ITERATIVE CHANNEL ESTIMATION AND TURBO EQUALIZATION FOR TIME-VARYING OFDM SYSTEMS

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ABSTRACT

We develop a new receiver for orthogonal frequency division multiplexing (OFDM) systems in time-varying channels by embedding channel estimation in a low-complexity block turbo equalizer. A linear minimum mean squared error (MMSE) pilot-assisted channel estimator is presented, and the soft data estimates from the turbo equalizer are used to improve the quality of the channel estimates.

***Index Terms*—** OFDM, channel estimation, turbo equalization, intercarrier interference, time-varying channels

1. INTRODUCTION

OFDM is one of the most important modulation schemes for wireless communications, since it is widely used in many standards such as DVB-T/H, DAB, IEEE 802.11 and IEEE 802.16. OFDM can eliminate intersymbol interference (ISI) introduced by a frequency-selective channel by turning it into a set of parallel frequency-flat channels, and therefore renders simple one-tap equalization for each subcarrier. However, high-mobility terminals and scatterers induce a different Doppler shift on each propagation path, giving rise to a time-selective or time-varying channel, thereby destroying the orthogonality among subcarriers. This for instance occurs in DVB-H and IEEE 802.16 as well as in OFDM for underwater communications.

In order to counteract the effects of a time-varying channel, low-complexity iterative MMSE equalization algorithms have been proposed [1, 2], where soft information is used in an iterative fashion to improve the bit error rate (BER) performance. These methods exploit both the banded structure of the frequency-domain channel matrix and receiver windowing. Optimal joint processing of equalization and decoding at the receiver is prohibitive due to the heavy computational burden. Instead, the equalization and decoding tasks can be performed separately and carried out iteratively, with soft information being interchanged between these two parts. The basis for the turbo equalizer is either a serial linear equalizer [2] or a block linear equalizer [1]. Although both have a comparable complexity, the block version seems to outperform the serial version in case windowing is used [1].

The Doppler shift caused by the high mobility also makes the channel estimation problem more challenging. In practice, the pilot-assisted channel estimation algorithm developed in [6] can be used to model and estimate the frequency- and time-selective channel. However, soft information can be used to improve the quality of channel estimation, as shown in [7] in a different context.

In this paper, we improve the block turbo equalizer of [1], by making the extrinsic information independent from the a priori information, in order to obtain a lower BER than the equalizer of [1]. The complexity is still linear in the number of subcarriers. Further, the pilot-assisted channel estimator of [6] is included in the iterative equalization and decoding loop, where soft data estimates are used to improve the quality of channel estimation. Simulation results show the validity of the equalization algorithm when channel state information is not available.

2. SYSTEM MODEL

We consider a single-user OFDM system with N subcarriers, and a channel that is both frequency- and time-selective. The structure of the transmitter and the receiver is shown in Fig. 1. At the transmitter, a sequence of bits is encoded with error correction coding, and the coded bits are interleaved and mapped into N_d complex symbols, represented by the $N_d \times 1$ vector $\mathbf{s}_d$. We define $\mathbf{s}_p$ as the $N_p \times 1$ vector that stands for the pilot symbols, which are multiplexed with $\mathbf{s}_d$ to form a block of $N = N_d + N_p$ transmitted symbols $\mathbf{s}$. For simplicity, we only consider unit-energy quaternary phase-shift keying (QPSK), and we adopt the standard assumption that the maximal channel order is equal to the OFDM cyclic prefix (CP) length, both denoted by L. This way, the equalizer can be designed separately for each OFDM block, and we can omit the OFDM block index from our notation. At the receiver, after removing the CP, the $N \times 1$ received vector $\mathbf{y}_t$ can be expressed as

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{F}^H \mathbf{s} + \mathbf{n}_t, \tag{1}$$

where $\mathbf{H}_t$ is the $N \times N$ time-domain channel matrix, $\mathbf{F}$ denotes the $N \times N$ unitary DFT matrix, $\mathbf{s}$ represents the $N \times 1$

