Emitter Position and Velocity Estimation Given Time and Frequency Differences of Arrival

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***Abstract*—Consider the problem of estimating the position and the velocity of an emitter given time and frequency differences of arrival acquired by a passive sensor array. By jointly eliminating the non-linear nuisance parameters of the model using an appropriate orthogonal projection, we obtain a least squares estimator of the parameters of interest. The advantage of the proposed estimator is the reduction of the computational complexity by an order of the number of sensors in the array.**

1. Introduction

Emitter localization using a passive sensor array continues to attract attention and interest due to its variety of applications including radar, sonar, wireless communications, satellites, airborne systems, and acoustics [1], [2], [3], [4]. The conventional estimation approach is based on two steps: First, one or more parameters (e.g., angle or time of arrival, signal strength or Doppler shift) are measured by the array. Then, the central processing unit of the array determines the emitter parameters (position and/or velocity) by exploiting their mathematical relations to the former intermediate parameters. The approach in [5], [6], on the other hand, estimates the emitter parameters directly from the observations. The focus of the current work is on the former approach.

We consider the case of a moving emitter observed by a stationary passive sensor array. The transmitted signal of the emitter is observed with different times of arrival and Doppler frequency shifts by each of the sensors of the array. Since the transmitted signal of the emitter is assumed to be unknown, a common approach to determine its position and velocity is by measuring the time differences of arrival (TDOAs) and frequency differences of arrival (FDOAs) between pairs of observed signals (for example, by maximizing the ambiguity function [7]). By selecting a reference sensor (e.g., the first sensor), the goal is to estimate the source position and velocity given the differences with respect to that sensor.

Several approaches were suggested to this estimation problem. Weinstein proposed an estimation technique which is applicable for a linear array only and assumes a source in the far-field region [8]. Ho and Xu proposed a two-step estimation procedure [9]. By introducing two nuisance parameters (the range and range rate associated with the reference sensor and the source), they obtain a set of linear equations. In the first step, a weighted least squares (WLS) solution is proposed to estimate the position and velocity of the source together with these nuisance parameters, and in the second step, the relations between the nuisance parameters and the parameters of interest are used to solely estimate the position and velocity using another WLS minimization. The performance of this method was shown to be close to the Cramér-Rao lower bound (CRLB) [9]. Friedlander suggested to estimate the source position and velocity by extending his least squares (LS) method which was developed to locate a stationary source given TDOAs only [10]. The LS position estimate of a stationary source relies on an orthogonal projection matrix to eliminate the nuisance parameter (range between the reference sensor and the source). The idea of Friedlander's extension was to use two similar orthogonal projections in a subsequent manner as follows: first obtain the LS source position as previously explained, and then eliminate the second nuisance parameter (range-rate between the reference sensor and the source) using the same orthogonal projection to get the LS velocity estimate.

In the current work we propose a LS estimator of the source position and velocity which is obtained from using a joint elimination (a single orthogonal projection) of the two nuisance parameters. We develop a model which linearly depends on the parameters of interest (position and velocity vectors of the source), and the nuisance parameter vector (range between the source and the first (reference) sensor, and the corresponding range rate). The proposed approach is based on eliminating the term in the model which depends on the nuisance parameter vector by pre-multypling with an orthogonal projection matrix. This mathematical operation leads to a linear model which solely depends on the parameter vector of interest (position and velocity vectors of the source), and we thus obtain a LS estimate of it. We note that this LS estimator is closely related to the first step of the WLS estimate in [9] following the results in [11].

The performance of the proposed estimator is evaluated with simulations for a source in the near-field and the far-field regions versus the noise variance using a circular sensor array configuration, and versus the number of sensors in the array. The simulations show good results for the proposed estimator

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compared with the two-step approach [9], the subsequent projection technique [10], and the CRLB. In the full version of our work [12] we also provide a detailed analysis of the theoretical covariance matrix of the proposed estimator and we show that the estimates are asymptotically unbiased. We also compare in [12] the computational complexities of the proposed estimator and the two-step estimator, and we show that the complexity of the proposed estimator increases quadratically with respect to the number of sensors, whereas the complexity of the two-step method [9] increases cubically.

