QUANTIZED MULTI-MODE PRECODING FOR SPATIAL MULTIPLEXING MIMO-OFDM SYSTEMS

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ABSTRACT

Spatial multiplexing with multi-mode precoding can achieve both high capacity and high reliability in multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. Multi-mode precoding uses linear transmit precoding but adapts the number of transmit streams or modes according to the channel conditions. Multi-mode precoding typically requires complete knowledge of the transmit precoding matrices for each subcarrier at the transmitter. In this paper we propose to reduce the feedback requirements by sending back the quantized precoding matrices of a fraction of the subcarriers and obtaining the other precoders using interpolation. Two algorithms are proposed for the interpolation of unitary matrices. Bit error rates simulations demonstrate the performance improvements of the proposed algorithms as a function of the feedback rate.

1. INTRODUCTION

Spatial multiplexing for MIMO systems can achieve high spectral efficiencies with moderate complexity. Linear precoding for spatial multiplexing has been proposed that increases the resilience of the system to channel ill-conditioning. This improves the BER performance without guaranteeing full diversity order. Multi-mode precoding, as proposed in [1, 2], varies the number of spatial multiplexing streams, assuming a fixed data rate and exhibits substantial gains over precoded spatial multiplexing with fixed number of data-streams.

The full gains of multi-mode precoding can be achieved only in the presence of perfect CSI at the transmitter. The design of linear precoding matrices with perfect CSI is studied in [3, 4]. In the absence of perfect CSI at the transmitter a feedback channel may be used to provide quantized CSI to the transmitter. In the context of narrowband channels, quantization of the precoding matrices for multi-mode is investigated in [1]. Linearly precoded spatial multiplexing and beamforming for MIMO-OFDM has been studied in the context of quantized CSI such as in [5] but these methods do not naturally apply to multi-mode precoding. This paper proposes a strategy to achieve substantial gains using multi-mode precoding for MIMO-OFDM with quantized CSI at the transmitter. In this paper we focuss on the framework of precoder quantization proposed in [1]. In particular, we consider full CSI at the receiver and a zero-delay, error-free, low-rate feedback channel to convey channel information to the transmitter updated once every channel realization.

Linear precoder designs proposed for narrowband channels may be applied directly to each subcarrier independently, since the marginal distribution of each subcarrier channel resembles the much-studied narrowband channels. Thus, precoder quantization strategies for narrowband channels, for example [1] can be applied to OFDM. But, the trivial extension of narrowband precoder quantization strategies [1] to OFDM results in a linear increase in the feedback cost (in terms of bit-rate on the feedback channel) with the number of subcarriers. Thus per-instance feedback may become prohibitively expensive in the case of OFDM. A ray of hope is provided by the fact that the subcarriers in OFDM are, in general, correlated and thus independent quantization for each subcarrier is suboptimal. This motivates the present research of compressing precoder information for feedback, utilizing the correlation of the adjacent subcarriers.

We propose to feedback information of quantized precoding matrices for a subset of the subcarriers and derive the precoders for the remaining subcarriers at the transmitter by employing smart interpolation strategies. A similar concept has been used in [5] where substantial reductions in feedback rate were demonstrated but does not extend naturally to the case of multi-mode. An important issue is the interpolation of two precoding matrices. It may be observed that the precoding matrices are unitary and the group of unitary matrices does not form a vector space. In other words, linear interpolation will result in a matrix that do not lie in the space of unitary matrices and this necessitates the use of machinery that includes optimization on non-linear manifolds (see [6] for example).

The aspect of quantization of the precoding matrices is discussed in part in [1, 5]. In the context of multi-mode, the codebook design proposed in [1] is not appropriate with precoder interpolation, because the interpolated precoders may exhibit different optimal number of spatial streams than the used pilots precoders. This requires the formulation of a quantization problem that enables multi-mode precoding as well as precoder interpolation.