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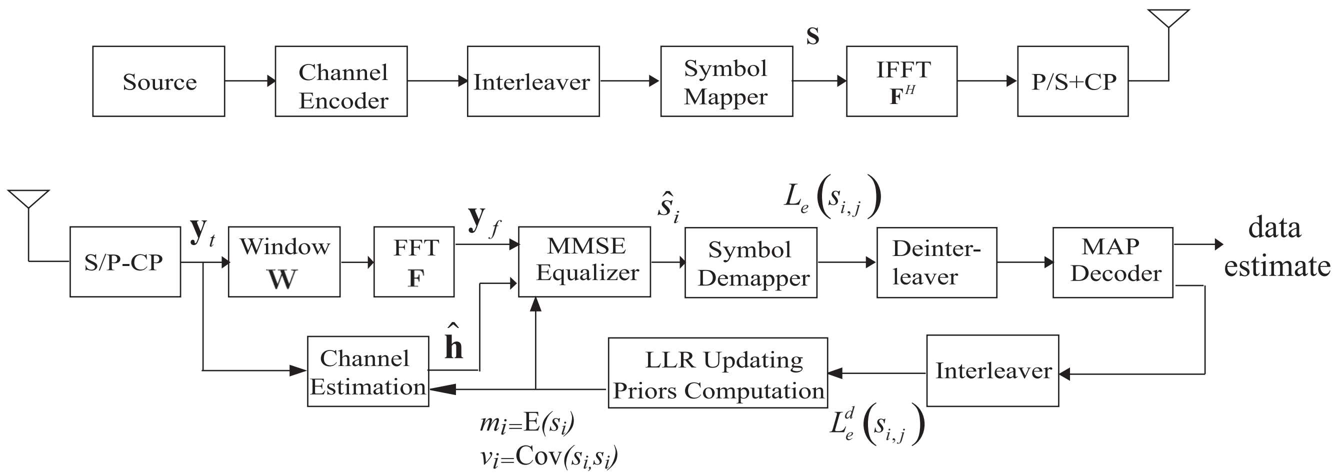

Fig. 1. System model

OFDM symbol consisting of the multiplexed pilot and data symbols, and $\mathbf{n}_t$ stands for the $N \times 1$ noise vector. For simplicity, we assume that $\mathbf{n}_t$ is a circularly symmetric complex Gaussian noise vector, with zero mean and covariance matrix $\mathbf{R}_{\mathbf{n}_t} = E(\mathbf{n}_t\mathbf{n}_t^H) = \sigma_n^2\mathbf{I}_N$. At the receiver, a time-domain window can be applied after CP removal and before the FFT operation. In this case, the output vector after the FFT operation can be expressed as

$$\mathbf{y}_f = \mathbf{F}\mathbf{W}\mathbf{H}_t\mathbf{F}^H\mathbf{s} + \mathbf{F}\mathbf{W}\mathbf{n}_t = \mathbf{H}_f\mathbf{s} + \mathbf{n}_f, \tag{2}$$

where $\mathbf{y}_f = \mathbf{F}\mathbf{W}\mathbf{y}_t$, $\mathbf{n}_f = \mathbf{F}\mathbf{W}\mathbf{n}_t$, $\mathbf{H}_f = \mathbf{F}\mathbf{W}\mathbf{H}_t\mathbf{F}^H$, and $\mathbf{W} = \mathrm{diag}(\mathbf{w})$, with $\mathbf{w}$ the $N \times 1$ vector denoting the time-domain receiver window.

In a time-varying scenario, $\mathbf{H}_t$ is no longer circulant as in the time-invariant case, and $\mathbf{H}_f$ becomes a non-diagonal matrix, giving rise to ICI that corresponds to the non-zero off-diagonal elements of $\mathbf{H}_f$. Fortunately, with a proper window design, $\mathbf{H}_f$ is almost banded, with the most significant elements around the main diagonal [2, 4].

