# SPACE-TIME COMPRESSIVE SAMPLING ARRAY

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Abstract—We propose a space-time compressive sampling (STCS) array architecture by exploiting the sparsity in the angle and frequency domain. Two Doppler-DoA estimation methods are designed to efficiently reconstruct the two-dimensional (2D) sparse signal. The STCS array together with the 2D reconstruction methods allow for accurate yet low-power location and speed estimation of moving targets. The proposed methods are not only tested by simulations but also using an experimental ultrasonic sensor array setup.

#### I. INTRODUCTION

For security and surveillance systems, detecting moving targets (e.g. humans) and their locations is of particular interest. An array based radar can obtain the movement information (velocity) via Doppler processing and the location or direction-of-arrival (DoA) information via spatial beamforming, which is referred to as the Doppler-DoA (DDoA) estimation. Fig. 1 shows a typical receiver front-end for a conventional array. Each antenna or sensor element of the array is connected to a separate chain of front-end circuits, which takes the analog RF signal as input and then performs amplifying, mixing, filtering, analog-to-digital conversion (ADC) and outputs the digital baseband (BB) signal for further processing, i.e., DDoA estimation.

For the power consumption of the array based architecture, the front-end processing done by analog circuits is dominant over the digital signal processing done by a microprocessor. Generally, a higher angle (DoA) resolution requires a larger number of elements in the array, for a fixed element spacing, which means a larger number of front-end circuit chains. For each front-end chain, the ADC needs to sample at the Nyquist rate determined by the maximum velocity (Doppler) of the moving targets. The key challenge is to lower the front-end complexity, i.e., reduce the number of front-end circuit chains and the ADC sampling rate, while maintaining the anglevelocity resolution in the DDoA estimation.



Fig. 1: Conventional array front-end architecture.

Compressive sampling (CS) is a method of acquisition and reconstruction of sparse signals [3]. Usually, the acquisition is done by a compression matrix drawn from a certain random distribution and the reconstruction is done by  $l_1$ -norm minimization [3]. Two CS applications of interest here are the analog-to-information convertor (AIC) that can sample below the Nyquist-rate for wideband spectrum estimation [5], and the CS array with spatial compression that can reduce the number of front-end chains for DoA estimation [6]. It is also shown in [6] that for the compressed signals, the minimum variance distortionless response (MVDR) estimator [1] can achieve similar angle resolution as that of the  $l_1$ -norm minimization based reconstruction algorithms.

In this paper, we propose a space-time compressive sampling (STCS) array with compression in both the spatial and temporal domain, by exploiting the sparsity in the angle and frequency respectively. For the STCS array based DDoA estimation, we propose two methods: one based on two-dimensional (2D) sparse reconstruction, and one based on an iterative procedure that jointly updates the beamformer and the frequency estimator using the MVDR approach.

## II. SIGNAL MODEL



Fig. 2: Geometry of the DDoA estimation problem.

The geometry of our DDoA estimation problem is illustrated in Fig. 2. In this paper an element refers to an omnidirectional antenna or sensor. A single element transmitter is located at the origin. The receiver is a uniform linear array (ULA) of  $N_l$  elements with element spacing d, located on the y-axis and centered at the origin. The transmitter radiates a continuous-wave (CW) sinusoidal signal at a carrier frequency  $f_c$  propagating at a velocity c (wavelength  $\lambda = c/f_c$ ). The transmitted signal hits K moving targets in the far-field which reflect the signal to the receiver. The k-th  $(k = 0, 1, \dots, K - 1)$  target is moving with a velocity  $v_k$  ( $v_k \ll c$ ), which is considered to be constant within a short observation window T. The received RF signal is down-mixed and low-pass filtered to obtain the complex baseband signal. After filtering out the DC components due to the reflections from stationary objects, the baseband signal contains only the reflections from the moving targets. On the *l*-th  $(l = 0, 1, ..., N_l - 1)$ receive element at the t-th  $(t = 0, 1, ..., N_t - 1)$  time instant, the baseband signal (ignoring the additive noise term) has the form

$$x(l,t) = \sum_{k=0}^{K-1} \beta_k e^{j2\pi (f_k/F_s)t} e^{-j2\pi (d/\lambda)\sin(\theta_k)l},$$
 (1)

where  $F_s$  is the Nyquist sampling rate determined by the maximum  $v_k$ , and  $N_t = TF_s$ . For the reflected signal from the k-th target,  $\beta_k$  is the complex amplitude,  $f_k = 2v_k/\lambda$  is the Doppler shift, and  $\theta_k$  is the DoA w.r.t. the x-axis. Given the received signals, we perform DDoA estimation to obtain the movement information  $v_k$  (or  $f_k$ ) and the associated spatial information  $\theta_k$  about the targets.