2. SYSTEM MODEL

We consider a spatial-multiplexing MIMO-OFDM wireless communication system that consists of an M_T-antenna transmitter, an M_R-antenna receiver and N subcarriers [1]. On the n^{th} subcar-

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[1]A word on notation: normal letters designate scalar quantities, boldface lower case letters indicate vectors and boldface capitals represent matrices. $\mathbf{I}_p$ is the $p \times p$ identity matrix. Moreover, $[\mathbf{M}]_{:,1:j}$ denotes the first

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rier, the transmitter optimally maps the $M_s[n]$-dimensional spatial data vector $\mathbf{s}[n] = [s_1[n] \cdots s_{M_s[n]}[n]]^T$, where $M_s[n] \leq \text{Min}(M_T, M_R)$, onto the M_T transmit antennas using a linear precoder $\mathbf{F}[n]$. Thus for the n^{th} subcarrier we can write

$$\mathbf{y}[n] = \mathbf{H}[n]\mathbf{F}[n]\mathbf{s}[n] + \mathbf{n}[n] \tag{1}$$

where $\mathbf{y}[n]$ is the received signal vector, $\mathbf{H}[n]$ is the channel in the frequency-domain, $\mathbf{n}[n]$ is the zero-mean spatially-white complex Gaussian receiver noise vector of covariance matrix $\sigma_n^2 \mathbf{I}_{M_R}$. We also consider equal transmit power and rate allocation across subcarriers. Then $E\{\mathbf{s}[n]\mathbf{s}[n]^H\} = \frac{\mathcal{E}_s}{M_s}\mathbf{I}_{M_s}$. The symbols that form the vector $\mathbf{s}[n]$ are drawn from a chosen constellation $\mathcal{S}[n]$ to satisfy the desired transmission rate. On any given subcarrier n, $\hat{\mathbf{s}}[n]$ may be detected using the MMSE receiver

$$\mathbf{G}[n] = \left(\frac{M_s\sigma_n^2}{\mathcal{E}_s}\mathbf{I}_{M_s} + \mathbf{F}[n]^H\mathbf{H}[n]^H\mathbf{H}[n]\mathbf{F}[n]\right)^{-1}\mathbf{F}[n]^H\mathbf{H}[n]^H.$$

Let $\mathbf{H}[n] = \mathbf{U}[n]\Sigma[n]\mathbf{V}[n]^H$ be the SVD of the MIMO channel on the n^{th} subcarrier. When perfect CSI is available at the transmitter and the receiver is linear, it is well-known [3, 4, 1] that the optimal precoder $\mathbf{F}[n]$ consists of the M_s first columns of $\mathbf{V}[n]$, i.e $\mathbf{F}[n] = [\mathbf{V}[n]]_{.,1:M_s}$. Note that the optimal precoders satisfy the unitary constraints, i.e. $\mathbf{F}[n]^H\mathbf{F}[n] = \mathbf{I}_{M_s}$.

3. PROPOSED PRECODER QUANTIZATION AND INTERPOLATION

We propose to construct a codebook of unitary matrices for the quantization of the complete $M_T \times M_T$-dimensional unitary right singular matrix $\mathbf{V}[n]$. Then we feedback the indices of the square quantized precoders on the U pilot subcarriers $\{\mathbf{V}[n_i]\}_{1\leq i\leq U}$. The U pilot precoders are chosen to be positioned at regular intervals in frequency. The optimal number of modes on each subcarrier, as determined by the receiver is also included in the feedback information. The transmitter interpolates the quantized unitary matrices of the U pilots to reconstruct the precoders on the remaining subcarriers and enforces the optimal mode on every subcarrier. The above strategy enables multi-mode precoding with reduced feedback overhead, through exploiting subcarrier correlation to deploy interpolation.

The selection of the optimal mode for each subcarrier at the receiver is detailed in Subsection 3.1, the design of the precoder codebook is examined in Subsection 3.2 and the precoder interpolation is discussed in Subsection 3.3.

3.1. Subcarrier Mode selection

This section describes the strategy of selecting the mode (determined by the number of datastreams $M_s[n]$ and the constellation $\mathcal{S}[n]$) for the n-th subcarrier. This information is used for selecting the first $M_s[n]$ columns of the interpolated quantized precoder matrix for each subcarrier. The optimality in determining the mode pertains to the minimization of an upper-bound on the symbol-vector error rate [7, 8], which was shown to be achieved through the maximization of the signal-to-noise ratio (SNR) on the weakest spatial stream [9]. More specifically, the mode-selection criterion, for the n^{th} subcarrier is

$$\begin{cases} M_s[n] = \max_{M_s}\{\lambda_{M_s}(\mathbf{H}[n]\mathbf{F}_{:,1:M_s}[n])\frac{\mathcal{E}_s}{M_s\sigma_n^2}\} \\ M_s[n] \cdot log_2(card(\mathcal{S}[n])) = \frac{R}{N} \end{cases} \tag{2}$$