To simplify equalization, the matrix $\mathbf{H}_f$ is further approximated by its banded version

$$\mathbf{H} = \mathbf{H}_f \circ \mathbf{\Theta}, \tag{3}$$

where we use the symbol $\circ$ to denote the Hadamard (element-wise) product between matrices, and $\mathbf{\Theta}$ is the $N \times N$ Toeplitz matrix, which has ones on the main diagonal, the B_c super- and B_c sub-diagonals in a circular sense, and zeros on the remaining entries.

3. LOW-COMPLEXITY TURBO EQUALIZATION

In this section, we improve the block turbo equalizer of [1], by making the extrinsic information independent from the a priori information, thereby improving the BER performance. A detailed discussion can be found in [3]. Let us define s_i as the QPSK symbol on the ith subcarrier, and $(s_{i,1}, s_{i,2})$ as the related bits. The means and the variances of the symbols are denoted as $m_i = \mathrm{E}(s_i)$ and $v_i = \mathrm{Cov}(s_i, s_i)$. For each of the N_p pilot symbols, the mean and variance are set to the pilot symbol value and zero, respectively. As far as the N_d data symbols are concerned, the means and variances are initialized with zeros and ones, respectively. But in every iteration of the turbo equalizer, they are updated using soft information from the estimated symbols, as explained next. Given $\{m_i\}$ and $\{v_i\}$ as prior information, the linear MMSE equalizer leads to

$$\hat{s}_i = \mathbf{g}_i^H(\mathbf{y} - \mathbf{H}\mathbf{m} + m_i\mathbf{h}_i), \tag{4}$$
$$\mathbf{g}_i = (\mathbf{A} + (1-v_i)\mathbf{h}_i\mathbf{h}_i^H)^{-1}\mathbf{h}_i, \tag{5}$$

where $\mathbf{h}_i$ is the ith column of $\mathbf{H}$, $\mathbf{V} = \mathrm{diag}([v_1, \ldots, v_N]^T)$, $\mathbf{m} = [m_1, \ldots, m_N]^T$, $\mathbf{A} = \mathbf{H}\mathbf{V}\mathbf{H}^H + \mathbf{R}_{\mathbf{n}_f}$, and $\mathbf{R}_{\mathbf{n}_f} = E(\mathbf{n}_f\mathbf{n}_f^H)$. At a first glance, this block MMSE equalizer seems very complicated, because a matrix inverse for each subcarrier is required in (5). However, it is possible to show that this equalizer can use a unique shared inverse. Indeed, from the matrix inversion lemma, we obtain

$$\begin{aligned}&(\mathbf{A} + (1-v_i)\mathbf{h}_i\mathbf{h}_i^H)^{-1}\\ =&\mathbf{A}^{-1} - \frac{1-v_i}{1+(1-v_i)t_i}\mathbf{A}^{-1}\mathbf{h}_i\mathbf{h}_i^H\mathbf{A}^{-1},\end{aligned} \tag{6}$$

where t_i is defined as $t_i = \mathbf{h}_i^H\mathbf{A}^{-1}\mathbf{h}_i$. Consequently, $\mathbf{g}_i$ becomes

$$\begin{aligned}\mathbf{g}_i &= \mathbf{A}^{-1}\mathbf{h}_i - \frac{1-v_i}{1+(1-v_i)t_i}t_i\mathbf{A}^{-1}\mathbf{h}_i\\ &= \frac{1}{1+(1-v_i)t_i}\mathbf{A}^{-1}\mathbf{h}_i.\end{aligned} \tag{7}$$

Hence, from (4), the estimated symbol becomes

$$\hat{s}_i = \frac{1}{1+(1-v_i)t_i}\mathbf{h}_i^H\mathbf{A}^{-1}(\mathbf{y}_f - \mathbf{H}\mathbf{m}) + \frac{t_i m_i}{1+(1-v_i)t_i}. \tag{8}$$

From (8), it is clear that the same inverse $\mathbf{A}^{-1}$ can be used for every subcarrier. We highlight that a similar procedure has also been presented in [10] but in a CDMA context.

