COMPRESSED SENSING TECHNIQUES FOR DYNAMIC RESOURCE ALLOCATION IN WIDEBAND COGNITIVE NETWORKS

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ABSTRACT

For multi-user cognitive networks, joint dynamic resource allocation (DRA) and waveform adaptation techniques have been developed that effectively represent, manipulate and utilize the physical-layer radio resources by synthesizing both transmitter and receiver waveforms from generalized signal expansion functions. To effect distributed DRA games, this paper discusses the intertwined sensing task and develops compressed sensing techniques that simultaneously estimate all the channel and interference links using only a small number of samples collected from a sparse set of expansion functions. By properly identifying and utilizing the sparsity properties of a wideband environment, the proposed schemes considerably reduce both sensing time and implementation costs.

1. INTRODUCTION

In wireless cognitive networks adopting open spectrum access, radio users dynamically decide the allocation of available radio resources to improve the overall spectrum utilization efficiency, also known as dynamic resource allocation (DRA) [1]. In the absence of a centralized spectrum controller, DRA can be carried out in a distributed fashion using multiuser games [2, 3].With the exception of orthogonal frequency division multiplexing (OFDM) systems, most works treat DRA and waveform adaptation as two separate tasks: DRA deals with frequency band allocation and power control on the allocated bands, while waveform design aims for rapid adjustment of the transmitted waveform spectra in order to comply with the dynamically allocated spectrum and power. Because it can be difficult or costly to generate a transmitted waveform that perfectly matches the allocated spectra of any flexible shape, the separate treatment may not offer desired DRA solutions for practical radios.

In a preceding work [4], we have developed a joint DRA and waveform adaptation framework for efficient spectrum utilization in multi-user CR networks. Therein, physical-layer radio resources over a very wide spectrum are represented by transmitter and receiver signal-expansion functions, which can be judiciously chosen to enable various agile platforms, such as frequency, time, or code division multiplexing (FDM, TDM, CDM). Based on such a radio-resource representation, distributed multi-user DRA games have been developed that iteratively adjust the usage of the expansion functions based on available resources [4]. Clearly, knowledge of the dynamically available resources needs to be acquired via sensing.

To fulfill DRA needs, this paper develops efficient sensing algorithms for both channel estimation of the desired links and interference sensing of the aggregate effects from multiple-access channels. Because of the signal-expansion framework adopted for wideband processing, each CR faces a large number of channel links arising from all the transmitter-receiver function pairs. To reduce the sensing complexity, this paper proposes compressive sampling techniques that simultaneously estimate all the channel and interference links using only a small number of samples collected from a sparse set of expansion functions, with the aid of an auxiliary wideband filter. The sparseness of both the wideband channels and the interferences on a certain domain is identified and then utilized for sparse signal recovery, which considerably reduce both sensing time and implementation costs.

Notations. $(\cdot)^*$ denotes conjugate, $(\cdot)^H$ denotes conjugate transpose, $(\cdot)^\dagger$ denotes pseudoinverse, $\otimes$ denotes Kronecker product, $\star$ stands for convolution, diag$(\cdot)$ converts an $N \times 1$ vector into an $N \times N$ diagonal matrix, and $E(\cdot)$ is expectation.

2. SIGNAL MODEL

Consider a wireless network consisting of Q active CR users, where each CR refers to a pair of one transmitter and one receiver. Adopting a block transmission structure, the q-th CR transmits a $K \times 1$ coded data vector $\mathbf{u}_q = \mathbf{F}_q \mathbf{s}_q$ in each block, where $\mathbf{s}_q \triangleq [s_{q,0}, \ldots, s_{q,K-1}]^T$ consists of K *i.i.d.* information symbols $\{s_{q,k}\}_{k=0}^{K-1}$ and $\mathbf{F}_q$ is a $K \times K$ linear precoding matrix. The symbol $u_{q,k}$ is modulated onto the transmitter function $\psi_k(t)$, $\forall k \in [0, K-1]$, yielding the transmitted waveform $u_q(t) = \sum_k u_{q,k} \psi_k(t)$. The CR sends $u_q(t)$ over a dispersive channel with impulse response $g_q(t)$, and pre-processes it at the receiver using the functions $\{\varphi_l(t)\}_{l=0}^{K-1}$ to