where $\lambda_{M_s}(\mathbf{B})$ denotes the M_s^{th} largest singular value of matrix $\mathbf{B}$, $card(\mathcal{S}[n])$ is the constellation size used to modulate the $M_s[n]$ spatial-multiplexing data streams, such that the rate constraint of R/N per-subcarrier is fulfilled. The mode selection is carried out at the receiver, where the perfect and complete CSI is available. The resulting optimal number of spatial multiplexing streams, to be used on each subcarrier, $\{M_s[n]\}_{1\leq n\leq N}$ are then fed-back to the transmitter together with the precoder information. Finally, the transmitter enforces the optimal spatial multiplexing mode, $\{M_s[n], \mathcal{S}[n]\}$, on each subcarrier n.

3.2. Precoder Quantizer Design

The quantizer design for the unitary matrices $\mathbf{V}[n]$ includes the construction of a codebook $\mathcal{V} = \{\mathbf{V}_1, \cdots, \mathbf{V}_{card(\mathcal{V})}\}$, and determination of a mapping from $\mathbf{V}[n]$ to one of the codebook entries. This also defines Voronoi partitions $\{\mathcal{R}_1, \cdots, \mathcal{R}_{card(\mathcal{V})}\}$ in the space of unitary matrices $\mathcal{U}(M_T, M_T)$, and identifying the optimal codebook entry to represent each of these regions, the codeword selection rule being

$$\mathbf{V}_Q[n] = \mathbf{V}_j \quad \text{if} \quad \mathbf{V}[n] \in \mathcal{R}_j, \quad j = 1, \cdots, card(\mathcal{V}). \tag{3}$$

A natural approach to determine the Voronoi regions is to minimize the average quantization distortion, measured by the mean squared error between the optimal unquantized precoder $\mathbf{V}[n]$ and its quantized version $\mathbf{V}_Q[n]$,

$$\{\mathcal{R}_j, \mathbf{V}_j\}_{1\leq j\leq card(\mathcal{V})} = \arg\min_{\mathbf{V}_Q[n]} \quad E\left\{||\mathbf{V}[n] - \mathbf{V}_Q[n]||_F^2\right\} \tag{4}$$

where the expectation is over the distribution of $\mathbf{V}[n]$. The optimization problem of (4) cannot be solved in closed-form due to the geometrical complexity of the partition regions $\{\mathcal{R}_j\}_{1\leq j\leq card(\mathcal{V})}$. Thus, we resort to the Lloyd's algorithm as in [10, 11], which sequentially optimizes the quantization regions $\{\mathcal{R}_j\}_{1\leq j\leq card(\mathcal{V})}$ and $\{\mathbf{V}_j\}_{1\leq j\leq card(\mathcal{V})}$. It may be noted that the centroid $\mathbf{V}_j$ for a given Voronoi region $\mathcal{R}_j$ may be identified as [10]

$$\mathbf{V}_j = \arg\min_{\mathbf{V}\in\mathcal{U}(M_T,M_T)} E\{||\mathbf{V}[n] - \mathbf{V}||_F^2 | \mathbf{V}[n] \in \mathcal{R}_j\}. \tag{5}$$

In other words, each $\mathbf{V}_j$ is simply the *mean unitary matrix in the Euclidean sense* [2] *over the quantization region* $\mathcal{R}_j$. This mean unitary matrix conveniently turns out to be the orthogonal projection of the arithmetic mean over $\mathcal{R}_j$ in the linear space $\mathcal{C}^{M_T\times M_T}$, onto the space of unitary matrices $\mathcal{U}(M_T, M_T)$ [12]. This crucial result is exploited to easily identify the codebook entries. The optimal mapping for selection of codeword is then given by

$$\mathbf{V}_Q[n] = \arg\min_{\{\mathbf{V}_k\}_{1\leq k\leq card(\mathcal{V})}} ||\mathbf{V}[n] - \mathbf{V}_k||_F^2. \tag{6}$$

A similar quantization criterion was previously used in [10] to quantize the waterpouring transmit covariance matrix. However, the resulting quantization regions and codebook were only used

j columns of the matrix $\mathbf{M}$. $()^H$, $(.)^T$ denote the conjugate transpose and the transpose of a matrix, respectively.