Dividing the DoA search range (e.g., of  $180^{\circ}$ ) into  $N_{\theta}$  angles denoted by  $\theta_p, p = 1, 2, \dots, N_{\theta}$ , we can define a basis matrix  $\Psi_{\theta}$ 

in the angle domain of size  $N_l \times N_{\theta}$   $(N_{\theta} \gg N_l)$  as

$$\Psi_{\theta} = [\theta_1 \ \theta_2 \ \cdots \ \theta_{N_{\theta}}], \qquad (2)$$
  

$$\theta_p = [1 \ e^{j\alpha_p} \ \cdots \ e^{j\alpha_p(N_l-1)}]^T / \sqrt{N_l}, \qquad (2)$$
  

$$\alpha_p = -2\pi (d/\lambda) \sin(\theta_p),$$

where  $\theta_p$  is an array steering vector for  $\theta_p$ . Similarly, dividing the Doppler search range (e.g., of  $F_s$  Hz) into  $N_f$  frequencies denoted by  $f_q, q = 1, 2, \ldots, N_f$ , we define a basis matrix  $\Psi_f$  in the frequency domain of size  $N_t \times N_f$  ( $N_f \ge N_t$ ) as

$$\Psi_{f} = [\mathbf{f}_{1} \ \mathbf{f}_{2} \ \cdots \ \mathbf{f}_{N_{f}}],$$

$$\mathbf{f}_{q} = [1 \ e^{j\omega_{q}} \ \cdots \ e^{j\omega_{q}(N_{t}-1)}]^{T} / \sqrt{N_{t}},$$

$$\omega_{q} = 2\pi (f_{q}/F_{s}),$$
(3)

where  $\mathbf{f}_q$  is a Fourier basis vector for  $f_q$ . Suppose that the angle and frequency grids are fine enough (if  $N_{\theta}$  and  $N_f$  are large enough), the k-th target has its DoA  $\theta_k$  and Doppler  $f_k$  aligned on the search grids, indexed by  $p_k$  and  $q_k$  respectively. We rewrite the signal of (1) in matrix form, by defining an  $N_l \times N_t$  matrix **X** with x(l, t) as its coefficient on the *l*-th row and the *t*-th column,

$$\mathbf{X} = \boldsymbol{\Psi}_{\theta} \mathbf{Z}_{\theta f} \boldsymbol{\Psi}_{f}^{T}, \tag{4}$$

where  $\mathbf{Z}_{\theta f}$  of size  $N_{\theta} \times N_{f}$  is a sparse matrix containing only K non-zero coefficients, i.e.,  $\mathbf{Z}_{\theta f}(p_{k}, q_{k}) = \beta_{k}$ .

*Notation*: In this paper we will use the following notations.  $\otimes$  denotes the Kronecker product, and \* denotes the complex conjugate.  $\mathbf{1}_n$  denotes an all one vector of size  $n \times 1$ , and  $\mathbf{I}_n$  denotes an identity matrix of size  $n \times n$ .

#### III. SPACE-TIME COMPRESSIVE SAMPLING

The proposed STCS array front-end architecture is illustrated in Fig. 3. We first define a spatial domain compression matrix  $\Phi_a$  of size  $M_l \times N_l$  ( $M_l < N_l$ ), which transforms the original array of dimension  $N_l$  to a smaller array of dimension  $M_l$ . After the spatial compression, only  $M_l$  (instead of originally  $N_l$ ) front-end circuit chains are needed to convert the analog RF signal to the digital baseband signal. In each front-end chain, we replace the original Nyquist-rate ADC with a sub-Nyquist-rate AIC for analog-to-digital conversion. Since the AIC performs block-wise compression, the original time domain signal of  $N_t$  samples (at Nyquist rate) is divided into B blocks such that  $N_t = BN_{\tau}$ . We define a temporal domain compression matrix  $\Phi_b$ of size  $M_{\tau} \times N_{\tau}$  ( $M_{\tau} < N_{\tau}$ ), which represents an AIC sampling at  $M_{\tau}/N_{\tau}$  of the Nyquist rate. The AIC transforms the  $N_{\tau}$  samples per block to  $M_{\tau}$  samples, and outputs a total number of  $M_t = BM_{\tau}$ samples. The compression matrices ( $\Phi_a$  and  $\Phi_b$ ) contain coefficients drawn i.i.d. from a random distribution, e.g., Gaussian, Bernoulli.



