution is well approximated by

$$|\mathbf{r}_i(k+1)| = |\mathbf{r}_i(k)| - \frac{2\mu_i}{\sqrt{\pi}}. \quad (30)$$

It means that the amplitude of the residual interference coefficients decrease linearly. It is thus very important to choose the stepwise wisely. Equation (30) allows us to do so if a target number of symbols is fixed and the initial level of interference is known (see below). In tracking, the noise is dominant and the evolution is well approximated by

$$|\mathbf{r}_i(k+1)|^2 = |\mathbf{r}_i(k)|^2\left(1 - \frac{2\mu_i}{\sqrt{\pi}}\frac{|C_{ii}|}{\sqrt{\sigma_n^2}}\right)^2 \quad (31)$$
$$= |\mathbf{r}_i(k)|^2\left(1 - \frac{2\mu_i}{\sqrt{\pi}}\sqrt{\mathrm{SaNR}_i}\right)^2 \quad (32)$$

where "SaNR_i" denotes the signal to additive noise ratio for line i, that is the ratio between the power of the received signal and the noise not taking into account the influence of crosstalk (or in other terms, the SNR that would be reached if all crosstalk was removed perfectly).[2] In our model it is given, for line i, by

$$\mathrm{SaNR}_i = \frac{\sigma_{u_i}^2|C_{ii}|^2}{\sigma_{n_i}^2} = \frac{|C_{ii}|^2}{\sigma_{n_i}^2}. \quad (33)$$

In this tracking case, the evolution is exponential. Both behaviors are well confirmed by simulations as it is shown later on, but they provide *average* behavior only.

Asymptotically, the residual interference does not cancel out completely. The asymptotic performance is evaluated by taking the expectation of the square norm of (27), and then assuming that the variance of the residual interference *asymptotically* stabilizes to a constant value

$$\mathrm{E}\left[|\mathbf{r}_i(k+1)|^2\right] = \mathrm{E}\left[|\mathbf{r}_i(k)|^2\right]. \quad (34)$$

The computation of all expectations is tedious but straightforward and is skipped here. Finally, we obtain the following variance in *asymptotic* behavior:

$$\mathbf{E}\,|\mathbf{r}_i(k)|^2 = \frac{\mu_i(N-1)\sqrt{\pi}}{2\sqrt{\mathrm{SaNR}_i}}. \quad (35)$$

It can be seen that this depends on the SaNR of the line. Hence, in practice, it is useful to consider different stepsizes for the different lines as suggested in (21). All the results here are given for normalized symbol variance $(\sigma_u^2 = 1)$. If the variance is not unity, stepsize(s) need to be scaled accordingly.

V. Setting the Stepsizes

The various analytical results presented above enable to efficiently choose the stepsizes on the different lines. Two conflicting objectives need to be taken into account. The stepsize

[2]We use this notation here as the term "SNR" is sometimes used with or without taking into account the crosstalk. To avoid any ambiguity we make a clear distinction between SaNR which only takes into account the additive noise and SINR which also takes into account the effect of crosstalk.

needs to be large enough to guarantee a fast convergence in acquisition, and small enough to obtain a good asymptotic performance. Following the performance analysis derived above, each line can be studied independently. It is assumed that the SINR for each line, before any precoding is applied, is available at the transmitter. This should practically always be the case, as systems using crosstalk cancelation will also use rate adaptation, which requires the knowledge of the SINR at the transmitter.

For the convergence aspect, the average behavior given by (30) can be used. Assuming that the initial SINR on line i is mostly limited by crosstalk (which is the worst case), we have $|\mathbf{r}_i(0)|^2 \approx 1/\mathrm{SINR}_i$, and in order to obtain a convergence in roughly K_{conv} symbols, the stepsize must be set as

$$\mu_i \geq \frac{\sqrt{\pi}}{2K_{\mathrm{conv}}\sqrt{\mathrm{SINR}_i}}. \tag{36}$$

Obviously, if the crosstalk is lower than expected, the convergence will even be faster. Regarding the asymptotic performance, (35) is used, again independently for each line. Using this requires a few additional assumptions. First it is assumed that the central office has a rough idea of the SaNR on the line of interest. We also define the *target interference rejection* as a design parameter from the CO point of view. It sets the desired level of remaining interference after convergence of the algorithm at T_r times below the additive noise. Based on these definitions, and on the asymptotic performance expression (35), the desired stepsize is given by

$$\mu_{fin,i} = \frac{2}{(N-1)T_r\sqrt{\pi}\sqrt{\mathrm{SaNR}_i}}. \tag{37}$$

Now this last value does not always satisfy the minimum convergence speed required by (36). Hence, it might be useful to use a varying stepsize, at least on the duration of the acquisition, in order to satisfy both criteria simultaneously. In that case, (36) needs to be satisfied on the *average* stepsize across the K_{conv} symbols. The condition on asymptotic performance obviously applies to the final value of the stepsize. Many decreasing patterns can be used as long as both conditions are fulfilled. Note that a wrong guess of the SaNR on the line of interest will not prevent the algorithm from converging, but the performance might deviate from the expected value.