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collect a block of K data samples $\mathbf{x}_q \triangleq [x_{q,0}, \ldots, x_{q,K-1}]^T$. The transmitter functions include a cyclic prefix that is then removed by the receiver functions, which avoids any inter-block interference [4]. The receiver is inflicted with an additive noise signal $\nu_q(t)$, which accounts for the aggregate interference from other CRs, primary users and ambient noise.

Assume that each CR pair is synchronized. For CR q, the link gains among all the transmitter and receiver functions are organized into a $K \times K$ channel matrix $\mathbf{H}_q$, whose (k,l)-th element is given by $h_{q,k,l} \triangleq [g_q(t) \star \psi_k(t) \star \varphi_l^*(-t)]|_{t=0}$. Meanwhile, the filtered noise sample vector at the receiver is $\mathbf{v}_q \triangleq [v_{q,0}, \ldots, v_{q,K-1}]^T$ with $v_{q,l} \triangleq [\nu_q(t) \star \varphi_l^*(-t)]|_{t=0}$, whose covariance matrix is $\mathbf{R}_q \triangleq E(\mathbf{v}_q \mathbf{v}_q^H)$. Hence, the discrete-time data model is given by

$$\mathbf{x}_q = \mathbf{H}_q \mathbf{u}_q + \mathbf{v}_q, \quad q = 1, \ldots, Q. \tag{1}$$

A few remarks on the above model are in order.

1. Different CRs do not have to be synchronized among one another, namely, the CR network is non-cooperative.
2. The sets of (bandlimited) transmitter and receiver filters $\{\psi_k(t)\}_{k=0}^{K-1}$ and $\{\varphi_k(t)\}_{k=0}^{K-1}$, the same for all CRs, represent the physical-layer radio resources that the CRs can manipulate. The size K is chosen large enough on the order of the time-bandwidth product of the wideband system, in order to adequately represent available resources.
3. A proper choice of the transmitter and receiver functions enables well-known multiple access scenarios, in the general form of carriers, pulses, codes, wavelets, and so on. Exemplary sets of transmitter and receiver functions are illustrated in [4], for FDM, TDM and CDM systems. Also, redundant sets of non-orthogonal functions are suggested [4, 7], e.g., using combinations of the functions used in FDM, TDM and/or CDM, which are useful for exploring the *sparsity property* of CR networks.

3. SPARSITY-CONSTRAINED DRA GAMES

This section briefly reviews the formulation of distributed DRA games as introduced in [4]. This framework integrates DRA, waveform adaptation and dynamic sensing to give rise to a truly distributed implementation, which also motivates the development in Section 4 on efficient sensing and acquisition of the channel and interference parameters $\mathbf{H}_q$ and $\mathbf{R}_q$.

The multi-user DRA problem aims to optimally design, at the transmitter side of the qth CR, the linear precoder $\mathbf{F}_q$ and the length-K power loading vector $\mathbf{a}_q$, whose k-th element is $a_{q,k} = \sqrt{E(|s_{q,k}|^2)}$, such that the spectrum utilization efficiency of the overall CR network is maximized. The network spectral efficiency is closely related to the per-user capacity $C(\mathbf{a}_q, \mathbf{F}_q)$, which, for a given channel realization $\mathbf{H}_q$, is:

$$C(\mathbf{a}_q, \mathbf{F}_q) = \frac{1}{K} \log_2 \left| \mathbf{I}_K + \text{diag}(\mathbf{a}_q) \mathbf{F}_q^H \mathbf{B}_q \mathbf{F}_q \text{diag}(\mathbf{a}_q) \right| \tag{2}$$