Fig. 3: Space-time CS array front-end architecture.

For notational simplification, in all the equations we will use the digital baseband version of the signal. Note that the spatial compression  $\Phi_a$  is applied to the analog RF signal whereas the temporal compression  $\Phi_b$  is applied to the analog baseband signal. The multiplication with the coefficients from  $\Phi_a$  can be implemented by an attenuator or a phase shifter or simply by an on-off selection switch. The AIC sampling is conceptually described as a Nyquistrate ADC followed by a compression matrix  $\Phi_b$ . In practice, the AIC operates on the analog signal and outputs the digital signal, where the multiplication with  $\Phi_b$  can be implemented using mixers and integrators [5]. For a fixed compression ratio, small values of  $M_{\tau}$ and  $N_{\tau}$  are preferred due to the AIC implementation complexity.

Letting i = 1, 2, ..., B be the block index, the received signal of the conventional array expressed by (4) can be rewritten as

$$\mathbf{X} = [\mathbf{X}_1 \ \mathbf{X}_2 \ \cdots \ \mathbf{X}_B]. \tag{5}$$

Then the received signal of the proposed STCS array is

$$\mathbf{Y} = [\mathbf{Y}_1 \ \mathbf{Y}_2 \ \cdots \ \mathbf{Y}_B], \tag{6}$$
$$\mathbf{Y}_i = \mathbf{\Phi}_a \mathbf{X}_i \mathbf{\Phi}_b^T.$$

where  $\mathbf{X}_i$  is the original signal of size  $N_l \times N_{\tau}$  and  $\mathbf{Y}_i$  is the spacetime compressed signal of size  $M_l \times M_{\tau}$ . In terms of the Doppler-DoA coefficients  $\mathbf{Z}_{\theta f}$ , (6) becomes

$$\mathbf{Y} = \mathbf{\Phi}_a \mathbf{X} (\mathbf{I}_B \otimes \mathbf{\Phi}_b)^T = \tilde{\mathbf{\Psi}}_{\theta} \mathbf{Z}_{\theta f} \tilde{\mathbf{\Psi}}_f^T,$$
(7)  
$$\tilde{\mathbf{\Psi}}_{\theta} = \mathbf{\Phi}_a \mathbf{\Psi}_{\theta}, \quad \tilde{\mathbf{\Psi}}_f = (\mathbf{I}_B \otimes \mathbf{\Phi}_b) \mathbf{\Psi}_f,$$

where  $\tilde{\Psi}_{\theta}$ ,  $\tilde{\Psi}_{f}$  are the angle basis matrix and the frequency basis matrix respectively for the compressed signal. If we divide  $\tilde{\Psi}_{f}$  into *B* sub-matrices, each with  $M_{\tau}$  rows, then  $\tilde{\Psi}_{f}^{T} = [\tilde{\Psi}_{f,1}^{T} \ \tilde{\Psi}_{f,2}^{T} \cdots \tilde{\Psi}_{f,B}^{T}]$ , where the *i*-th sub-matrix is denoted as  $\tilde{\Psi}_{f,i}$ . The  $\mathbf{Y}_{i}$  in (6) can be expressed by  $\tilde{\Psi}_{f,1}$  (the first  $M_{\tau}$  rows of  $\tilde{\Psi}_{f}$ ) as

$$\mathbf{Y}_{i} = \tilde{\boldsymbol{\Psi}}_{\theta} \mathbf{Z}_{\theta f} \tilde{\boldsymbol{\Psi}}_{f,i}^{T} = \tilde{\boldsymbol{\Psi}}_{\theta} \bar{\mathbf{Z}}_{\theta f,i} \tilde{\boldsymbol{\Psi}}_{f,1}^{T}, \tag{8}$$

where  $\bar{\mathbf{Z}}_{\theta f,i}$  is  $\mathbf{Z}_{\theta f}$  with some element-wise phase rotation, which can be ignored since only  $|\mathbf{Z}_{\theta f}|$  is of interest.