2. The TDOA and FDOA Measurements

We assume that M stationary sensors and a moving transmitter are located in a q-dimensional Cartesian coordinate system ($q = 2$ or $q = 3$). We denote by

$$\bar{\mathbf{p}}_s \triangleq [\mathbf{p}_s^T, \dot{\mathbf{p}}_s^T]^T \tag{1}$$

the $2q \times 1$ vector, where $\mathbf{p}_s$ and $\dot{\mathbf{p}}_s$ are the $q \times 1$ true unknown position and velocity vectors of coordinates of the source. We also define the known $q \times 1$ vector of coordinates of the mth sensor by $\mathbf{p}_m$, $m = 1, 2, ..., M$.

Let $\Delta t_{m,1}$ and $\Delta f_{m,1}$ be the true TDOA and FDOA between the signals received by the mth sensor and the first (reference) sensor. The signal propagation speed and the carrier frequency of the signal are given by c and f_c, respectively. The true range $r_{m,1}$ and range-rate $\dot{r}_{m,1}$ differences are then given as

$$\begin{aligned} r_{m,1} &\triangleq c\Delta t_{m,1} = d_{m,s} - d_{1,s} && (2)\\ \dot{r}_{m,1} &\triangleq \frac{c}{f_c}\Delta f_{m,1} = \dot{d}_{m,s} - \dot{d}_{1,s} && (3) \end{aligned}$$

where the range and range-rate between the mth sensor and the source, respectively, are defined as

$$\begin{aligned} d_{m,s} &\triangleq \|\mathbf{p}_s - \mathbf{p}_m\| && (4)\\ \dot{d}_{m,s} &\triangleq \frac{(\mathbf{p}_m - \mathbf{p}_s)^T\dot{\mathbf{p}}_s}{d_{m,s}} && (5) \end{aligned}$$

We next define the $2(M-1) \times 1$ vector $\bar{\mathbf{r}}$ as

$$\begin{aligned} \bar{\mathbf{r}} &\triangleq [\mathbf{r}^T, \dot{\mathbf{r}}^T]^T \\ \mathbf{r} &\triangleq [r_{2,1}, \ldots, r_{M,1}]^T \\ \dot{\mathbf{r}} &\triangleq [\dot{r}_{2,1}, \ldots, \dot{r}_{M,1}]^T \end{aligned} \tag{6}$$

where $\mathbf{r}$ and $\dot{\mathbf{r}}$ are $(M-1) \times 1$ vectors containing the true TDOAs and FDOAs, respectively.

In the presence of additive noise, we are given the noisy $2(M-1) \times 1$ measurements vector,

$$\hat{\bar{\mathbf{r}}} = \bar{\mathbf{r}} + \boldsymbol{\delta} \tag{7}$$

where

$$\begin{aligned} \hat{\bar{\mathbf{r}}} &\triangleq [\hat{\mathbf{r}}^T, \hat{\dot{\mathbf{r}}}^T]^T \\ \hat{\mathbf{r}} &\triangleq [\hat{r}_{2,1}, \ldots, \hat{r}_{M,1}]^T \\ \hat{\dot{\mathbf{r}}} &\triangleq [\hat{\dot{r}}_{2,1}, \ldots, \hat{\dot{r}}_{M,1}]^T \end{aligned} \tag{8}$$

where $\hat{\mathbf{r}}$ and $\hat{\dot{\mathbf{r}}}$ are vectors containing the noisy measurements of the range and range-rate differences, respectively. The $2(M-1) \times 1$ vector $\boldsymbol{\delta}$ representing the additive noise is deefined as

$$\begin{aligned} \boldsymbol{\delta} &\triangleq [\boldsymbol{\epsilon}^T, \boldsymbol{\xi}^T]^T \\ \boldsymbol{\epsilon} &\triangleq [\epsilon_{2,1}, \ldots, \epsilon_{M,1}]^T \\ \boldsymbol{\xi} &\triangleq [\xi_{2,1}, \ldots, \xi_{M,1}]^T \end{aligned} \tag{9}$$

where $\boldsymbol{\epsilon}$ and $\boldsymbol{\xi}$ are $(M-1) \times 1$ vectors representing the errors of the TDOA and FDOA measurements.

