Noise Suppression in UWB Transmitted Reference Systems[1]

Geert Leus and Alle-Jan van der Veen
Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science
Mekelweg 4, 2628 CD Delft, The Netherlands
Email: {leus,allejan}@cas.et.tudelft.nl

Abstract — **Transmitted reference (TR) systems have recently been proposed for ultra wideband (UWB) communications. They considerably simplify synchronization and channel estimation, which are known to be difficult problems in UWB communications. In this paper, we extend existing receivers for TR-UWB systems by replacing the correlation operation by a linear combination of specific parts of the correlation and weighting the parts that have a small noise contribution more than parts that have a large noise contribution. This turns out to improve the performance considerably.**

I. Introduction

The transmitted reference (TR) approach has been envisioned a long time ago as an effective means to avoid synchronization and channel estimation (see for instance [2]). In [3], it has been re-introduced in the field of ultra wideband (UWB) communications, since for this application, synchronization and channel estimation constitute major challenges. Many researchers picked up on the idea, and by now a plethora of TR-UWB systems have been proposed.

We can basically distinguish two possible types of TR-UWB systems. One type uses a single pulse per frame, where some frames represent a reference and others represent a data symbol. The corresponding receivers generate a template using the reference frames only [5] or using both the reference and data frames [1], and employ this template to correlate it with the frame containing the data symbol that we want to detect. The second type uses multiple pulses per frame, where all pulses together make up the data symbol. The corresponding receivers correlate the frame with a number of delayed versions thereof, and process all outputs to obtain an estimate of the data symbol [3, 4, 6]. Note that the first type of TR-UWB systems has a lower spectral efficiency than the second type of TR-UWB systems. In addition, the first type has to implement longer delays than the second type, but has a better performance than the second type.

Generally, we can assume that the delay spread of the received pulse is much smaller than the frame duration. As a result, a large part of the frame only consists of noise. However, none of the above receivers exploit this property. In this paper, we try to exploit it by replacing the correlation operation by a linear combination of specific parts of the correlation and giving the parts that have a small noise contribution a higher weight than parts that have a large noise contribution. However, since these noise contributions are unknown, the weighting coefficients are derived in a blind fashion based on the received signal.

[1]This research was supported in part by NWO-STW under the VICI program (DTC.5893).

To simplify the presentation, we will only focus on the Hoctor-Tomlinson TR-UWB system [3, 4], which is a special case of the second type of TR-UWB systems. However, the proposed ideas can be extended to other TR-UWB systems as well.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts ^{T}, ^{H}, and † represent transpose, Hermitian, and pseudo-inverse, respectively. Continuous-time (discrete-time) variables are denoted as $x(\cdot)$ ($x[\cdot]$). Finally, $\text{sign}(\cdot)$ denotes the sign operator and $\text{E}(\cdot)$ represents the expectation. All other notation should be self-explanatory.

II. Data Model

As mentioned in the introduction, we consider the Hoctor-Tomlinson TR-UWB system [3, 4] in this work. Each data symbol is represented by N_f frames of duration T_f. Each frame consists of a reference pulse $p(t)$ of duration T_p and a modulated/delayed version thereof. More specifically, the reference pulse is modulated with $b[\lfloor n/N_f \rfloor]c[n]$ and delayed by $d[n]T_d$, where $b[\lfloor n/N_f \rfloor] \in \{+1, -1\}$ is the data symbol related to the nth frame, $c[n] \in \{+1, -1\}$ and $d[n] \in \{1, 2, \dots, D\}$ are the user codes with spreading gain N_f, and T_d is the minimal delay between a reference pulse and a data pulse. Hence, the transmitted signal is given by

$$s(t) = \sum_{n=-\infty}^{\infty} p(t - nT_f) + b[\lfloor n/N_f \rfloor]c[n]p(t - nT_f - d[n]T_d).$$