#### IV. THE STCS ARRAY BASED DDOA ESTIMATION

To estimate the DDOA spectrum using sparse reconstruction, one straightforward way is to vectorize the signal model of (7) and rewrite  $\mathbf{Z}_{\theta f}$  into a big sparse vector of size  $(N_{\theta}N_f) \times 1$ , and then solve it using the single measurement vector (SMV) based  $l_1$ -norm minimization. But  $N_{\theta}N_f$  is typically so large that the computational complexity is usually unaffordable for practical (especially real-time) implementations. Hence, we will propose two other approaches.

## A. 2D sparse reconstruction

The multiple measurement vectors (MMV) sparse reconstruction is addressed in [2], which computes the sparse solutions given MMV sharing a common sparsity structure. We refer to [2] as the 1D MMV sparse reconstruction, since it exploits the sparsity in one single domain. The 1D MMV sparse reconstruction problem is formulated as solving arg min<sub>**Z**</sub>  $||\mathbf{Z}||_s$  s.t.  $\mathbf{Y} = \mathbf{\Psi}\mathbf{Z}$ , where **Y** collects the  $N_c$ MMV,  $\mathbf{\Psi}$  is the common sparsity structure among the  $N_c$  MMV, **Z** of size  $N_r \times N_c$  is the sparse solution. The cost function to MMV, **Z** of size  $n_r \times N_c$  is the sparse solution. The cost function to minimize is defined as the row norm given by  $||\mathbf{Z}||_s = \sum_{r=1}^{N_r} (\sum_{c=1}^{N_c} |z_{r,c}|^2)^{s/2}$ , where  $z_{r,c}$  is the (r, c)-th element of **Z**, and  $s \in [0, 1]$  is a diversity measure parameter. To minimize  $||\mathbf{Z}||_s$ , a smaller *s* penalizes more the non-zero rows of **Z**.

To estimate  $\mathbf{Z}_{\theta f}$ , we first utilize the sparsity in the angle ( $\theta$ ) domain by solving a 1D MMV problem of

$$\hat{\mathbf{Z}}_{\theta t} = \arg \min_{\mathbf{Z}_{\theta t}} \| \mathbf{Z}_{\theta t} \|_{s} \quad \text{s.t.} \quad \mathbf{Y} = \tilde{\mathbf{\Psi}}_{\theta} \mathbf{Z}_{\theta t}, \tag{9}$$

where by rewriting (7),  $\mathbf{Z}_{\theta t} = \mathbf{Z}_{\theta f} \tilde{\boldsymbol{\Psi}}_{f}^{T}$  is a sparse matrix with (at most) K non-zero rows corresponding to  $\theta_k$ . Given  $\mathbf{Z}_{\theta t}$ , we obtain a coarse estimation of the angle spectrum and select some candidate angle indices  $\{p_c\}$  as,

$$p_{c} = \{p|P_{\theta}(p) > h \cdot \max P_{\theta}(p)\}, P_{\theta}(p) = \sum_{t=1}^{M_{t}} |\hat{z}_{p,t}|^{2}, \quad (10)$$

where  $\hat{z}_{p,t}$  is the (p,t)-th element of  $\hat{\mathbf{Z}}_{\theta t}$  and h is a threshold value. Then for each  $p \in \{p_c\}$ , we solve a 1D SMV problem of

$$\hat{\mathbf{z}}_{\theta_p f} = \arg\min_{\mathbf{z}_{\theta_p f}} \|\mathbf{z}_{\theta_p f}^T\|_s \quad \text{s.t.} \quad \hat{\mathbf{z}}_{\theta_p t}^T = \tilde{\boldsymbol{\Psi}}_f \mathbf{z}_{\theta_p f}^T, \tag{11}$$

where  $\mathbf{z}_{\theta_p f}$  and  $\hat{\mathbf{z}}_{\theta_p t}$  are the *p*-th rows of  $\mathbf{Z}_{\theta f}$  and  $\hat{\mathbf{Z}}_{\theta t}$  respectively. Finally, the estimated  $\mathbf{Z}_{\theta f}$  has  $\hat{\mathbf{z}}_{\theta p f}$  as its *p*-th row for  $p \in \{p_c\}$ , and zeros elsewhere.