The problem we consider herein is: Given the vector of measurements $\hat{\bar{\mathbf{r}}}$, determine the vector of interest $\bar{\mathbf{p}}_s$.

3. The Linear Model and the Proposed Estimator

The development of the linear model is based on the derivations introduced in [10]. Define the $(M-1) \times q$ matrix $\mathbf{S}$ and the $(M-1) \times 1$ vector $\mathbf{u}$ as,

$$\begin{aligned} \mathbf{S} &\triangleq [\mathbf{p}_2 - \mathbf{p}_1, \cdots \mathbf{p}_M - \mathbf{p}_1]^T && (10)\\ \mathbf{u} &\triangleq \frac{1}{2}[\|\mathbf{p}_2\|^2 - \|\mathbf{p}_1\|^2 - r_{2,1}^2, \ldots, \\ &\qquad \|\mathbf{p}_M\|^2 - \|\mathbf{p}_1\|^2 - r_{M,1}^2]^T && (11) \end{aligned}$$

Follwoing [10, Eq. (7a)] we get that,

$$\mathbf{S}\mathbf{p}_s = \mathbf{u} - d_{1,s}\mathbf{r} \tag{12}$$

Next, we define the $(M-1) \times 1$ time derivative vector of $\mathbf{u}$, denoted by $\dot{\mathbf{u}}$, and the 2×1 vector $\bar{\mathbf{d}}_{1,s}$ as

$$\begin{aligned} \dot{\mathbf{u}} &\triangleq [-r_{2,1}\dot{r}_{2,1}, \ldots, -r_{M,1}\dot{r}_{M,1}]^T && (13)\\ \bar{\mathbf{d}}_{1,s} &\triangleq [d_{1,s}, \dot{d}_{1,s}]^T && (14) \end{aligned}$$

Taking the derivative of (12) with respect to time results in [10, Eq. (60)],

$$\mathbf{S}\dot{\mathbf{p}}_s = \dot{\mathbf{u}} - [\ \dot{\mathbf{r}} \quad \mathbf{r}\ ]\bar{\mathbf{d}}_{1,s} \tag{15}$$

The two models in (12) and (15) were considered separately in [10]. We emphasize that most of the work in [10] focused on estimating the position of a stationary source, and therefore the first model was mainly discussed, while the second model was presented as a possible extension to the case of a moving source.

In the current work, on the other hand, we note that these two models contain the vectors of interest, i.e., the position and the velocity of the source. Thus, by combining (12) and (15) we get a linear model with respect to $\bar{\mathbf{p}}_s$ given as,

$$\mathbf{F}\bar{\mathbf{p}}_s = \bar{\mathbf{u}} - \mathbf{H}\bar{\mathbf{d}}_{1,s} \tag{16}$$

where the $2(M-1) \times 1$ vector $\bar{\mathbf{u}}$, the $2(M-1) \times 2q$ matrix $\mathbf{F}$, and the $2(M-1) \times 2$ matrix $\mathbf{H}$ are defined as

$$\begin{aligned} \bar{\mathbf{u}} &\triangleq [\mathbf{u}^T\ ,\ \dot{\mathbf{u}}^T]^T && (17)\\ \mathbf{F} &\triangleq \mathbf{I}_2 \otimes \mathbf{S} && (18)\\ \mathbf{H} &\triangleq \begin{bmatrix} \mathbf{r} & \mathbf{0}_{M-1} \\ \dot{\mathbf{r}} & \mathbf{r} \end{bmatrix} && (19) \end{aligned}$$

where $\mathbf{I}_n$ is an $n \times n$ identity matrix, $\otimes$ is a Kronecker product, and $\mathbf{0}_n$ is an $n \times 1$ vector of zeros.