[2] the mean is associated with the Euclidean distance $\text{d}(\mathbf{A}, \mathbf{B}) = ||A - B||_F$.

as “good” initial values that are further on redefined based on a capacity criterion.

This codebook fulfills two fundamental requirements to enable interpolation-based quantized multi-mode precoding. First, it preserves the ordering of the quantized right singular vectors. Second, through quantizing the complete singular matrix, it provides sufficient information to the transmitter for successful interpolation and mode selection. It is computed off-line and known to both the transmitter and the receiver.

3.3. Precoder Interpolation

Based on the pilot unitary precoders, our goal is to recover the precoders on the remaining subcarriers through interpolation. This is reasonable because the frequency correlation exhibited by the MIMO channels across subcarriers also extend to the precoders on these subcarriers [5, 13]. We propose two interpolation strategies that exploit this frequency correlation to interpolate the available unitary pilot precoders $\{\mathbf{V}_Q[n_i]\}_{1\leq i\leq U}$, under a unitary constraint. The first interpolation strategy performs the interpolation directly in $\mathcal{U}(M_T, M_T)$ whereas the second one is inspired from pilot-aided linear MIMO channel estimation for OFDM.

3.3.1. Geodesic interpolation

This approach considers two unitary precoders on successive pilot subcarriers, for instance $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]$ with $1 \leq i \leq U-1$, and interpolates to recover the unitary precoders on all subcarriers in between. It considers these two pilot precoders as two frames on the special unitary group $\mathcal{SU}(M_T, M_T)$ [3] [6], and tries to identify the smoothest trajectory on $\mathcal{SU}(M_T, M_T)$ between these two frames. The rotations constructing this trajectory, referred to as a *geodesic*, are the desired interpolated unitary precoders on the subcarriers between the two successive pilots. This so-called *geodesic interpolation* is widely known in the computer vision literature [14, 15], where it is the optimal way to perform grand tours of 3-D objects. Before detailing the proposed interpolation solution, we highlight that every right singular matrix $\mathbf{V}[n]$, and by extension its quantized version $\mathbf{V}_Q[n]$, is ambiguous up to a diagonal unitary matrix $\mathbf{\Theta}[n]$, which determines the orientation of the right singular vectors. In fact, each orientation matrix $\mathbf{\Theta}[n]$ represents M_T additional degrees of freedom, which we propose to exploit in order to optimize the proposed interpolation solution.

We propose to optimize the orientation matrix $\mathbf{\Theta}[n_i]$ related to the two successive pilot precoders $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]$, with $1 \leq i \leq U-1$, such that the two frames $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]\mathbf{\Theta}[n_i]$ are as close as possible in Frobenius norm

$$\mathbf{\Theta}[n_i] = \min_{\mathbf{D}} ||\mathbf{V}_Q[n_i] - \mathbf{V}_Q[n_{i+1}]\mathbf{D}||_F^2, \tag{7}$$

where $\mathbf{D}$ is diagonal and unitary. This additional optimization aims at identifying the optimal orientation of the quantized singular vectors in $\mathbf{V}_Q[n_{i+1}]$ that minimizes their Euclidean distance to $\mathbf{V}_Q[n_i]$, such that the subsequent geodesic interpolation can then be successfully used to identify the smoothest path between these two pilot precoders. Let $\mathbf{A}[n_i] = \mathbf{V}_Q[n_{i+1}]^H\mathbf{V}_Q[n_i]$, it can be shown, based on [16, p. 431-432], that the optimal orientation matrix $\mathbf{\Theta}[n_i]$ is given by

$$\mathbf{\Theta}[n_i] = diag\left(\frac{\mathbf{A}_{1,1}[n_i]}{|\mathbf{A}_{1,1}[n_i]|}, \cdots, \frac{\mathbf{A}_{M_T,M_T}[n_i]}{|\mathbf{A}_{M_T,M_T}[n_i]|}\right).$$

[3]The set of $M_T \times M_T$-dimensional rotations, or, equivalently, the set of $M_T \times M_T$ unitary matrices with determinant $+1$.

Our proposed solution to the problem of optimizing the orientation matrices, exhibits the attractive feature of solely depending on the quantized pilot precoders, as such it can be carried out at the transmitter and does not require additional feedback, in constrast to [13].