To compute a new value for m_i and v_i, we make the standard assumption that the probability density function (PDF) $p(\hat{s}_i|s_i = \alpha_k)$ is Gaussian. From (8), we can compute the mean and variance of this PDF as

$$\begin{aligned} \mu_{i,k} &= \frac{1}{1+(1-v_i)t_i} t_i \alpha_k, \\ \sigma_{i,k}^2 &= \mathbf{g}_i^H (\mathbf{A} - v_i \mathbf{h}_i \mathbf{h}_i^H)^{-1} \mathbf{g}_i \\ &= \frac{1}{[1+(1-v_i)t_i]^2} t_i (1 - v_i t_i). \end{aligned} \tag{9}$$

Therefore, the *extrinsic* log-likelihood ratio (LLR) $L_e(s_{i,j}) = L(s_{i,j}|\hat{s}_i) - L(s_{i,j})$, where $L(s_{i,j})$ is the *a priori* LLR and $L(s_{i,j}|\hat{s}_i)$ is the *a posteriori* LLR, can be calculated as [5]

$$\begin{aligned} L_e(s_{i,1}) &= \frac{[1+(1-v_i)t_i]\sqrt{8}\mathrm{Re}(\hat{s}_i)}{1 - v_i t_i}, \\ L_e(s_{i,2}) &= \frac{[1+(1-v_i)t_i]\sqrt{8}\mathrm{Im}(\hat{s}_i)}{1 - v_i t_i}. \end{aligned} \tag{10}$$

The extrinsic LLR $L_e(s_{i,j})$ is passed to the decoder to generate a new extrinsic LLR $L_e^d(s_{i,j})$, which is added to the a priori LLR to form the a posteriori LLR or the new version of the a priori LLR, which is used to update the means and the variances of the estimated symbol as in [5]:

$$\begin{aligned} L_{new}(s_{i,j}) &= L(s_{i,j}) + L_e^d(s_{i,j}), \\ m_{i,new} &= \frac{\tanh(\frac{L_{new}(s_{i,1})}{2}) + i \cdot \tanh(\frac{L_{new}(s_{i,2})}{2})}{\sqrt{2}}, \\ v_{i,new} &= 1 - |m_{i,new}|^2. \end{aligned} \tag{11}$$

The whole procedure described in this subsection can then be repeated, depending on the chosen number of iterations.

In order to calculate $\hat{s}_i$, we need $\mathbf{A}^{-1}(\mathbf{y}_f - \mathbf{Hm})$ and t_i. First, we exploit the banded structure of the approximated frequency-domain channel matrix $\mathbf{H}$ to reduce the complexity of $\mathbf{A}^{-1}(\mathbf{y}_f - \mathbf{Hm})$ by applying a band LDL^H factorization of $\mathbf{A}$ [4]. To reduce the complexity of the t_i calculations, we exploit the fact that the vector $\mathbf{h}_i$ is characterized by $2B_c + 1$ non-zero entries. Hence, to calculate a specific $t_i = \mathbf{h}_i^H \mathbf{A}^{-1} \mathbf{h}_i$, we only need a square subblock of $\mathbf{A}^{-1}$ of dimension $2B_c + 1$. To compute all t_i's, we thus only need to know $4B_c + 1$ diagonals of $\mathbf{A}^{-1}$ which can easily be computed using again the band LDL^H factorization of $\mathbf{A}$ [9]. For more details, we refer the interested reader to [3]. The overall equalization algorithm described above has a complexity of $\mathcal{O}(B_c^2 N)$, where the parameter B_c is usually much smaller than the number of subcarriers N, e.g., $1 \leq B_c \leq 4$.

4. ITERATIVE CHANNEL ESTIMATION

The turbo equalizer presented earlier requires channel state information (CSI) at the receiver. For this purpose, we modify the linear MMSE pilot-assisted channel estimator of [6] in a turbo fashion. Besides the pilots, the soft data estimates originating from the turbo equalizer and the decoder can in subsequent iterations be used as auxiliary pilot symbols in order to improve the quality of the channel estimates [8].