with $\mathbf{B}_q \triangleq \mathbf{H}_q^H \mathbf{R}_q^{-1} \mathbf{H}_q$. As apparent from (2), the DRA problem is intertwined with waveform design and adaptation. To show this, let $S_q(f; \mathbf{a}_q, \mathbf{F}_q)$ denote the power spectral density (PSD) of the transmitted signal $u_q(t)$, which depends on $(\mathbf{a}_q, \mathbf{F}_q)$. In the CR scenario, the spectral shape of each transmitted waveform has to comply with a spectral mask $S_c(f)$ that depends on both policy-based long-term regulatory issues and cognition-based dynamic frequency notch masks for interference control and protection of primary users, that is,

$$S_q(f; \mathbf{a}_q, \mathbf{F}_q) \leq S_c(f), \qquad \forall f. \tag{3}$$

Meanwhile, the average power P_q of the q-th CR has to be confined below a predefined upper limit $P_{q,\max}$, as follows:

$$P_q(\mathbf{a}_q, \mathbf{F}_q) = \int S_q(f; \mathbf{a}_q, \mathbf{F}_q) df \leq P_{q,\max}. \tag{4}$$

To solve the DRA problem, let us keep in mind the following two basic remarks.

1. Due to the huge complexity arising from a centralized optimization of the collective actions $\{(\mathbf{a}_q, \mathbf{F}_q)\}_{q=0}^{Q-1}$ maximizing the sum-rate of all users subject to (3)-(4), a decentralized approach to DRA is motivated.
2. Since transmission opportunities in wideband networks are typically searched by active CRs over a large spectrum range, the actual resources needed for reliable transmission are sparse compared to the total available ones.

The above facts suggest a distributed noncooperative game framework for joint DRA and waveform adaptation [4], wherein each CR q acts as a game player and seeks to maximize the per-user capacity utility in (2) by taking allocation actions on $(\mathbf{a}_q, \mathbf{F}_q)$ from its own set of permissible strategies. Actions on $(\mathbf{a}_q, \mathbf{F}_q)$ directly shape the transmitted waveform spectra, which in turn define the action space via (3) and (4). Further, some sparsity constraints are imposed to confine the action space to a few (unknown) effective expansion functions, rather than on the entire function space representing the total opportunities. Through an iterative game implementation, the above design principle boils down to solving the following per-user formulation at each game iteration, $\forall q \in [1, Q]$:

$$\max_{\mathbf{a}_q \succeq \mathbf{0}, \mathbf{F}_q} \quad C(\mathbf{a}_q, \mathbf{F}_q) \tag{5a}$$

$$\text{s.t.} \quad (3) \text{ and } (4) \tag{5b}$$

$$||\mathbf{a}_q||_l \leq L_{q,\max}^{(l)}. \tag{5c}$$

In (5c), the l-norm $||\mathbf{a}_q||_l$ measures the sparsity order of $\mathbf{a}_q$, with $0 \leq l < 2$ [6]. By confining it to be below a predefined threshold $L_{q,\max}^{(l)}$, (5c) induces a sparse solution to $\mathbf{a}_q$, in the sense that only a few entries of $\mathbf{a}_q$ will be nonzero and the rest of the elements will be zero. Through sparse power loading, this strategy effectively deactivates most transmitter expansion functions to reduce implementation costs, with little loss of attained utility [4]; see [4] for details of the implementations and characterization of the DRA games at steady state.

The optimal solution to (5) has been derived in [4], which hinges on the knowledge of the channel matrix $\mathbf{H}_q$ and the covariance matrix $\mathbf{R}_q$ at each iteration, in order to obtain $\mathbf{B}_q$ in (2). Next section provides such dynamic sensing solutions.

4. COMPRESSED SENSING FOR DRA

This section develops channel estimation and interference sensing methods for acquiring $\mathbf{H}_q$ and $\mathbf{R}_q$, as required by iterative DRA games. This task takes place during the training phase by sending out training symbols $\mathbf{s}_q$, so that the transmitted symbols $\mathbf{u}_q = \mathbf{F}_q\mathbf{s}_q$ are known. First, we will estimate $\mathbf{H}_q$ by considering the interference $\mathbf{v}_q$ as nuisance noise. Then, after removing the contribution of $\mathbf{H}_q\mathbf{u}_q$ from $\mathbf{x}_q$, we will sense the interference and estimate its covariance matrix from the sample average. In both steps, new compressed sampling techniques will be developed to reduce the required number of samples and hence improve computational efficiency.