Capitalizing on the previous optimization, we now apply the geodesic interpolation to $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]\mathbf{\Theta}[n_i]$, instead of the original version that simply interpolated between $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]$ [14, 15]. To do so, we first make the following transformation on the pilot frames, to position the start frame on the identity element of the special unitary group $\mathcal{SU}(M_T, M_T)$, where the geodesic is known

$$\begin{cases} \mathbf{V}_Q[n_i] & \rightarrow \quad \mathbf{I}_{M_T} \\ \mathbf{V}_Q[n_{i+1}]\mathbf{\Theta}[n_i] & \rightarrow \quad \mathbf{M} = \mathbf{V}_Q^{-1}[n_i]\mathbf{V}_Q[n_{i+1}]\mathbf{\Theta}[n_i] \end{cases}. \tag{8}$$

It was shown that the geodesic (tangent at the identity element) is defined as

$$\mathbf{\Phi}_\mathbf{I}(t) = \exp(t\mathbf{S}) \quad t \in [0, 1], \tag{9}$$

where $\mathbf{S}$ is skew-hermitian (i.e $\mathbf{S}^H = -\mathbf{S}$) and $\mathbf{M} = \exp(\mathbf{S}) = \mathbf{\Phi}_\mathbf{I}(1)$. This form is known as the exponential map of the unitary matrix $\mathbf{M}$. In fact, every unitary matrix can be written like this, where the matrix exponent is skew-Hermitian [14, 15]. To determine $\mathbf{S}$ starting from $\mathbf{M}$, we use the eigenvalue decomposition $\mathbf{M} = \mathbf{A}\mathbf{\Sigma}\mathbf{A}^{-1}$. Since $\mathbf{\Sigma}$ is a diagonal matrix, we can easily define its exponential map $\mathbf{\Sigma} = \exp(\mathbf{S}_\Sigma)$. Consequently, $\mathbf{M}$ can be re-written as

$$\mathbf{M} = \mathbf{A}\exp(\mathbf{S}_\Sigma)\mathbf{A}^{-1} = \exp(\mathbf{S} = \mathbf{A}\mathbf{S}_\Sigma\mathbf{A}^{-1}). \tag{10}$$

Finally, we can determine the skew-Hermitian matrix of the exponential map of $\mathbf{M}$ in (9) as $\mathbf{S} = \mathbf{A}\mathbf{S}_\Sigma\mathbf{A}^{-1}$. After having determined the exponential map of $\mathbf{M}$, we can reverse the initial transformation of the pilot frames in (8) and consequently identify the geodesic or set of rotations between $\mathbf{V}_Q[n_i]$ and $\mathbf{V}_Q[n_{i+1}]\mathbf{\Theta}[n_i]$ as:

$$\Phi_{\mathbf{V}_Q}(t) = \mathbf{V}_Q[n_i]\mathbf{\Phi}_\mathbf{I}(t) = \mathbf{V}_Q[n_i]\exp(t\mathbf{S}) \quad t \in [0, 1], \tag{11}$$

where $\mathbf{S}$ is given by (10) and the step in the definition of t is determined by the number of subcarriers between the two successive pilot subcarriers.

3.3.2. Conditional interpolation

This approach first considers channel interpolation, based on the MIMO channel matrices acquired on the U pilot subcarriers. Based on the results of this channel interpolation and the knowledge of the structure of the optimal precoders on the remaining subcarriers, this approach tries to identify an “inherited” precoder interpolation. Based on the MIMO channels on the U pilots, it is easy to reconstruct the MIMO channel on the k^{th} subcarrier as [17]

$$\mathbf{H}[n] = \sum_{i=1}^{U}(\mathbf{T}\mathbf{T}_U^\dagger)_{k,n_i}\mathbf{H}[n_i], \tag{12}$$

where $\mathbf{T}$ represents the $N \times N$ DFT matrix and $\mathbf{T}_U$ is the $U \times N$ partial DFT matrix which corresponds to the U pilot positions. For notational brevity, we subsequently write $\alpha_{k,n_i} = (\mathbf{T}\mathbf{T}_U^\dagger)_{k,n_i}$. Since U as well as the position of the pilots are known both at the transmitter and the receiver, the parameters $(\alpha_{k,n_i})_{1\leq k\leq N_c; 1\leq i\leq U}$ are also known at the transmitter. Based on the previous interpolation expression, we try to extract the optimal precoder on the