The linear model in (16) contains both the unknown non-linear nuisance vector $\bar{\mathbf{d}}_{1,s}$ (range and range-rate of the source with respect to the reference sensor) and the unknown vector of interest $\bar{\mathbf{p}}_s$. In [9] the approach is to first estimate $\bar{\mathbf{d}}_{1,s}$ together with $\bar{\mathbf{p}}_s$, and then to use the relation between the two vectors to further refine the previous estimate of $\bar{\mathbf{p}}_s$. In [10] the estimation is based on two subsequent steps: 1) eliminating the term associated with $d_{1,s}$ in (12), with an orthogonal projection matrix [10, Eq. (8)], and obtaining the LS solution for $\mathbf{p}_s$; 2) eliminating the term associated with $\dot{d}_{1,s}$ in (15), using the same orthogonal projection matrix [10, Eq. (8)], and then obtaining the LS solution for $\dot{\mathbf{p}}_s$ (where $d_{1,s}$ involved in the latter solution is calculated using the estimate of $\mathbf{p}_s$ obtained after the first step).

We adopt a different estimation approach. The notion is to jointly eliminate the unknown non-linear nuisance vector $\bar{\mathbf{d}}_{1,s}$ in (16) using an appropriate orthogonal projection matrix which leads to an equation that solely depends on the unknown vector of interest $\bar{\mathbf{p}}_s$. We emphasize that this operation considers the two vectors $\bar{\mathbf{d}}_{1,s}$ and $\bar{\mathbf{p}}_s$ as independent, and therefore ignores the fact that they are mathematically related.

We define the $2(M-1) \times 2(M-1)$ orthogonal projection matrix of $\mathbf{H}$ as,

$$\mathbf{P}^{\perp} = \mathbf{I}_{2(M-1)} - \mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T \tag{20}$$

By pre-multiplying (16) with $\mathbf{P}^{\perp}$ we obtain a linear model which only depends on the vector of interest $\bar{\mathbf{p}}_s$,

$$\mathbf{P}^{\perp}\mathbf{F}\bar{\mathbf{p}}_s = \mathbf{P}^{\perp}\bar{\mathbf{u}} \tag{21}$$

In the presence of noise we replace the true vectors and matrices in (21) by their noisy versions, since we will adopt the noisy measurements vector $\hat{\bar{\mathbf{r}}}$ given in (7). For notation simplicity we denote by $\hat{\mathbf{x}}$ the noisy version of the true vector (or matrix) $\mathbf{x}$. This results in the approximated model

$$\hat{\mathbf{P}}^{\perp}\mathbf{F}\hat{\bar{\mathbf{p}}}_s \cong \hat{\mathbf{P}}^{\perp}\hat{\bar{\mathbf{u}}} \tag{22}$$

The LS estimate of $\bar{\mathbf{p}}_s$ is obtained by the following minimization,

$$\hat{\bar{\mathbf{p}}}_s = \underset{\bar{\mathbf{p}}_s}{\operatorname{argmin}} \left\| \hat{\mathbf{P}}^{\perp}(\mathbf{F}\bar{\mathbf{p}}_s - \hat{\bar{\mathbf{u}}}) \right\|^2 = \hat{\mathbf{Q}}\hat{\bar{\mathbf{u}}} \tag{23}$$

where $\hat{\mathbf{Q}}$ is a $2q \times 2(M-1)$ matrix defined as,

$$\hat{\mathbf{Q}} \triangleq (\mathbf{F}^T\hat{\mathbf{P}}^{\perp}\mathbf{F})^{-1}\mathbf{F}^T\hat{\mathbf{P}}^{\perp} \tag{24}$$

This concludes the derivation of the proposed position and velocity estimator.