The channel estimator estimates the time-domain channel matrix $\mathbf{H}_t$, and then transforms it to the frequency-domain banded matrix $\mathbf{H}$. We define $h_{n,l}$ as the lth channel tap at the nth time-instance, and $h_{n,l} = 0$ for $l < 0$ or $l > L$, since the maximal channel order is assumed to be L. Thus the elements of $\mathbf{H}_t$ can be expressed as

$$[\mathbf{H}_t]_{p,q} = h_{L+1+p,mod(p-q,N)}, \tag{12}$$

which means there are $N(L+1)$ unknowns to estimate. The basis expansion model (BEM) can be used to reduced the number of unknowns from $N(L+1)$ to $(Q+1)(L+1)$, where $Q+1$ is the number of basis functions [6].

By stacking all the channel taps within the block in one $N(L+1) \times 1$ vector $\mathbf{h}_t = [h_{L+1,0}, \ldots, h_{L+1,L}, \ldots, h_{L+N,0}, \ldots, h_{L+N,L}]^T$, a BEM models this vector as

$$\mathbf{h}_t = (\mathbf{B} \otimes \mathbf{I}_{L+1})\mathbf{h}, \tag{13}$$

where $\mathbf{B} = [\mathbf{b}_0, \ldots, \mathbf{b}_Q]$ is an $N \times (Q+1)$ matrix that has $Q+1$ orthonormal basis functions $\mathbf{b}_q$ as columns, and $\mathbf{h}$ is a $(Q+1)(L+1)$ vector that collects all the BEM coefficients of all the channel taps. Similarly to [6], we can write the received signal $\mathbf{y}_t$ as a function of $\mathbf{h}$

$$\begin{aligned} \mathbf{y}_t &= \mathbf{D}\left\{\mathbf{I}_{Q+1} \otimes [\mathrm{diag}(\mathbf{m})\mathbf{F}_{\mathrm{L}}]\right\}\mathbf{h} \\ &\quad + \mathbf{D}\left\{\mathbf{I}_{Q+1} \otimes [\mathrm{diag}(\mathbf{s}-\mathbf{m})\mathbf{F}_{\mathrm{L}}]\right\}\mathbf{h} + \mathbf{n}_t \\ &= \mathbf{Ph} + \mathbf{d}_t + \mathbf{n}_t, \end{aligned} \tag{14}$$

where $\mathbf{F}_L$ represents the first $L+1$ columns of the matrix $\sqrt{N}\mathbf{F}$, and $\mathbf{D} = [\mathbf{D}_0, \ldots, \mathbf{D}_Q]$, with $\mathbf{D}_q = \mathbf{F}\mathrm{diag}\{\mathbf{b}_q\}\mathbf{F}^H$.

The linear MMSE channel estimate can then be written as

$$\hat{\mathbf{h}} = \mathbf{R}_{\mathbf{h}}\mathbf{P}^H(\mathbf{P}\mathbf{R}_{\mathbf{h}}\mathbf{P}^H + \mathbf{R}_{\mathbf{d}_t} + \mathbf{R}_{\mathbf{n}_t})^{-1}\mathbf{y}_t, \tag{15}$$

where $\mathbf{R}_{\mathbf{h}} = E(\mathbf{h}\mathbf{h}^H)$ and $\mathbf{R}_{\mathbf{d}_t} = E(\mathbf{d}_t\mathbf{d}_t^H)$. We can express $\mathbf{R}_{\mathbf{d}_t}$ as $\mathbf{R}_{\mathbf{d}_t} = \mathbf{D}\mathbf{R}_x\mathbf{D}^H$, where

$$[\mathbf{R}_x]_{m,n} = \begin{cases} v_{\mathrm{mod}(m,N)}[\mathbf{X}]_{m,n} & \text{if } \mathrm{mod}(m-n,N) = 0 \\ 0 & \text{otherwise} \end{cases}, \tag{16}$$

with $\mathbf{X} = (\mathbf{I}_{Q+1} \otimes \mathbf{F}_L)$.