4.1. Channel Estimation

Let us model the channel impulse response $g_q(t)$ for the q-th CR as a tapped delay line with impulse response

$$g_q(t) = \sum_{n=0}^{N-1} g_{q,n}\delta(t-nT), \tag{6}$$

where N is the number of taps depending on the propagation scenario and T is the tap spacing corresponding to the inverse of the essential bandwidth. As described in Section 2, the channel coefficient $h_{q,k,l}$ can be re-written as

$$h_{q,k,l} = \sum_{n=0}^{N-1} g_{q,n}\left[\psi_k(t-nT)\star\varphi_l^*(-t)\right]\big|_{t=0} = \sum_{n=0}^{N-1} g_{q,n}\vartheta_{k,l,n}, \tag{7}$$

where $\vartheta_{k,l,n} \triangleq [\psi_k(t)\star\varphi_l^*(-t)]\big|_{t=-nT}$. From (7), the K^2 composite channel coefficients $\{h_{q,k,l}\}_{k,l}$ in $\mathbf{H}_q$ can be acquired by estimating the $N\times 1$ multipath tap vector $\mathbf{g}_q = [g_{q,0},\ldots,g_{q,N-1}]^T$, where $N \le K \ll K^2$ typically holds.

For complexity considerations during the training phase, let us make use of a (small) number L of transmitter and receiver functions with indices from the set $\mathcal{K} = \{k_0,\ldots,k_{L-1}\}$, with $N \le L \ll K$ and $k_0 < k_1 < \cdots < k_{L-1}$. With (7), the data model in (1) can be written as ($\forall l \in \mathcal{K}$)

$$x_{q,l} = \sum_{k\in\mathcal{K}} u_{q,k}\sum_{n=0}^{N-1} g_{q,n}\vartheta_{k,l,n} + v_{q,l} = \bar{\mathbf{u}}_q^T\mathbf{\Theta}_l\mathbf{g}_q + v_{q,l}, \tag{8}$$

where $\bar{\mathbf{u}}_q \triangleq [u_{q,k_0},\ldots,u_{q,k_{L-1}}]^T$, and $\mathbf{\Theta}_l$ is a known $L\times N$ matrix with its (m,n)-th element given by $[\mathbf{\Theta}_l]_{m,n} = \vartheta_{k_m,l,n}$. Upon stacking the L outputs $\{x_{q,k_m}\}_{m=0}^{L-1}$ into the vector $\bar{\mathbf{x}}_q \triangleq [x_{q,k_0},\ldots,x_{q,k_{L-1}}]^T$, we obtain

$$\bar{\mathbf{x}}_q = \left(\mathbf{I}_L\otimes\bar{\mathbf{u}}_q^T\right)\mathbf{\Theta}\mathbf{g}_q + \bar{\mathbf{v}}_q, \tag{9}$$

where $\bar{\mathbf{v}}_q \triangleq [v_{q,k_0},\ldots,v_{q,k_{L-1}}]^T$ and $\mathbf{\Theta}$ is an $L^2\times N$ matrix defined as $\mathbf{\Theta} \triangleq [\mathbf{\Theta}_0^T,\ldots,\mathbf{\Theta}_{L-1}^T]^T$. Since the interference vector $\bar{\mathbf{v}}_q$ is unknown within the channel estimation step (as well as its statistics), we solve for $\mathbf{g}_q$ resorting to a least-squares (LS) approach, thus getting

$$\hat{\mathbf{g}}_q = \left[\left(\mathbf{I}_L\otimes\bar{\mathbf{u}}_q^T\right)\mathbf{\Theta}\right]^\dagger\bar{\mathbf{x}}_q. \tag{10}$$