#### B. 2D MVDR reconstruction

Let  $\{\tilde{\boldsymbol{\theta}}_p\}, p = 1, 2, \dots, N_{\theta}$ , be the columns of the angle basis matrix  $\tilde{\Psi}_{\theta}$ , and  $\{\tilde{\mathbf{f}}_q\}, q = 1, 2, \dots, N_f$ , be the columns of the frequency basis matrix  $\tilde{\Psi}_{f,1}$ . Define  $\mathbf{Y}' = [\mathbf{Y}_1^T \ \mathbf{Y}_2^T \ \cdots \ \mathbf{Y}_B^T]$  of size  $M_{\tau} \times (BM_l)$ . First we select some candidate angle indices  $\{p_c\}$ and candidate frequency indices  $\{q_c\}$  based on a coarse estimation of the angle spectrum  $P_{\theta}(p)$  and the frequency spectrum  $P_f(q)$ , respectively:

$$p_{c} = \{p|P_{\theta}(p) > h_{1} \cdot P_{\theta}^{m}\}, P_{\theta}(p) = 1/(\tilde{\boldsymbol{\theta}}_{p}^{H} \mathbf{R}_{y}^{-1} \tilde{\boldsymbol{\theta}}_{p}), \qquad (12)$$
$$q_{c} = \{q|P_{f}(q) > h_{2} \cdot P_{f}^{m}\}, P_{f}(q) = 1/(\tilde{\mathbf{f}}_{q}^{H} \mathbf{R}_{y'}^{-1} \tilde{\mathbf{f}}_{q}),$$

where  $\mathbf{R}_y = (\mathbf{Y}\mathbf{Y}^H)/M_t$ ,  $\mathbf{R}_{y'} = (\mathbf{Y}'\mathbf{Y}'^H)/(BM_l)$ ,  $P_{\theta}^m = \max P_{\theta}(p)$ ,  $P_f^m = \max P_f(q)$ , and  $h_1$  and  $h_2$  are two threshold values.

Then for each selected angle-frequency pair  $(p,q), p \in \{p_c\}$  and  $q \in \{q_c\}$ , we perform an iterative procedure to find jointly the optimum MVDR beamformer  $\mathbf{w}_p$  and the optimum MVDR frequency estimator  $\mathbf{v}_q$ .

• INITIALIZATION: 
$$\mathbf{w}_{p}^{(0)} = \mathbf{1}_{M_{l}}, \, \mathbf{v}_{q}^{(0)} = \mathbf{1}_{M_{\tau}}.$$

• ITERATION: j = 0, 1, ...Calculate  $\mathbf{w}_p^{(j+1)}$  by solving  $\arg\min \mathbf{w}^H \mathbf{R}_v^{(j)} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \tilde{\boldsymbol{\theta}}_p = 1,$ 

where  $\mathbf{R}_v^{(j)} = (\mathbf{Y}_v^{(j)} \mathbf{Y}_v^{(j)H})/B$ , and  $\mathbf{Y}_v^{(j)} = \mathbf{Y}(\mathbf{I}_B \otimes \mathbf{v}_q^{(j)*})$ . The solution is explicitly given by the MVDR estimator as

$$\mathbf{w}_{p}^{(j+1)} = (\mathbf{R}_{v}^{(j)})^{-1} \tilde{\boldsymbol{\theta}}_{p} / (\tilde{\boldsymbol{\theta}}_{p}^{H} (\mathbf{R}_{v}^{(j)})^{-1} \tilde{\boldsymbol{\theta}}_{p}),$$
(13)

Calculate  $\mathbf{v}_q^{(j+1)}$  by solving

$$\arg\min \mathbf{v}^H \mathbf{R}_w^{(j)} \mathbf{v}$$
 s.t.  $\mathbf{v}^H \tilde{\mathbf{f}}_q = 1$ 

where  $\mathbf{R}_w^{(j)} = (\mathbf{Y}_w^{(j)} \mathbf{Y}_w^{(j)H})/B$ , and  $\mathbf{Y}_w^{(j)} = \mathbf{Y}'(\mathbf{I}_B \otimes \mathbf{w}_p^{(j)*})$ . The solution is explicitly given by the MVDR estimator as

$$\mathbf{v}_{q}^{(j+1)} = (\mathbf{R}_{w}^{(j)})^{-1} \tilde{\mathbf{f}}_{q} / (\tilde{\mathbf{f}}_{q}^{H} (\mathbf{R}_{w}^{(j)})^{-1} \tilde{\mathbf{f}}_{q}), \tag{14}$$