Note that it is not necessary to take all the received samples into account, so that the complexity of the channel estimator could be reduced, as proposed in [6].

5. SIMULATION RESULTS

We consider an OFDM system with $N = 128$ subcarriers, where 8 equidistant subcarriers are reserved for pilots. All

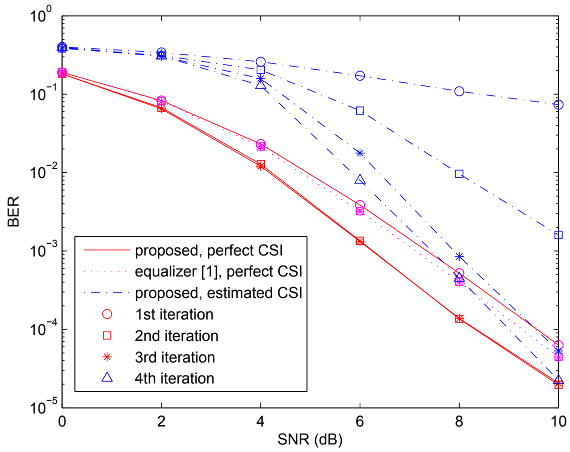

Fig. 2. BER performance, $f_d/\Delta f = 0.35$.

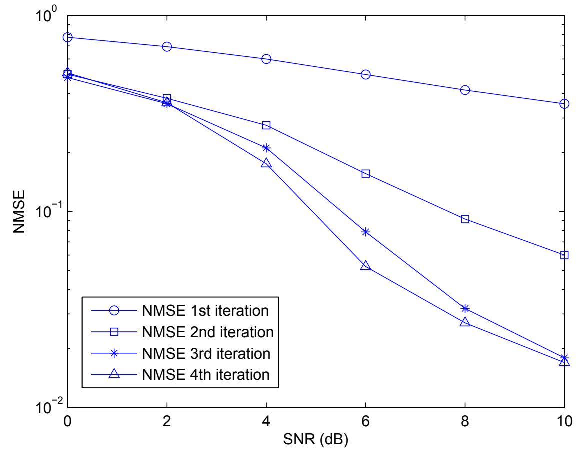

Fig. 3. Channel estimation performance, $f_d/\Delta f = 0.35$.

the pilot and data subcarriers have the same power. The channel order and the CP length are the same and equal to $L = 5$. The channel is assumed to be Rayleigh distributed with uniform power delay profile, and a U-shaped Doppler spectrum. We consider a high-mobility case where the normalized Doppler frequency is $f_d/\Delta f = 0.35$ with f_d the Doppler frequency and Δf the subcarrier spacing. We use the generalized complex-exponential BEM with $Q = 4$ to model the time-varying channel [6].

The time-domain receiver window [4] as well as the equalizer are designed for a bandwidth parameter $B_c = 3$. A rate 1/2 convolutional code with generator matrix [1 0 1;1 1 1] and a block length of 8192 is used. We employ random interleaving. The decoder employs a linear approximation to the log-MAP decoding algorithm.

Fig. 2 shows the BER performance of [1] and the proposed iterative channel estimation and equalization algorithm. It can be seen that the proposed algorithm outperforms [1] in case of perfect CSI. Further, in the case of unknown CSI, the proposed iterative algorithm highly outperforms the LMMSE channel estimator of [6], which coincides with the first iteration of our algorithm. In addition, by increasing the number of iterations, the performance of the proposed algorithm converges to the performance with perfect CSI, at least at medium to high SNR. Fig. 3 shows the normalized mean square error NMSE $= E\{||\mathbf{h}_t - (\mathbf{B} \otimes \mathbf{I}_{L+1})\hat{\mathbf{h}}||^2/N\}$ of the MMSE channel estimator. It can be seen that the channel estimation performance is consistent with the equalization performance.

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