Finally, (7) and (10) yield the channel matrix estimate as

$$\hat{\mathbf{H}}_q = \mathbf{\Gamma}\left(\mathbf{I}_K\otimes\hat{\mathbf{g}}_q\right), \tag{11}$$

where $\mathbf{\Gamma}$ is a $K\times(KN)$ matrix defined as $\mathbf{\Gamma} \triangleq [\mathbf{\Gamma}_0,\ldots,\mathbf{\Gamma}_{K-1}]$, and $\mathbf{\Gamma}_l$ is a $K\times N$ matrix with its (k,n)-th element given by $[\mathbf{\Gamma}_l]_{k,n} = \vartheta_{k,l,n}$, $\forall l \in [0, K-1]$.

Some remarks about (10) now follow.

1. In the special case of OFDM signalling, the orthogonality condition among the transmitter and receiver functions yields $\vartheta_{k,l,n} = 0, \forall k \neq l$. Hence, (10) simplifies to

$$\hat{\mathbf{g}}_q = \left[\mathrm{diag}(\bar{\mathbf{u}}_q)\bar{\mathbf{\Theta}}\right]^\dagger\bar{\mathbf{x}}_q, \tag{12}$$

   where $\bar{\mathbf{\Theta}}$ is an $L\times N$ matrix with its (m,n)-th element given by $[\bar{\mathbf{\Theta}}]_{m,n} = \vartheta_{k_m,k_m,n}$.
2. The channel tap vector $\mathbf{g}_q$ is often sparse for a wideband multipath channel. The sparsity feature can be incorporated as prior knowledge to enhance the estimation accuracy, via the following ℓ_1-regularized LS formulation:

$$\hat{\mathbf{g}}_q = \arg\min_{\mathbf{g}_q}\left\{\|\mathbf{g}_q\|_1 + \lambda\left\|\bar{\mathbf{x}}_q - \left(\mathbf{I}_L\otimes\bar{\mathbf{u}}_q^T\right)\mathbf{\Theta}\mathbf{g}_q\right\|_2^2\right\}. \tag{13}$$

   where the ℓ_1-norm term imposes sparsity on the recovered channel vector, and the positive weight λ balances the bias-variance tradeoff in the channel estimate [6].
3. Under the assumption of zero-mean white Gaussian interference, it is known from [9] that minimum mean square error (MMSE) training is based on using L equi-powered and equi-spaced training symbols. In practice, however, whenever the interference covariance matrix is available after interference sensing, the channel estimate can be further improved, for instance following the method in [10], and possibly, by iterating between the channel and interference sensing steps. For briefness, details of this enhancement technique are skipped.

4.2. Interference Sensing

For sensing the interference covariance matrix $\mathbf{R}_q$ at low complexity, we develop a compressive sampling mechanism that only requires a small number of samples.

Let us adopt the expansion model for describing $\nu_q(t)$:

$$\nu_q(t) = \sum_{k=0}^{K-1}\nu_{q,k}\varphi_k(t) \tag{14}$$

where $\boldsymbol{\nu}_q \triangleq [\nu_{q,0},\ldots,\nu_{q,K-1}]^T$ is the vector representation of $\nu_q(t)$. Accordingly, the filtered interference sample $v_{q,l}$ is:

$$v_{q,l} = \sum_{k=0}^{K-1}\nu_{q,k}\left[\varphi_k(t)\star\varphi_l^*(-t)\right]\big|_{t=0} = \sum_{k=0}^{K-1}\nu_{q,k}\xi_{l,k},$$

or equivalently in vector-matrix form: $\mathbf{v}_q = \mathbf{\Xi}\boldsymbol{\nu}_q$, where $\mathbf{\Xi}$ is a $K\times K$ matrix with its (l,k)-th element $\xi_{l,k} \triangleq [\varphi_k(t)\star\varphi_l^*(-t)]\big|_{t=0}$ known. The sensing task of acquiring $\mathbf{v}_q$ is now equivalent to estimating $\boldsymbol{\nu}_q$.