VI. Simulation Results

To verify the performance of the algorithm in a standard VDSL scenario, we investigate a situation with four interfering users. Crosstalk channels are generated with a log-normal model. The power, channel and noise conditions are set to obtain different SNR (or more precisely SaNR) and crosstalk levels. Unless otherwise stated, the different users all have the same SaNR.

This paragraph presents a few simulations with the considered method. In all these simulations, we assume a system with five users, but only two lines are shown. First, it is interesting to point out that long acquisition times, as used in [8], are not necessary. In fact, if the stepsize is chosen carefully, much quicker acquisition can be obtained. An example of simulation is shown in Fig. 2 for a situation where the SaNR is 40 dB and the initial interference is at 23 dB below the useful signal on all lines. The evolution of the remaining interference (crosstalk) is shown

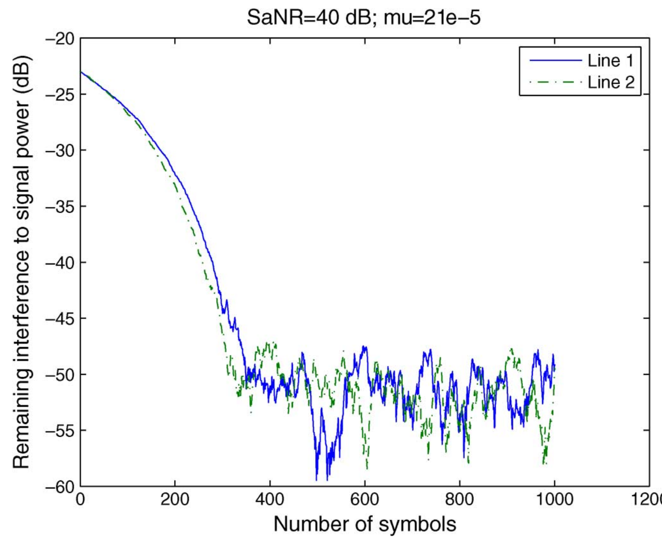

Fig. 2. Example simulation ($\mathrm{SaNR} = 40$ dB, initial crosstalk at 23 dB).

through the various iterations of the algorithm. The interference is represented as a ratio (in decibels) with respect to the signal power. The stepsize has been fixed according to (30) in order to converge in roughly 300 symbols. Only two lines are shown. On the chosen duration, the interference is rejected to almost 10 dB below the noise.

Now we compare the theoretical performance expressions given above with several simulations in order to validate them. Fig. 3 compares the theoretical acquisition performance (dashed curve) with the simulation in a case with high initial crosstalk ($\mathrm{SaNR} = 40$ dB, and the initial crosstalk is at 14 dB below the signal). The performance prediction is quite accurate. Fig. 4 compares the theoretical and simulation results for the tracking performance of the considered method. The 2 displayed lines respectively have an SaNR of 30 and 25 dB. The initial crosstalk is set below the noise to start in a tracking mode. Again, the match between theory and simulation is very good. Finally, Fig. 5 provides a similar comparison for the asymptotic performance. The horizontal dash lines represent the expected remaining level of interference corresponding to the theoretical asymptotic variance of the residual interference $\mathrm{E}[|\mathbf{r}_i|^2]$. Even though there remains a lot of variation, the predictions seem reliable. Note that all simulations performed here use classical square constellations. Despite the approximations of Gaussian symbols used to derive the performance expressions, the prediction is still accurate.