4. Approximated Bias and Covariance Matrix

In [12] we provide detailed analysis of the bias and of the covariance matrix of the estimator in (23). Herein, we present the final results only. In the presence of small errors, the position and the velocity estimates in (23) are

$$\hat{\bar{\mathbf{p}}}_s \cong \bar{\mathbf{p}}_s + \Delta\bar{\mathbf{p}}_s \tag{25}$$

where $\Delta\bar{\mathbf{p}}_s$ is the first order approximation of $\hat{\bar{\mathbf{p}}}_s$ (we neglect higher order error terms of $\hat{\bar{\mathbf{p}}}_s$ which depend on products of errors). The first order approximation of the bias is $E[\Delta\bar{\mathbf{p}}_s]$, and the first order approximation of the covariance matrix is $E[(\Delta\bar{\mathbf{p}}_s - E[\Delta\bar{\mathbf{p}}_s])(\Delta\bar{\mathbf{p}}_s - E[\Delta\bar{\mathbf{p}}_s])^T]$.

To obtain these first order approximations we consider the estimate in (23) and perform the following steps. We express the noisy matrix $\hat{\mathbf{Q}}$ and the noisy vector $\hat{\bar{\mathbf{u}}}$ using first order approximations as $\hat{\mathbf{Q}} = \mathbf{Q} + \tilde{\mathbf{Q}}$ and $\hat{\bar{\mathbf{u}}} = \bar{\mathbf{u}} + \tilde{\bar{\mathbf{u}}}$, respectively. After several mathematical steps we get that [12]

$$\begin{aligned} \Delta\bar{\mathbf{p}}_s &= \tilde{\mathbf{Q}}\mathbf{H}\bar{\mathbf{d}}_{1,s} + \mathbf{Q}\tilde{\bar{\mathbf{u}}} \\ &= \mathbf{QJ}\boldsymbol{\delta} \end{aligned} \tag{26}$$

where in the second passing we used the explicit expressions that we derive for the first order approximations $\tilde{\mathbf{Q}}$ and $\tilde{\bar{\mathbf{u}}}$ in [12], and the $2(M-1) \times 2(M-1)$ matrix $\mathbf{J}$ is defined as

$$\mathbf{J} \triangleq -\begin{bmatrix} \mathbf{D}(\mathbf{r} + d_{1,s}\mathbf{1}) & \mathbf{0}_{M-1}\mathbf{0}_{M-1}^T \\ \dot{\mathbf{D}}(\mathbf{r} + d_{1,s}\mathbf{1}) & \mathbf{D}(\mathbf{r} + d_{1,s}\mathbf{1}) \end{bmatrix} \tag{27}$$

where $\mathbf{D}(\mathbf{x})$ is a diagonal matrix with the elements of the vector $\mathbf{x}$ on the main diagonal, and $\dot{\mathbf{D}}(\mathbf{x})$ is the derivative of the matrix $\mathbf{D}(\mathbf{x})$ with respect to time.

Assume that $\boldsymbol{\delta}$ is a zero mean Gaussian random vector. We conclude that the first order approximation of the bias of the estimate $\hat{\bar{\mathbf{p}}}_s$ is zero. The first order approximation of the covariance matrix of $\hat{\bar{\mathbf{p}}}_s$ is therefore given by

$$E[\hat{\bar{\mathbf{p}}}_s\hat{\bar{\mathbf{p}}}_s^T] = \mathbf{QJ}E[\boldsymbol{\delta}\boldsymbol{\delta}^T]\mathbf{J}^T\mathbf{Q}^T \tag{28}$$

Notice that this covariance matrix can be used as a weighting matrix to refine the estimate in (23) using a weighted LS approach.

5. Simulation Results

To demonstrate the performance of the proposed method, we present the results of simulated experiments. We show the root mean square error (RMSE) of the position and velocity estimates using independent Monte-Carlo trials (5000 trials) where in each trial a different noise realization is considered. We compare the position RMSE and the velocity RMSE of the proposed LS estimator with those of the two-step method [9]. We evaluate the CRLB according to the derivation in [9, Appendix C], and the theoretical position RMSE and velocity RMSE of the proposed estimator [12], and those of the two-step estimator according to [9, Eq. (25)].