The rationale of our sensing mechanism hinges on the observation that the signal $\nu_q(t)$ is sparse in the large space spanned by all $\{\varphi_k(t)\}_{k=0}^{K-1}$. This means that $\boldsymbol{\nu}_q$ is a sparse vector with only a small number of nonzero elements (measured by the l_0-norm $||\boldsymbol{\nu}_q||_0$), whose locations are clearly unknown. The sparse nature of $\boldsymbol{\nu}_q$ results from both the CR context of interest and the sparsity-constrained DRA problem that we have formalized in (5). Specifically: *i*) the set of expansion functions $\{\varphi_k(t)\}_k$ we adopt at the receiver can be a redundant non-orthogonal set or a combination of sets of orthogonal functions tailored for communication signals, e.g., a combination of the functions used in FDM and TDM; accordingly, it provides an over-complete representation of the signal space; *ii*) sparsity constraints are imposed to limit the number of transmitter functions, which results in sparse resource occupancy by CR users after DRA optimization; *iii*) the sparsity of $\boldsymbol{\nu}_q$ induced by *i*) and *ii*) can be further reduced by using a sparsifying basis $\mathbf{T}_q$ for $\boldsymbol{\nu}_q$, such that $\tilde{\boldsymbol{\nu}}_q \triangleq \mathbf{T}_q^{-1}\boldsymbol{\nu}_q$ has a low sparsity order. With the use of $\mathbf{T}_q$, $\mathbf{v}_q = (\boldsymbol{\Xi}\mathbf{T}_q)\tilde{\boldsymbol{\nu}}_q$ has a sparse representation $\tilde{\boldsymbol{\nu}}_q$ on the transformed receiver waveforms $\boldsymbol{\Xi}\mathbf{T}_q$. We focus on recovering $\boldsymbol{\nu}_q$, while $\tilde{\boldsymbol{\nu}}_q$ can be recovered similarly when stronger sparsity is desired.

Thanks to its sparseness, $\boldsymbol{\nu}_q$ can be recovered from a small number C of measurements, with $||\boldsymbol{\nu}_q||_0 < C \leq K$, in line with the recent results in compressive sampling [5], [6]. Overall, a compressive sampler can be implemented as follows.

- The received signal $x(t)$ is processed by an *auxiliary wideband filter* $s(t)$ of bandwidth $1/T$ (chosen to be "universal" regardless of the signal structure) to get $y_q(t)$.
- After channel estimation of $\{g_{q,n}\}$, the signal component $y_q^{(s)}(t) = u_q(t) \star g_q(t) \star s(-t)$ can be removed from $y_q(t)$, yielding $\zeta_q(t) = \nu_q(t) \star s(-t)$. Specifically in digital form, $y_q^{(s)}(t)$ at a given sampling point $t = t_c$ is

$$y_q^{(s)}(t_c) = \sum_k u_{q,k} \sum_{n=0}^{N-1} g_{q,n} \left[\psi_k(t-nT) \star s(-t)|_{t=t_c}\right],$$

  which can be removed from the received filtered sample $y_q(t) \star s(-t)|_{t=t_c}$ to obtain the interference sample $\zeta_q(t_c)$.
- $\zeta_q(t)$ is sampled at C time instances $\{t_c\}_{c=0}^{C-1}$ within each block after skipping the cyclic prefix, yielding samples

$$\zeta_{q,c} = \zeta_q(t_c) = \sum_{k=0}^{K-1} \nu_{q,k} \left[\varphi_k(t) \star s(t)\right]\big|_{t=t_c},$$

  or, in vector-matrix form,

$$\boldsymbol{\zeta}_q = \boldsymbol{\Lambda}\boldsymbol{\nu}_q, \qquad (15)$$

  where $\boldsymbol{\zeta}_q \triangleq [\zeta_{q,0}, \ldots, \zeta_{q,C-1}]^T$ and $\boldsymbol{\Lambda}$ is a $C \times K$ known measurement matrix with its (c,k)-th element given by $[\boldsymbol{\Lambda}]_{c,k} = [\varphi_k(t) \star s(t)]\big|_{t=t_c}$. Compressive sampling theory suggests several choices for both the filter $s(t)$ and the sampling instances $\{t_c\}_{c=0}^{C-1}$, which enable effective recovery of the sparse unknowns; see, e.g., [5], [6], [11].