Assuming $g(t)$ is the propagation channel with delay spread T_g, the received signal is given by

$$y(t) = \sum_{n=-\infty}^{\infty} h(t-nT_f)+b[\lfloor n/N_f \rfloor]c[n]h(t-nT_f-d[n]T_d)+\nu(t),$$

where $h(t) = p(t) \star g(t)$ is the composite channel with delay spread $T_h = T_p + T_g$ and $\nu(t)$ is the additive noise, which includes the received signals from the other users. In the following, we assume that there is no interframe interference between $y(t)$ and $y(t + DT_d)$, i.e., $T_f \geq 2DT_d + T_h$. For the sake of simplicity, we also assume that there is no interpulse interference, i.e., $T_d \geq T_h$, as done in [3]. However, in case there is interpulse interference, we can adapt the proposed ideas following the approach introduced in [4].

III. Autocorrelation Receiver

Under the assumptions made in the previous section, we can use the well-known autocorrelation receiver [3] to detect the data. This receiver correlates each frame with a shifted version thereof:

$$z[n] = \int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} y(t)y(t + d[n]T_d)dt, \tag{1}$$

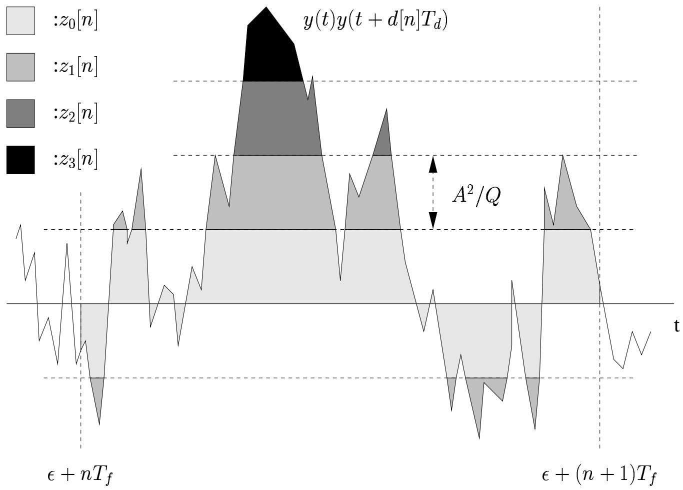

Figure 1: Splitting of the integrand of the correlation into multiple amplitude sections (option 1).

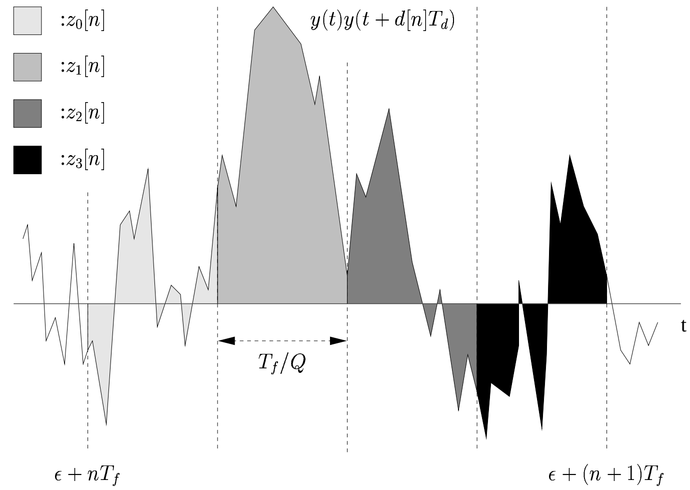

Figure 2: Splitting of the integration interval of the correlation into multiple time sections (option 2).

where ϵ is the timing offset. If this timing offset is chosen such that $\epsilon \in [-T_f + DT_d + T_h, -DT_d)$, it is easy to show that

$$z[n] = \alpha b[\lfloor n/N_f \rfloor]c[n] + \nu[n],$$

where α is given by

$$\alpha = \int_{-\infty}^{+\infty} h^2(t)dt, \tag{2}$$

and $\nu[n]$ can be written as $\nu[n] = \nu^{(1)}[n]b[\lfloor n/N_f \rfloor]c[n] + \nu^{(2)}[n]$, with $\nu^{(1)}[n]$ and $\nu^{(2)}[n]$ given by