k^{th} subcarrier based on the knowledge of the precoders on the U pilots. Since the optimal precoder on the k^{th} subcarrier is given by $\mathbf{V}[n]$, where $\mathbf{V}[n]$ contains the eigenvectors of $\mathbf{H}^H[n]\mathbf{H}[n]$, we now detail the expression of $\mathbf{H}^H[n]\mathbf{H}[n]$ based on (12)

$$\mathbf{H}^H[n]\mathbf{H}[n] = \sum_{i=1}^{U} |\alpha_{k,n_i}|^2 \mathbf{V}[n_i]\mathbf{\Sigma}^2[n_i]\mathbf{V}^H[n_i] + \sum_{i \neq j} \alpha_{k,n_i}^* \alpha_{k,n_j} \mathbf{H}^H[n_i]\mathbf{H}[n_j]. \quad (13)$$

Clearly, the calculation of the optimal precoder on the k^{th} subcarrier would require not only the knowledge of the precoders on the pilots $(\mathbf{V}[n_i])_{1 \leq i \leq U}$ but also the knowledge of the corresponding eigenvalues $(\mathbf{\Sigma}^2[n_i])_{1 \leq i \leq U}$ and that of the complete SVD of $(\mathbf{H}^H[n_i]\mathbf{H}[n_j])_{i<j}$. Since the former information is the only one available at the transmitter, we propose to consider the optimal precoder conditioned on the knowledge of the precoders on the U pilots. Rather than considering (13), the right expression to be evaluated is $E_{cond}\{\mathbf{H}^H[n]\mathbf{H}[n]\}$, where $E_{cond}\{.\}$ denotes $E_{\mathbf{H}|(\mathbf{V}[n_i])_{1 \leq i \leq U}}\{.\}$. This expression can be shown to reduce to (see [18] for details)

$$E_{cond}\{\mathbf{H}^H[n]\mathbf{H}[n]\} = \sum_{i=1}^{U} |\alpha_{k,n_i}|^2 \mathbf{V}[n_i] E_{cond}\{\mathbf{\Sigma}^2[n_i]\}\mathbf{V}^H[n_i]. \quad (14)$$

Since the calculation of $E_{cond}\{\mathbf{\Sigma}^2[n_i]\}$ only requires the knowledge of the channel statistics, it can easily be acquired or made available beforehand at the transmitter. Finally, the optimal precoder, given only the knowledge of the precoders on the pilots, is given by

$$\mathbf{T}_{opt} = \text{eigenvectors of } \mathbf{J}, \text{ where,}$$

$$\mathbf{J} = \left(\sum_{i=1}^{U} |\alpha_{k,n_i}|^2 \mathbf{V}[n_i] E_{cond}\{\mathbf{\Sigma}^2[n_i]\}\mathbf{V}^H[n_i] \right). \quad (15)$$

As a final step, $E_{cond}\{\mathbf{\Sigma}^2[n_i]\}$ or equivalently $E_{\mathbf{H}[n_i]}\{\mathbf{\Sigma}^2[n_i]\}$ is calculated using the joint probability density function of the ordered eigenvalues of $\mathbf{H}^H[n_i]\mathbf{H}[n_i]$ [19]. It is worthwhile mentioning that this conditional interpolation is invariant with respect to the orientation of the singular vectors. As such, it avoids the additional optimization of these orientations needed by the geodesic interpolation for optimal performance.

4. PERFORMANCE RESULTS

In the following, we consider a MIMO-OFDM system with 2 transmit and 2 receive antennas and $N = 64$ subcarriers. We use the MIMO channel model provided by the IEEE 802.11 TGn [20] assuming the following parameters: channel model B for the downlink and non line-of-sight propagation, antenna spacing at both the transmitter and the receiver is $\lambda = 5.8cm$, and a sampling rate of 20 MHz. At this sampling rate, channel model B (rms delay spread 15 ns) exhibits $L = 10$ samples. The BER plots were obtained by averaging over more than 500 channel realizations.

Quantization performance: We consider a scenario where the feedback bandwidth is large enough to accommodate the indices of the quantized precoders for all the subcarriers, as well as the mode information on all $N = 64$ subcarriers. Figure 1 illustrates the BER performance degradation due to the quantization of the unitary precoders using $\{2, 3, 4\}$ bits per subcarrier. In the case of multi-mode precoding, as $card(\mathcal{V})$ increases the BER degradation decreases. It is also observed that the dominant mode is single-stream transmission. In the fixed-mode precoding, the uncoded BER performance degrades when the quantization accuracy is increased. This can be explained by the fact that a worse accuracy, in fact, means an imperfect diagonalization of the channel, which basically amounts to a kind of linear precoding of the two transmit streams across the two eigen-modes of the channel. Such "unintentional" linear precoding was shown to exhibit a better uncoded BER performance, yet, a worse coded BER performance [7].