All simulations above have been done with fixed stepsize. In practice a fixed stepsize is not necessarily the best choice. As described above, a higher starting value will benefit from a quicker acquisition, and the value can then be decreased for better asymptotic performance. Fig. 6 compares the algorithm behavior for an exponentially decreasing pattern (plain line) with what can be obtained for fixed stepsizes (dashed and dashed–dotted lines). The exponentially decreasing pattern used here starts at $\mu = 2e-3$ and decreases to $\mu = 1e-4$ around 500 symbols. It is then fixed for the remainder of the algorithm. The algorithm has been run 15 times for each case and the average behavior is represented. It clearly appears that

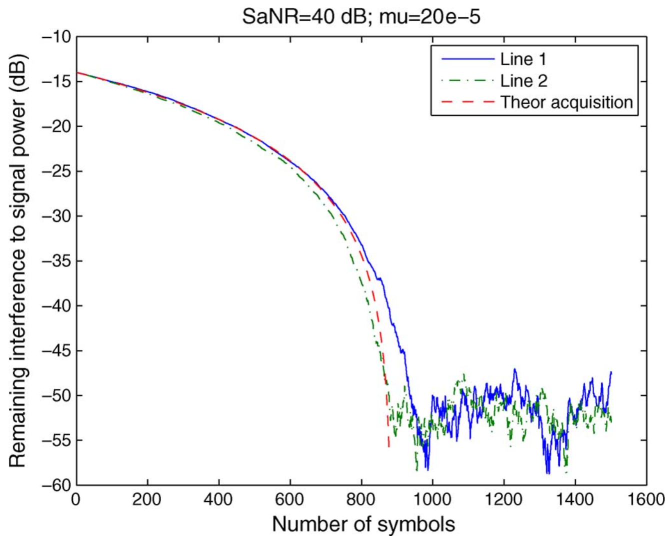

Fig. 3. Comparison between predicted (dash line) and simulated acquisition speed ($\mathrm{SaNR} = 40$ dB, initial crosstalk at 14 dB).

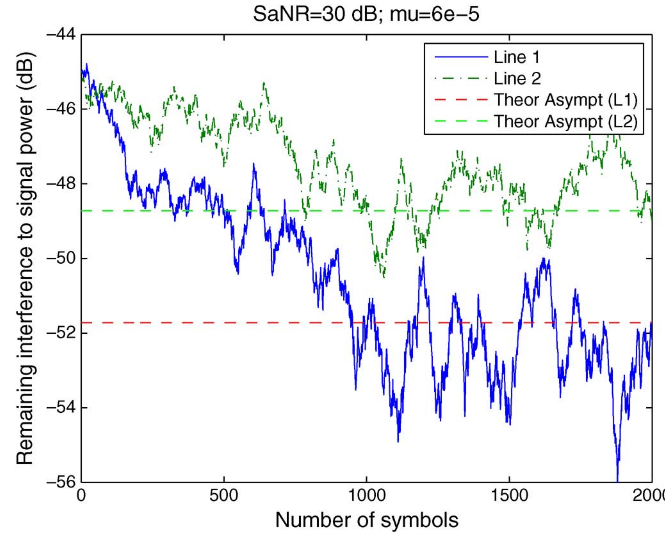

Fig. 5. Comparison between predicted (horizontal dashed lines) and simulated asymptotic performance.

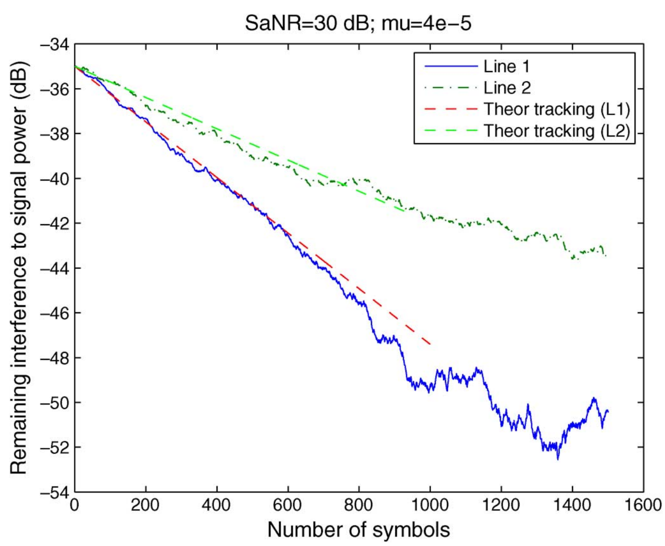

Fig. 4. Comparison between predicted (dashed line) and simulated tracking speed.