$$\begin{aligned}\nu^{(1)}[n] = &\int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} h(t-nT_f-d[n]T_d)\nu(t+d[n]T_d)dt \\ &+ \int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} h(t-nT_f)\nu(t)dt, \end{aligned}\tag{3}$$

$$\begin{aligned}\nu^{(2)}[n] = &\int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} h(t-nT_f)\nu(t+d[n]T_d)dt \\ &+ \int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} h(t-nT_f+d[n]T_d)\nu(t)dt \\ &+ \int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} \nu(t)\nu(t+d[n]T_d)dt. \end{aligned}\tag{4}$$

When $\nu(t)$ is considered to be a zero-mean i.i.d. random variable, $\nu^{(1)}[n]$ and $\nu^{(2)}[n]$ are mutually independent zero-mean i.i.d. random variables, and thus $\nu[n]$ is a zero-mean i.i.d. random variable. Hence, applying a matched filter to $z[n]$ will yield the best performance. Denoting $\mathbf{c}[k] = [c[kN_f], \dots, c[(k+1)N_f - 1]]^T$ and $\mathbf{z}[k] = [z[kN_f], \dots, z[(k+1)N_f - 1]]^T$, an estimate for $b[k]$ is therefore computed as:

$$\hat{b}[k] = \mathrm{sign}\{\mathbf{c}^T[k]\mathbf{z}[k]\}.$$

IV. Proposed Receiver

Since the delay spread of the composite channel T_h is much smaller than the frame duration T_f, a large part of the frame only consists of noise. However, the autocorrelation receiver does not exploit this. In this paper, we try to use this knowledge by replacing the correlation operation by a linear combination of specific parts of the correlation and weighting the parts that have a small noise contribution more than parts that have a large noise contribution. More specifically, we compute $z[n]$ as

$$z[n] = \sum_{q=0}^{Q-1} a_q z_q[n], \tag{5}$$

where $z_q[n]$ represents a specific part of the correlation in (1). Two options are investigated.

Option 1: We can divide the correlation of (1) into different parts by splitting the integrand into multiple amplitude sections (see Figure 1). In other words, we define $z_q[n]$ as

$$z_q[n] = \int_{\epsilon+nT_f}^{\epsilon+(n+1)T_f} f_q(y(t)y(t+d[n]T_d))dt,$$

where

$$f_q(x) = \begin{cases} 0, & \text{if } |x| < qA^2/Q \\ x - \mathrm{sign}(x)qA^2/Q, & \text{if } qA^2/Q \le |x| < (q+1)A^2/Q \\ \mathrm{sign}(x)A^2/Q, & \text{if } |x| \ge (q+1)A^2/Q \end{cases},$$

if $q = 0, 1, \dots, Q-2$, and

$$f_q(x) = \begin{cases} 0, & \text{if } |x| < qA^2/Q \\ x - \mathrm{sign}(x)qA^2/Q, & \text{if } |x| \ge qA^2/Q \end{cases},$$

if $q = Q-1$, with A an indication of the maximum amplitude of the composite channel $h(t)$.

Option 2: We can also divide the correlation of (1) into different parts by splitting the integration interval into multiple time sections. In other words, we define $z_q[n]$ as

$$z_q[n] = \int_{\epsilon+nT_f+qT_f/Q}^{\epsilon+nT_f+(q+1)T_f/Q} y(t)y(t+d[n]T_d)dt.$$

Note that in order for this to work, we have to make sure that for each n the peak in the integrand of (1) always appears on the same position within the window $[\epsilon + nT_f, \epsilon + (n+1)T_f)$, irrespective of $d[n]$. By choosing the integrand as $y(t)y(t + d[n]T_d)$, as we do in this work, this clearly is the case.

A combination of the two options is of course also a possibility. Note that if $a_q = 1$, for $q = 0, 1, \ldots, Q-1$, options 1 and 2 fall back to the autocorrelation receiver. However, by weighting terms that have a small noise contribution more than parts that have a large noise contribution, we can introduce some noise suppression effect, and therefore outperform the autocorrelation receiver as shown in Section VI.