We assume that the transmitted signal is a white process with variance σ_s^2, independent of the noise processes which are all white, independent processes with variance σ_n^2. Also, the attenuations of the intercepted signal at all sensors are identical. We consider the covariance matrix of the noise vector $\boldsymbol{\delta}$ as given in [8, Section II] [8, Eq. (8), Section II])

$$E[\boldsymbol{\delta}\boldsymbol{\delta}^T] = \mathrm{Diag}(E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T], \beta E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T]) \tag{29}$$

where $\mathrm{Diag}(\mathbf{Z}_1, \cdots, \mathbf{Z}_N)$ is a block diagonal matrix where the matrices $\mathbf{Z}_1, \cdots, \mathbf{Z}_N$ are on the main diagonal, $\beta \triangleq \frac{12}{T^2}$

and T is the observation time, and [8, Eq. (10), Eq. (14)]

$$E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T] \triangleq \gamma(\mathbf{I}_{M-1} + \mathbf{1}_{M-1}\mathbf{1}_{M-1}^T) \quad (30)$$

$$\gamma \triangleq \frac{3\pi c^2}{TW^3}\frac{1+M\text{SNR}}{M\text{SNR}^2} \quad (31)$$

$$\text{SNR} \triangleq \sigma_s^2/\sigma_n^2 \quad (32)$$

It is noteworthy to mention that this covariance matrix assumes that the transmitted signal is a Gaussian random process with a known power spectrum density. Other covariance matrices for different models, which are obtained from analyzing the CRLB, can also be used instead. For example, the covariance matrix given in [13] assumes that the transmitted signal and the attenuations to the sensors are known, and the covariance matrix given in [14] assumes that the signal is deterministic but unknown and also the attenuations to the sensors are unknown.

We evaluate the position RMSE and velocity RMSE versus the parameter γ for a sensor array with a circular configuration. We consider the case where a source is located in the far-field region and the case where the source is located in the near-field region. In the far-field case the position and the velocity vectors of the source are $\mathbf{p}_s = [10000\cos(\pi/3), 10000\sin(\pi/3)]^T$ [meter] and $\dot{\mathbf{p}}_s = [30\sin(\pi/3), 30\cos(\pi/3)]^T$ [meter/sec], respectively. While in the near-field the position of the source is $\mathbf{p}_s = [1000\cos(\pi/3), 1000\sin(\pi/3)]^T$ [meter] with the same velocity vector. The number of sensors in the array is eight and the positions of the sensors are $\mathbf{p}_m = 100 \cdot [\cos\left(\frac{2\pi m}{8}\right), \sin\left(\frac{2\pi m}{8}\right)]^T$ [meter], $m = 1, \ldots, 8$. We vary the parameter $10\log_{10}(\gamma)$ from -50[dB meter^2] to -20[dB meter^2] (in case the source is in the near-field region) and from -80[dB meter^2] to -50[dB meter^2] (in case the source is in the far-field region). We assume that $\beta = 0.1$ [Hz^2]. We normalize the position RMSE by the distance between the source position and the origin, and normalize the velocity RMSE by the Euclidean norm of the source velocity vector. The normalized RMSE of the position and the normalized RMSE of the velocity of the source using the proposed LS estimator, and the two step-approach are shown in Figure 1, where the CRLB is also plotted. We also simulated the approach suggested in [10, Section V]. The normalized theoretical position RMSE and velocity RMSE of the two-step method and the suggested approach obtained from their approximated covariance matrix are also plotted. As can be seen, the RMSE of the LS solution is close to that of the two-step approach and the CRLB, while the RMSE of the subsequent orthogonal projection approach in [10] are worse compared to the LS estimator and the two-step method.

We next evaluated the position RMSE and velocity RMSE versus the number of sensors in the array. We consider a circular configuration as in the first example and a source in the far-field region. The position and the velocity vectors of the source are $\mathbf{p}_s = [10000\cos(\pi/3), 10000\sin(\pi/3)]^T$ [meter] and $\dot{\mathbf{p}}_s = [30\sin(\pi/3), 30\cos(\pi/3)]^T$ [meter/sec], respectively. We vary the number of sensors in the configuration from 8 to 32 with a step of 4. We consider a source in the far-field region, and set $10\log_{10}(\gamma) = -40$[dB meter^2], and $\beta = 0.1$ [Hz^2]. We normalized the position RMSE and velocity RMSE as is explained for the previous simulation. The normalized RMSE of the position and the normalized RMSE of the velocity of the source using the proposed LS estimator and the two step-approach are shown in Figure 2, where the CRLB is also plotted, together with the normalized theoretical RMSE of the two-step method and the suggested method. Observe that compared to the two-step approach, the decrease of the RMSE of the LS method with respect to the number of sensors is smaller. In other words, the proposed approach provides increasingly worse accuracy (relative to the two-step approach) as the number of sensors in the array increases. On the other hand, as the number of sensors increases, the proposed approach becomes more computationally efficient.