• OUTPUT: after J iterations,  $\mathbf{w}_p = \mathbf{w}_p^{(J)}, \mathbf{v}_q = \mathbf{v}_q^{(J)}$ .

for  $\theta_p$ , and  $\mathbf{v}_q^{(j)}$  is a frequency estimator for  $f_q$ . To update the beamformer, we use  $\mathbf{v}_q^{(j)}$  as a bandpass filter centered at  $f_q$  to obtain the filtered signal  $\mathbf{Y}_v^{(j)}$ , based on which the spatial correlation  $\mathbf{R}_{v}^{(j)}$  is computed to get the new beamformer  $\mathbf{w}_{p}^{(j+1)}$ . To update the frequency estimator, we use  $\mathbf{w}_p^{(j)}$  as a spatial filter steered to  $\theta_p$  to obtain the beamformed signal  $\mathbf{Y}_w^{(j)}$ , based on which the temporal correlation  $\mathbf{R}_w^{(j)}$  is computed to get the new frequency estimator  $\mathbf{v}_q^{(j+1)}$ . Both  $\mathbf{w}_p^{(j+1)}$  and  $\mathbf{v}_q^{(j+1)}$  are solved using the MVDR approach, i.e., looking for the filter rejecting the maximum amount of out-of-band power while passing the component at angle  $\theta_p$  or frequency  $f_q$  with no distortion.

Finally, the estimated  $\mathbf{Z}_{\theta f}$  is

$$\hat{\mathbf{Z}}_{\theta f}(p,q) = \mathbf{w}_p^H \mathbf{R}_v \mathbf{w}_p, \quad p \in \{p_c\}, q \in \{q_c\},$$
(15)

where  $\mathbf{R}_v = (\mathbf{Y}_v \mathbf{Y}_v^H)/B$ , and  $\mathbf{Y}_v = \mathbf{Y}(\mathbf{I}_B \otimes \mathbf{v}_a^*)$ .

# V. SIMULATIONS AND EXPERIMENTS

To verify the proposed STCS array concept, we perform simulations and also experiments using ultrasonic sensor array. The following parameters are used for both simulations and experiments. The transmitted ultrasound signal is a CW sinusoid of  $f_c = 40$ kHz (c = 343 m/s and  $\lambda = 4.3$  mm). The target is a person walking indoors with the expected maximum velocity of  $v_m = 4$  m/s producing a maximum Doppler of  $f_m = 933$  Hz  $(2v_m/\lambda)$ , requiring a Nyquist sampling rate of  $F_s = 2$  kHz  $(2f_m)$ . One observation window of T = 32 ms ( $N_t = TF_s = 64$ ) is used to produce one DDoA spectrum estimate. The conventional receiver array has  $N_l = 8$  front-end chains, each equipped with an ADC sampling at  $F_s$ . The STCS receiver array is spatially compressed to  $M_l = 4$ chains, each equipped with an AIC sampling at  $0.5F_s$  ( $N_{\tau} = 8$ ,  $M_{\tau} = 4, B = N_t/N_{\tau} = 8$ ). The compression matrices  $\Phi_a$  and  $\Phi_b$ are generated using random Gaussian distribution. The DoA search range (from  $-90^{\circ}$  to  $90^{\circ}$ ) is divided uniformly into  $N_{\theta} = 90$  points, and the Doppler search range (from  $-F_s/2$  to  $F_s/2$ ) is divided uniformly into  $N_f = 256$  points.

The conventional array with Bartlett DDoA spectrum is used as the reference, i.e.,  $\mathbf{Z}_{\theta f}^{bart} = \boldsymbol{\Psi}_{\theta}^{H} \mathbf{X} \boldsymbol{\Psi}_{f}^{*}$ . For the STCS array, the 2D sparse reconstruction employs M-FOCUSS (s = 0.8, 16 iterations) to solve (9) and FOCUSS (s = 0.8, 32 iterations) to solve (11), and h = 0.5 in (10), and the 2D MVDR employs  $h_1 = h_2 = 0.5$  in (12) and J = 8 iterations for each selected angle-frequency pair.