As for the autocorrelation receiver, we can show that if the timing offset is chosen such that $\epsilon \in [-T_f + DT_d + T_h, -DT_d)$, we obtain

$$z_q[n] = \alpha_q b[\lfloor n/N_f \rfloor] c[n] + \nu_q[n], \tag{6}$$

where α_q is given as in (2), reducing the integrand to the qth amplitude section (option 1) or reducing the integration interval to the qth time section (option 2), and $\nu_q[n]$ can again be written as $\nu_q[n] = \nu_q^{(1)}[n] b[\lfloor n/N_f \rfloor] c[n] + \nu_q^{(2)}[n]$, with $\nu_q^{(1)}[n]$ and $\nu_q^{(2)}[n]$ given as in (3) and (4), reducing the integrand of each term to the qth amplitude section (option 1) or reducing the integration interval of each term to the qth time section (option 2). From (5) and (6), we obtain

$$\begin{aligned} z[n] &= \Big(\sum_{q=0}^{Q-1} a_q \alpha_q \Big) b[\lfloor n/N_f \rfloor] c[n] + \sum_{q=0}^{Q-1} a_q \nu_q[n] \\ &= \alpha b[\lfloor n/N_f \rfloor] c[n] + \nu[n]. \end{aligned}$$

For the two options introduced earlier, it is again possible to derive that if $\nu(t)$ is considered to be a zero-mean i.i.d. random variable, $\nu[n]$ also is a zero-mean i.i.d. random variable, and thus applying a matched filter to $z[n]$ will again result in the best performance. Denoting $\mathbf{c}[k] = [c[kN_f], \ldots, c[(k+1)N_f - 1]]^T$ and $\mathbf{z}[k] = [z[kN_f], \ldots, z[(k+1)N_f - 1]]^T$, an estimate for $b[k]$ is therefore again computed as:

$$\hat{b}[k] = \operatorname{sign}\{\mathbf{c}^T[k]\mathbf{z}[k]\}.$$

The question remains how to determine the weighting coefficients a_q. Since the noise contribution to each term $z_q[n]$ is unknown, we will estimate the weighting coefficients blindly based on the received signal, as outlined in the next section.

V. Receiver Optimization

Assume a burst of K data symbols is transmitted: $\mathbf{b} = [b[0], \ldots, b[K-1]]^T$. Defining $\mathbf{z} = [z[0], \ldots, z[KN_f - 1]]^T$ and $\boldsymbol{\nu} = [\nu[0], \ldots, \nu[KN_f - 1]]^T$, we can write

$$\mathbf{z} = \alpha \mathbf{C} \mathbf{b} + \boldsymbol{\nu}, \tag{7}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}[0] & & \\ & \ddots & \\ & & \mathbf{c}[K-1] \end{bmatrix}.$$

Defining $\mathbf{z}_q = [z_q[0], \ldots, z_q[KN_f - 1]]^T$, we can write

$$\mathbf{z} = \mathbf{Z}\mathbf{a},$$

where $\mathbf{Z} = [\mathbf{z}_0, \ldots, \mathbf{z}_{Q-1}]$ and $\mathbf{a} = [a_0, \ldots, a_{Q-1}]^T$. Hence, (7) can be expressed as

$$\mathbf{Z}\mathbf{a} = \alpha \mathbf{C} \mathbf{b} + \boldsymbol{\nu}.$$

We now optimize $\mathbf{a}$ by minimizing the least squares error between $\mathbf{Za}$ and $\alpha\mathbf{Cb}$ with unknowns $\mathbf{a}$ and $\alpha\mathbf{b}$:

$$\min_{\mathbf{a}, \alpha\mathbf{b}} \|\mathbf{Za} - \alpha\mathbf{Cb}\|^2. \tag{8}$$