It is worthwhile highlighting that the superior performance of multi-mode precoding over conventional fixed-mode precoding is maintained whatever the resolution of the precoder quantization. Consequently, these exhibited results assess the effectiveness of codebook-based precoder quantization as a means of efficiently representing the unitary precoders, in view of their feedback to the transmitter.

Interpolation with Quantization performance: We illustrate the BER performance of the two proposed interpolators, namely the geodesic and conditional interpolators along with precoder quantization. Figure 2 depicts the average BER performance of our multi-mode selection when the two proposed interpolation solutions are used to recover the unitary precoders, based on only $U = 8$ pilot precoders, drawn from a precoder codebook of cardinality $card(\mathcal{V}) = 4$, for IEEE 802.11 TGn channel B. It turns out that the geodesic and the conditional interpolators lead to very similar BER performances, and exhibits a 2.5 dB SNR degradation, compared to the unquantized multi-mode solution of Figure 1. The marginal BER performance loss, related to this 8-fold precoder down-sampling, illustrates the feedback-reduction potential of capitalizing on the precoders correlation across frequency, to deploy precoder frequency down-sampling combined with our two proposed interpolators.

5. SUMMARY

In this paper, we have proposed a framework of multi-mode precoding for MIMO-OFDM systems when the feedback rate available for conveying the channel state information is limited. The basic ingredients of the proposal are: quantization of the precoding unitary (square) matrices ; interpolation algorithms for reconstructing the precoding matrices of all the subcarriers from the quantized precoding matrices of only a few pilot subcarriers. Simulations with realistic channel models have shown that the proposed algorithms are effective and incur a small loss in performance with reasonable feedback overhead.

In this article, it is considered that the mode information for each subcarrier is fed back perfectly. It is worth mentioning that the mode information may be significantly compressed using simple ideas of clustering - grouping the mode information for adjacent subcarriers (see [18] for details).

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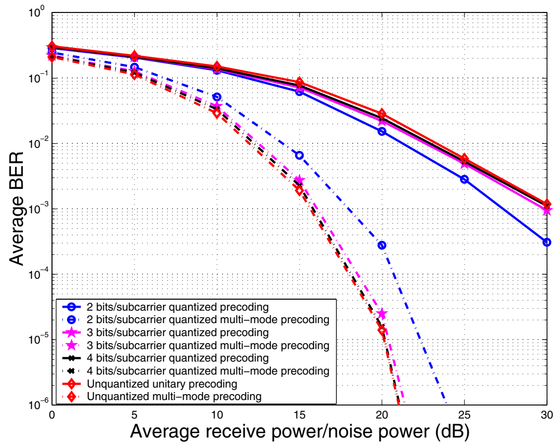

Fig. 1. Average uncoded BER comparison for a 2×2 MIMO-OFDM set-up at $R = 64$ Mbps with channel B and $U = 64$ pilot precoders and $card(\mathcal{V}) = \{4, 8, 16\}$

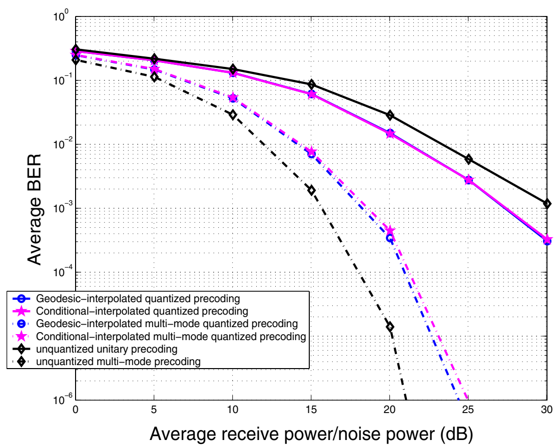

Fig. 2. Average uncoded BER comparison for a $(2, 2)$ MIMO-OFDM set-up at rate 64 Mbps with channel B and 8 pilot precoders and $card(\mathcal{V}) = 4$

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