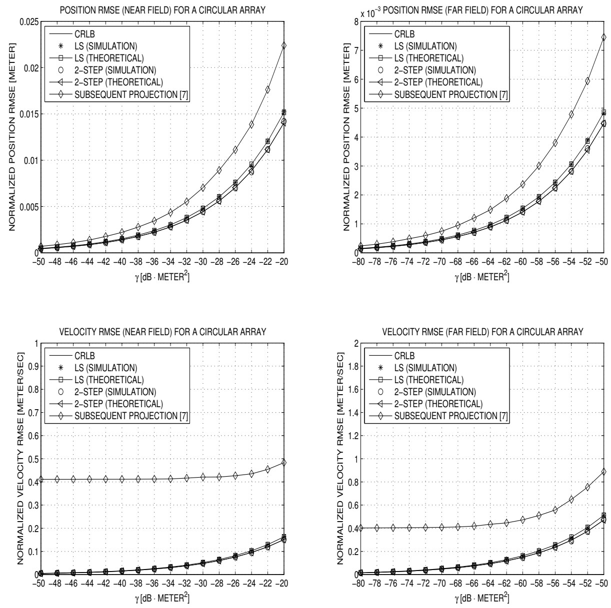

Fig. 1. Normalized theoretical and simulated RMSE of the estimated position and velocity of the source in the far-field and near-field regions versus γ for an array with eight elements in a circular configuration, using the LS proposed method, the two-step approach, and the subsequent projection method [10], all compared with the CRLB.

6. Conclusions

We presented a method to estimate the position and the velocity of a moving transmitting source. The source signal is intercepted by an array of sensors. By collecting the TDOAs and FDOAs measurements obtained from the intercepted signals, we proposed a closed-form least squares estimator which is based on jointly eliminating the unknown nuisance parameters (range and range-rate differences between the reference sensor and the source) using an orthogonal projection matrix. The main benefit of using the proposed estimator is reducing the computational complexity by an order of the number of sensors in the array.

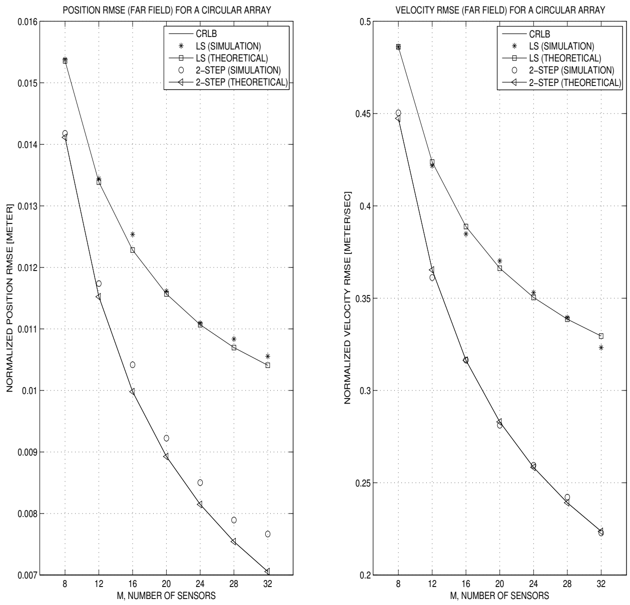

Fig. 2. Normalized theoretical and simulated RMSE of the estimated position and velocity of the source in the far-field and near-field regions versus the number of sensors in an array with a circular configuration, using the LS proposed method and the two-step approach, both compared with the CRLB.

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