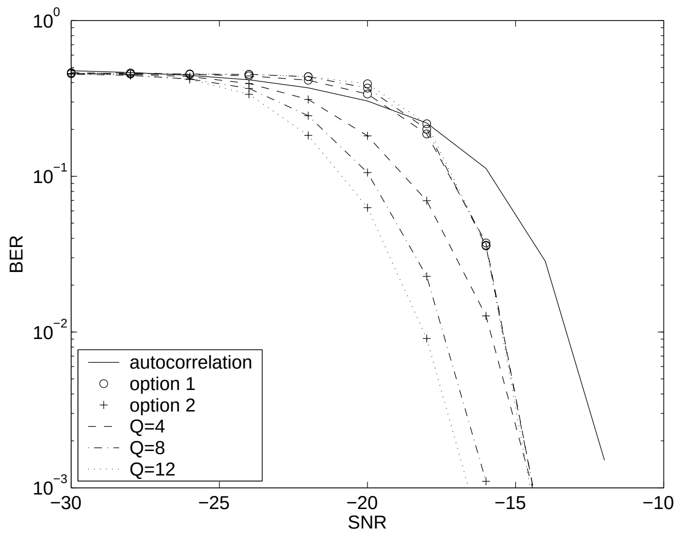

Figure 3: Performance comparison for the frequency-flat non-fading channel case.

It is clear that we need an extra constraint to avoid the trivial solution $\mathbf{a} = \mathbf{0}_{Q\times 1}$ and $\alpha\mathbf{b} = \mathbf{0}_{K\times 1}$. If only a constraint on $\mathbf{a}$ is applied, we can first solve (8) for $\alpha\mathbf{b}$, which leads us to

$$\widehat{\alpha\mathbf{b}} = \mathbf{C}^\dagger \mathbf{Za} = 1/N_f \mathbf{C}^H \mathbf{Za}.$$

Substituting this solution in (8), we obtain

$$\min_{\mathbf{a}} \|(\mathbf{I}_{KN_f} - 1/N_f \mathbf{CC}^H)\mathbf{Za}\|^2.$$

Now the question is what kind of constraint we should put on $\mathbf{a}$. It is important to note that we should choose a constraint that prevents $\mathbf{a}$ from lying in the right null-space of $\mathbf{Z}$. Since $\mathbf{Z}$ has rank 1 in the noiseless case, a unit-norm or monic constraint is not a good option. Therefore, we choose a unit-energy constraint: $\|\mathbf{Za}\|^2 = 1$, which clearly prevents $\mathbf{a}$ from lying in the right null-space of $\mathbf{Z}$. Hence, we will solve

$$\min_{\mathbf{a}} \|(\mathbf{I}_{KN_f} - 1/N_f \mathbf{CC}^H)\mathbf{Za}\|^2, \quad \text{s.t. } \|\mathbf{Za}\|^2 = 1. \tag{9}$$

Defining the 'economy size' singular value decomposition (SVD) of $\mathbf{Z}$ as $\mathbf{Z} = \mathbf{U\Sigma V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices of size $KN_f \times Q$ and $Q \times Q$, respectively, and $\mathbf{\Sigma}$ is a diagonal matrix of size $Q \times Q$, and denoting $\tilde{\mathbf{a}} = \mathbf{\Sigma V}^H \mathbf{a}$, we can rewrite (9) as

$$\min_{\tilde{\mathbf{a}}} \|(\mathbf{I}_{KN_f} - 1/N_f \mathbf{CC}^H)\mathbf{U}\tilde{\mathbf{a}}\|^2, \quad \text{s.t. } \|\tilde{\mathbf{a}}\|^2 = 1.$$

The solution for $\tilde{\mathbf{a}}$ is then given by the right singular vector corresponding to the lowest singular value of $(\mathbf{I}_{KN_f} - 1/N_f \mathbf{CC}^H)\mathbf{U}$. In the next section, we will illustrate that this procedure assigns higher weights to parts with a small noise contribution than to parts with a large noise contribution.