#### A. Simulation results

We simulate a scenario of K = 2 moving targets, -1 m/s at  $40^{\circ}$ , and 3 m/s at  $-30^{\circ}$ , respectively. The received reflections from the two targets have equal signal strength, with AWGN of 20 dB below the signal power. Fig. 4 shows the simulation results, where (4a) and (4b) show the Bartlett spectrum for the conventional array and the STCS array repectively, (4c) is the STCS array using 2D M-FOCUSS, and (4d) is the STCS array using 2D MVDR. The STCS array produces a Bartlett spectrum with aliasing in angles and frequencies (or velocities) due to the space-time compression, i.e., both the spatial dimension of the array and the temporal domain sampling rate are halved as compared to the conventional array. Using the two proposed methods, the STCS array produces a DDoA spectrum that correctly identifies the two moving targets with high resolution. The 2D M-FOCUSS and the 2D MVDR achieve similar resolutions in the reconstructed DDoA spectrum, while the 2D MVDR requires less computational complexity. Each iteration of the M-FOCUSS involves one matrix pseudo-inverse computation, i.e.,  $\tilde{\Psi}_{\theta}$  of size  $M_l \times N_{\theta}$  in (9) and  $\Psi_f$  of size  $M_t \times N_f$  in (11). Each iteration of the 2D MVDR

In the *j*-th iteration of the above algorithm,  $\mathbf{w}_{n}^{(j)}$  is a beamformer



involves two matrix inverse computations, i.e.,  $\mathbf{R}_w^{(j)}$  of size  $M_l \times M_l$ and  $\mathbf{R}_v^{(j)}$  of size  $M_\tau \times M_\tau$ . The 2D MVDR also converges fast as suggested by the simulation, where  $\mathbf{w}_p^{(j)}$  and  $\mathbf{v}_q^{(j)}$  do not change any more after J = 8 iterations.

## B. Experimental results using ultrasonic sensor array

The commercially available ultrasonic sensors from Knowles [4] are used. The experiment setup is shown in Fig. 5. The transmitter is a single sensor (model 400ST/R160). The receiver ULA is built with eight identical sensors (model SPM0404UD5). One person is moving (towards the sensors) around 0.8 m/s at approximately  $-40^{\circ}$ , at a distance of 2 m from the sensors. ( $0^{\circ}$  refers to the boresight of the ULA)



Fig. 5: The experimental setup.



Fig. 6: Emulated STCS array front-end processing.

Instead of a real circuit implementation, we emulate the STCS array front-end processing as shown in Fig. 6. The received signal from each one of the 8 ultrasonic sensors is sampled at 200 kHz



Fig. 7: Experimental results. 0.8 m/s at  $-40^{\circ}$ .

(with proper filtering around the 40 kHz carrier), which is viewed as the 'analog RF' signal. After  $\Phi_a$  we obtain the spatially-compressed 'analog RF' signal with 4 branches. After mixing and LPF (Nyquist rate bandwidth  $F_s$ ) we obtain the 'analog BB' signal. Finally the AIC  $\Phi_b$  produces the 'digital BB' signal at  $0.5F_s$ . Before the DDoA estimation, a high-pass filter ( $H(z) = 1 - 2z^{-1} + z^{-2}$ ) is applied to remove the reflections from stationary objects. The result is the space-time compressed signal of (7). Fig. 7 shows the experimental results, where similar conclusions to those for the simulation results can be drawn. The STCS array with the two proposed reconstruction methods can correctly identify the moving person.

# VI. CONCLUSION

We have proposed a space-time CS array architecture together with two reconstruction methods for DDoA estimation, by exploiting the sparsity in the angle and frequency domain. The STCS array concept and the algorithms have been tested by simulations as well as experiments using an ultrasonic sensor array. As compared to the conventional array, the STCS array can reduce greatly the frontend processing complexity, which is the dominant part in the power consumption. The proposed algorithms can also be extended to other 2D sparse reconstruction scenarios such as range-DoA and range-Doppler.

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