Having acquired the compressive measurements $\boldsymbol{\zeta}_q$, the sparse signal $\boldsymbol{\nu}_q$ can be recovered by solving an ℓ_1-norm regularized LS problem similar to (13), as follows (ρ is a weight):

$$\hat{\boldsymbol{\nu}}_q = \arg\min_{\boldsymbol{\nu}_q} \left\{ \|\boldsymbol{\nu}_q\|_1 + \rho \|\boldsymbol{\zeta}_q - \boldsymbol{\Lambda}\boldsymbol{\nu}_q\|_2^2 \right\}. \qquad (16)$$

Finally, after estimating $\hat{\mathbf{v}}_q = \boldsymbol{\Xi}\hat{\boldsymbol{\nu}}_q$ on a number of subsequent blocks, the interference covariance matrix $\mathbf{R}_q$ can be evaluated by its sample average.

As a final remark on the implementation costs, we note that, after having estimated $\mathbf{H}_q$, it would be possible to recover the filtered interference vector $\mathbf{v}_q$ as $\hat{\mathbf{v}}_q = \mathbf{x}_q - \hat{\mathbf{H}}_q\mathbf{u}_q$ directly. This is a conceptually simple approach, but its main drawback is the use of all K receiver functions in order to acquire $\mathbf{x}_q$. Sensing the interference through compressive sampling, instead, requires a small number of samples per block that can be made significantly smaller than K, which can considerably reduce hardware-related implementation costs.

5. SIMULATIONS RESULTS

We simulate a Q-user peer-to-peer CR network. There are Q^2 channel links, where Q of them are desired links and the others are interferences. Each link experiences frequency selective fading modelled by an N-tap tapped delay line, where each tap coefficient is independently generated as zero-mean and unit variance complex Gaussian. The link power gain is expressed by the scalar $\rho_{rq} > 0$, $\forall r, q \in [1, Q]$, which captures both the path loss and the fading power. The noise variance is set to be unitary. As case study, we focus on an OFDM platform consisting of $K = 32$ subcarriers as signal expansion functions. Given the literature on channel estimation, we focus on testifying the effectiveness of the compressed interference sensing technique in Section 4.2, assuming that channels have been accurately acquired. The auxiliary wideband filter $s(t)$ is chosen to be a rectangular pulse of time-span T.

5.1. Performance of Interference Sensing

The CR scenario is based on $Q = 3$ users transmitting the average powers $P_1 = 5$, $P_2 = P_3 = 10$, respectively, with propagation channels having gains $\rho_{qq} = 1$, $\forall q \in [1, Q]$, $\rho_{rq} = 5$ for $\forall r \neq q \in [1, Q]$, and $N = 8$ tap coefficients. Taking user $q = 1$ as an example, the snapshots of the recovered interference $\hat{\mathbf{v}}_q$ versus the function index are depicted in Fig. 1, for various compression ratios $C/K = 45\%, 70\%$ and 90% respectively. The case of full-rate sampling ($C/K = 100\%$) results in perfect recovery, and hence is not shown. It is apparent that even with a moderate compression ratio $C/K = 45\%$, the compressed sensing scheme can reliably recover the instantaneous interference, without having to collect K samples per block by activating all the K receiver functions.