VI. Simulation Results

In this section, we compare the proposed receiver with the autocorrelation receiver. We consider a Hoctor-Tomlinson TR-UWB system where each data symbol is represented by $N_f = 10$ frames of duration $T_f = 50$ ns. As pulse, we adopt the first

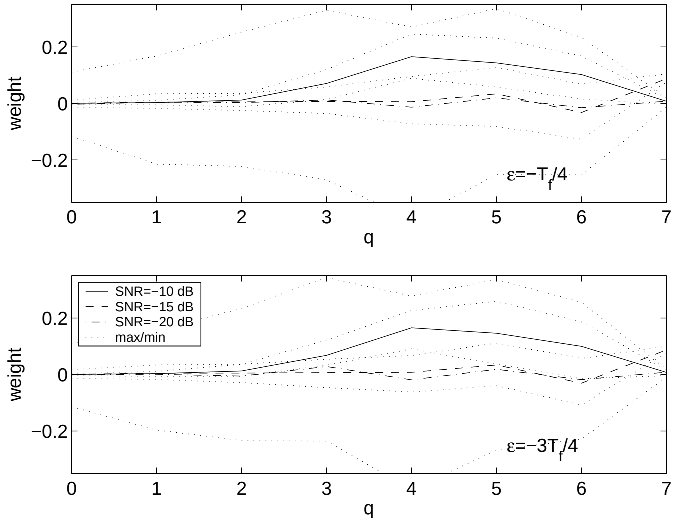

Figure 4: Weighting coefficients related to option 1 for two different timing offsets.

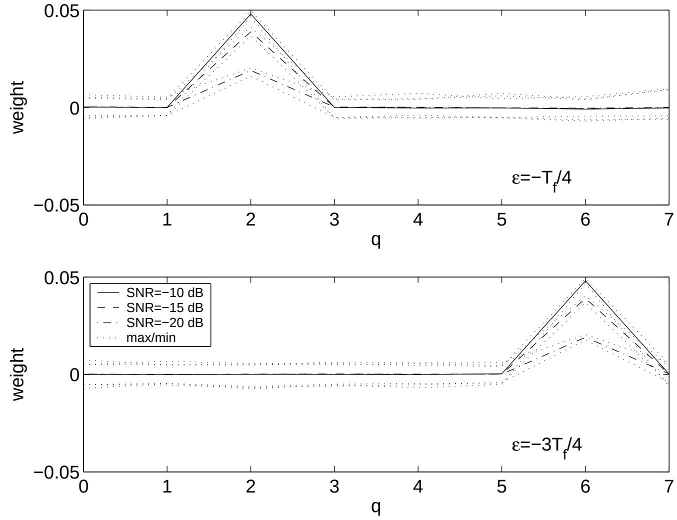

Figure 5: Weighting coefficients related to option 2 for two different timing offsets.

derivative of a Gaussian: $p(t) = e^{-s^2}s$, where $s = (t-T_p/2)5/T_p$ in order for the pulse to have a duration of about T_p. We select this duration to be $T_p = 0.5$ ns. We consider $D = 4$ possible delays between the reference pulse and the data pulse, with a minimal delay of $T_d = 2$ ns. For the sake of simplicity, we consider a single user system and model the additive noise $\nu(t)$ as additive white Gaussian noise (AWGN). The signal-to-noise ratio (SNR) is defined as $SNR = E_f/(T_f\sigma^2)$, where E_f is the average signal energy in a frame: $E_f = 2\mathrm{E}(\int_{-\infty}^{+\infty} h^2(t)dt)$, and σ^2 is the variance of the AWGN $\nu(t)$.