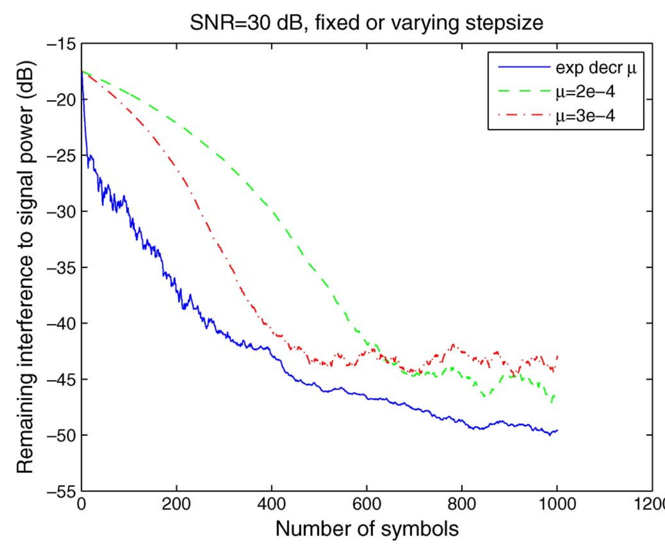

Fig. 6. Comparison between fixed stepsize and decreasing stepsize.

the exponentially decreasing pattern has superior performance, both in acquisition and asymptotically.

We also compare the performance of the proposed algorithm with the estimation method based on pilot symbols. In the pilot-based method, it is assumed that a given number K of pilot symbols are sent (the synchronization sequence may be used for instance) simultaneously on all lines. The received samples at the different line outputs corresponding to these pilots are fed back at the CO which computes the crosstalk coefficient estimates. The pilot sequences are assumed to be decorrelated among the different lines in order to perform the estimation efficiently. For this method, the achievable performance, in terms of signal to interference ratio on a given line i, can be shown to be well approximated by

$$\mathrm{SIR}_i = \frac{K\mathrm{SaNR}_i}{N} \tag{38}$$

where N is the total number of lines.

Fig. 7 shows the comparison between the proposed algorithm and the pilot method. The achievable interference rejection (interference to signal ratio) is represented as a function of the number of symbols used in the algorithm, for several situations (SaNRs and initial interference). For the proposed method, constant stepsize simulations are performed. The stepsize is optimized independently for each possible number of symbols in order to provide the best performance at the end of the given duration. The value represented is the average interference rejection obtained over 20 simulations for the chosen stepsize. As shown above, further improvements could be obtained by choosing varying stepsize patterns. Three situations are represented. Dashed curves correspond to the pilot method and plain lines are for the proposed method. The number of symbols used in the pilot methods has been corrected by a factor of 8 to take into account the fact that received symbols need to be fed back with sufficient precision (typically 8 bits) instead of the 1 bit per symbol in the proposed method. In the figure, the top curves are

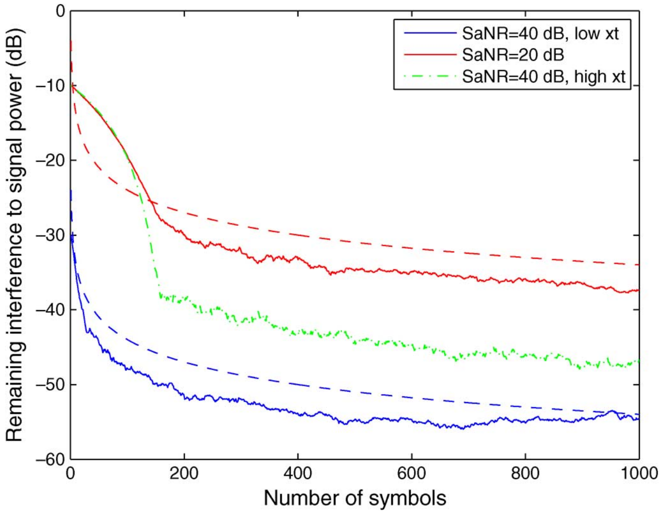

Fig. 7. Comparison between the proposed algorithm and the pilot methods.

for an SaNR of 20 dB (starting with crosstalk at 10 dB below the signal) and the bottom curves are for an SaNR of 40 dB (starting with crosstalk at either 10 or 30 dB below the signal). It can be seen that the proposed method is more efficient than the pilot method for moderate initial crosstalk. For high initial crosstalk (middle curve), it takes longer to reach the good performance region, although this could be corrected by the use of varying stepsizes. Note that the proposed method can potentially be faster as it is possible to feedback the sign on every symbol, instead of only using the synchronization sequence as with the pilot methods.

VII. Conclusion

We have studied a precoder design algorithm for decreasing the effect of crosstalk in downstream VDSL systems. The algorithm works on a per tone basis and assumes that each receiver send back the sign of the detection noise to the transmitter. An adaptive algorithm has been derived that computes the precoder iteratively using the information fed back by the receivers, without the need to estimate the crosstalk channel as an intermediate step. It has been shown by simulation that, despite the various approximations used in deriving the algorithm, it still provides good asymptotic and acquisition performance. This performance has been extensively analyzed both theoretically and with simulations. A comparison has been provided with another possible method for crosstalk estimation and precoder computation.

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