5.2. Sensitivity of DRA Efficiency to Sensing Errors

The impact of interference sensing errors on the capacity performance of DRA is now addressed when the actions of the active CR users are decided based on imperfect interference estimates $\{\hat{\mathbf{v}}_q\}_q$. We assume for simplicity that $\{\hat{\mathbf{v}}_q\}_q$ deviate from the true values $\{\mathbf{v}_q\}_q$ element-by-element by a zero-mean random component with standard deviation σ_v, whose

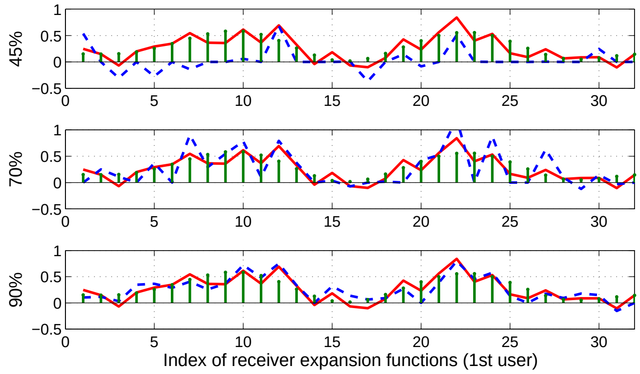

Fig. 1. Noise free interference profile (vertical lines), its noisy realization (dashed line) and the estimated interference (solid line) versus the function index, for user $q = 1$.

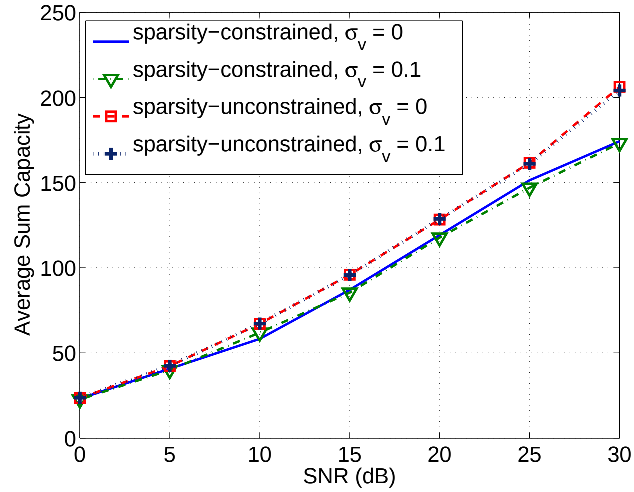

Fig. 2. Average sum capacity vs. SNR budget (decided by $P_{q,\max}$) for both sparsity constrained and unconstrained DRA games, when sensing errors are zero ($\sigma_v = 0$) or moderately large ($\sigma_v = 0.1$).

value is chosen as a SNR-independent upper bound derived from trials similar to that in Section 5.1. Further, we assume that the DRA optimization problem is solved only under the average power constraint (4), dropping the mask constraint (3) for simplicity. As performance metric, we consider the average sum capacity of all users (averaged over 100 sets of independent channel realizations) versus SNR for DRA under four scenarios: *a)* sparsity-constrained DRA via (5) with $l = 1$ in the l-norm constraint (5c), and no sensing errors ($\sigma_v = 0$); *b)* sparsity-constrained DRA with $l = 1$, and moderately large sensing errors ($\sigma_v = 0.1$); *c)* sparsity-unconstrained DRA via (5) in the absence of the l-norm constraint (5c), and no sensing errors ($\sigma_v = 0$); *d)* sparsity-unconstrained DRA and moderately large sensing errors ($\sigma_v = 0.1$).

Fig. 2 depicts the values of sum capacity attained by the above DRA strategies. As expected, sparsity-unconstrained DRA outperforms the sparsity-constrained counterpart, but the performance gaps are rather small. This means that the sparsity-constrained DRA design can dynamically select active expansion functions where the effective resources lie on, thus offering a noticeable saving in complexity yet at a tolerable cost of small performance loss. Regarding the impact of sensing errors, the capacity performance of the distributed DRA games reveals to be quite robust against imperfect (statistical) knowledge of interference. Such robustness allows for strong compression during sensing, which helps to keep the computational load at affordable levels.

6. SUMMARY

Focusing on cognitive radios with multiple transmitter and receiver expansion functions, this paper has developed compressed sensing techniques that estimate all the channel and interference links using only a small number of samples collected from a sparse set of expansion functions. The proposed sensing schemes effectively fulfill the sensing needs for joint DRA and waveform adaptation, resulting in a truly distributed implementation for spectral-efficient multiuser CR networks.

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