Test case 1: We first consider the frequency-flat non-fading channel case: $g(t) = 1$. Figure 3 shows the performance of the autocorrelation receiver and the two options of the proposed receiver. We consider $Q = 4$, $Q = 8$, and $Q = 12$. The BER results are obtained from 100 Monte Carlo runs of $K = 100$ data symbols, where in each run we consider a new data, code, and noise realization, and a new timing offset $\epsilon \in [-T_f + DT_d + T_h, -DT_d)$. First of all, we observe that both options outperform the autocorrelation receiver, especially at high SNR, and that option 1 is outperformed by option 2 at low to medium SNR. However, the slope of the BER curve for option 1 seems to be higher than for option 2, which means that at some high SNR value, option 1 could outperform option 2. In addition, we observe that option 1 does not really improve with increasing Q, whereas option 2 does, but the improvement gradually diminishes. The latter is caused by the fact that as Q increases, the number of unknowns in the blind receiver optimization algorithm also increases. To illustrate that the proposed receiver optimization algorithm indeed puts more emphasis on low noise parts than on high noise parts of the correlation, Figures 4 and 5 show the behavior of the weighting coefficients a_q for options 1 and 2, respectively. To generate these plots, we consider $Q = 8$. The results are obtained from 100 Monte Carlo runs of $K = 100$ data symbols, where in each run we consider a new data, code, and noise realization, but fix the timing offset to a specific value within the range $[-T_f + DT_d + T_h, -DT_d)$. We plot the mean coefficient profile as well as the maximum and minimum coefficient profiles for two timing offsets and different SNRs. As expected, the coefficient profile related to option 1 is independent of the timing offset ϵ, whereas the coefficient profile related to option 2 puts a higher weight on the time section where the peak of the intergand of (1) is present. On the other hand, we observe that the coefficient profile of option 2 simply scales with the SNR, whereas the coefficient profile of option 1 changes shape with the SNR, putting a high weight on the amplitude section that is important at that SNR. Note that at very low SNR, the gap between the mean coefficient profile and the maximum or minimum coefficient profile related to option 1 is large, which basically means that in this case all kinds of coefficient profiles are possible.

Test case 2: Let us now consider a frequency-selective Rayleigh fading channel case:

$$g(t) = \sum_{l=0}^{L} g_l \delta(t - \tau_l),$$

where g_l is Gaussian distributed with mean 0 and variance $e^{-5l/L}$, and $\tau_l = lT_p/10$ (i.e., $T_g = LT_p/10$ and $T_h = T_p + LT_p/10$). Figures 6 and 7 shows the performance of the autocorrelation receiver with the two options of the proposed receiver for $L = 10$ (i.e., $T_g = 0.5$ ns and $T_h = 1$ ns) and $L = 20$ (i.e., $T_g = 1$ ns and $T_h = 1.5$ ns), respectively. Note that for both values of L, we have no interpulse interference, i.e., $T_d \geq T_h$. We again consider $Q = 4$ and $Q = 8$. As a benchmark, we also show the performance of the optimal receiver, which corresponds to applying a matched filter to $y(t)$. The BER results are obtained from 100 Monte Carlo runs of $K = 100$ data symbols, where in each run we consider a new data, code, and noise realization, a new timing offset $\epsilon \in [-T_f + DT_d + T_h, -DT_d)$, and a new channel realization. Again, we observe that both options perform better than the autocorrelation receiver, and that option 1 is outperformed by option 2. However, this time the performance improvement for option 1 is small, and the slopes of the BER curves for the two options in the considered SNR range are more or less the same. We also observe, as before, that option 1 does not really improve with increasing Q, whereas option 2 does, but only up to a certain value of Q. Next, we notice that

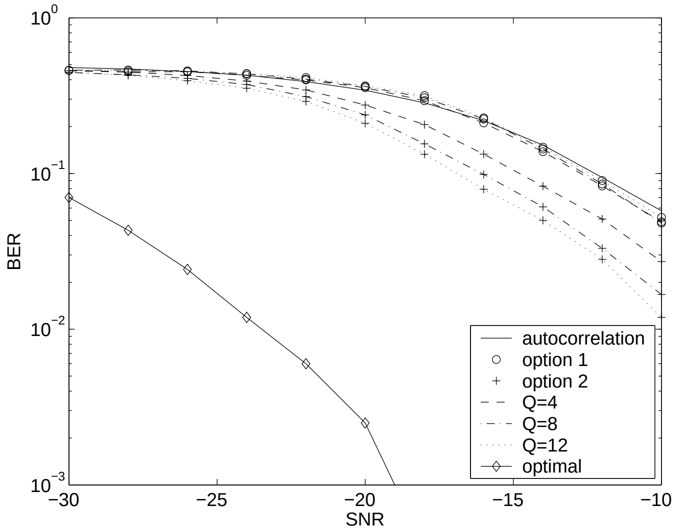

Figure 6: Performance comparison for the frequency-selective Rayleigh fading channel case ($L = 10$).

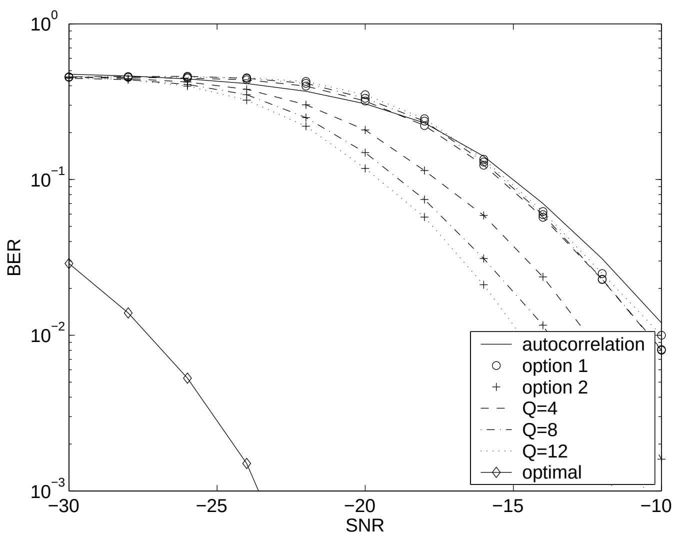

Figure 7: Performance comparison for the frequency-selective Rayleigh fading channel case ($L = 20$).

there is still a big gap between the performance of the proposed receiver and the optimal receiver. However, this is the price we have to pay for relaxing the synchronization and channel estimation requirements related to the optimal receiver. Finally, observe that the performance of all receivers slightly improves with increasing delay spread, due to the increasing multipath diversity. Of course, this only holds as long as $T_d \geq T_h$. As we already indicated, when interpulse interference is present, we can adapt the proposed receiver following the ideas introduced in [4]. This will be a topic for future research.

VII. Conclusions

In this paper, we have extended the autocorrelation receiver for the Hoctor-Tomlinson TR-UWB system by replacing the correlation operation by a linear combination of parts of the correlation and weighting the parts that have a small noise contribution more than parts that have a large noise contribution. Two options were studied. The first option splits the integrand of the correlation into multiple amplitude sections, whereas the second option splits the integration interval of the correlation into multiple time sections. The weighting coefficients are derived in a blind fashion based on the received signal. Both options clearly improve the performance compared to the autocorrelation receiver.

References

[1] S. Franz and U. Mitra, "On Optimal Data Detection for UWB Transmitted Reference Systems," In *Proc. of Global Commun. Conf. (GLOBECOM'03)*, San Francisco, CA, Dec. 2003.

[2] R. M. Gagliardi, "A Geometrical Study of Transmitted Reference Communication Systems," IEEE Trans. on Communication Technologies, pp. 118–123, Dec. 1964.

[3] R. T. Hoctor and H. W. Tomlinson, "Delay-Hopped, Transmitted-Reference RF Communications," In *Proc. of Conf. on UWB Systems and Technologies (UWBST'03)*, Baltimore, MA, May 2002.

[4] A. Trindade, Q. H. Dang, A.-J. van der Veen, "Signal Processing Model for a Transmit-Reference UWB Wireless Communication System," In *Proc. of Conf. on UWB Systems and Technologies (UWBST'03)*, Reston, VA, Nov. 2003.

[5] L. Yang and G. B. Giannakis, "Optimal Pilot Waveform Assisted Modulation for Ultra-Wideband Communications," In *Proc. of Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2002, pp. 733–737.

[6] H. Zhang and D. L. Goeckel, "Generalized Transmitted-Reference UWB Systems," In *Proc. of Conf. on UWB Systems and Technologies (UWBST'03)*, Reston, VA, Nov. 2003.