DISTRIBUTED MAX-SINR SPEECH ENHANCEMENT WITH AD HOC MICROPHONE ARRAYS

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ABSTRACT

In recent years, signal processing with ad hoc microphone arrays has attracted a lot of attention. Speech enhancement in noisy, interfered, and reverberant environments is one of the problems targeted by ad hoc microphone arrays. Most of the proposed solutions require knowledge of fingerprints, such as acoustic transfer functions, which may not be known as accurately as required in practical situations. In this paper, a distributed signal subspace filtering method is proposed which is not restricted to a special graph topology. Here, the maximum signal to interference-plus-noise ratio (max-SINR) criterion is used with the primal-dual method of multipliers for distributed filtering. The paper investigates the convergence of the algorithm in both synchronous and asynchronous schemes, and also discusses some practical pros and cons. The applicability of the proposed method is demonstrated by means of simulation results.

***Index Terms*—** Speech enhancement, max-SINR, ad hoc microphone array, distributed filtering, primal-dual method of multipliers.

1. INTRODUCTION

Speech enhancement is a necessity for goals such as listening comfort, better perception, and improved intelligibility. It is performed in different ways, e.g., by noise removal and interference rejection in which cases the enhancement process is usually done by filtering. The single-channel filtering is usually done in time-domain, frequency-domain, or short-time Fourier transform (STFT) domain [1]. In multichannel speech enhancement, microphone arrays, which can be viewed as multi-input multi-output (MIMO) acoustic systems [2], are used for their spatial filtering capability, namely beamforming, that usually result in better performance over the single-channel approach [3, 4].

Unfortunately, effective beamforming requires prior knowledge or estimation of spatial, or similar, fingerprints such as acoustic transfer functions (ATFs), relative transfer functions (RTFs), direction of arrivals (DOAs), pseudo-coherencies, etc. Luckily, subspace techniques can be used in enhancement algorithms for estimating the required fingerprints on the fly or implicitly in blind enhancement algorithms. Recently, RTFs have been estimated using the generalized eigen-value decomposition (GEVD) in multiple interference suppression [5], the multichannel Wiener filter (MWF) has been expanded to the assumption of low-rank speech signal subspace [6], and finally a family of variable span linear filters has been proposed for using the GEVD to trade–off between known techniques [7, 8].

The aforementioned methods are applicable to a centralized approach with a fusion center (FC); however, there are emerging applications for which such a centralized approach is inefficient (e.g., huge bandwidth or power requirements) or even inapplicable (e.g., irrealizable computational complexity). Ad hoc microphone arrays are among the acoustic systems in which microphones are embedded in distributed portable devices (nodes). Several challenges are inherent to ad hoc microphone arrays including power, computational, and bandwidth limitations, scalability, packet-loss, synchronization, etc. [9]. In recent years, various distributed solutions have been proposed to overcome these challenges. In [10], three distributed techniques, namely LC-DANSE [11], D-LCMV [12], and DGSC [13], have been compared with regard to their computational, communication, and adaptivity characteristics. Unfortunately, these algorithms assume fully connected wireless networks or partially connected networks with a tree structure, which may be violated in ad hoc scenarios, unless a tedious reconfiguration step is employed.

Other distributed algorithms have been proposed without such a restricting assumptions on the geometry, e.g., the distributed delay–and–sum beamformer has been implemented with the randomized gossip algorithm [14], and the distributed minimum variance distortionless response (MVDR) beamformer has been implemented with a message passing algorithm in [15], and with the diffusion adaptation paradigm in [16]. Recently, the primal-dual method of multipliers (PDMM) [17], formerly BiADMM [18, 19], has been proposed for distributed convex optimization. In [20], it has been shown that PDMM has superior convergence compared to the alternating direction method of multipliers (ADMM) [21]. Due to its advantageous properties, the PDMM algorithm has recently been used in various distributed speech enhancing algorithms, e.g., in the distributed ATF-based linearly constrained minimum variance (LCMV) beamforming [22], in the distributed coherency-based MVDR beamforming [23], and in the distributed sparse MVDR beamforming [24].

All of the aforementioned distributed enhancement techniques demand knowledge of the spatial fingerprints or equivalent cues, which are very hard to estimate in the ad hoc arrays, for their random and dynamic geometries. To overcome this challenge, the idea of using subspace methods in enhancement algorithms has been extended to the distributed scheme, e.g., the signal covariance matrix has been estimated using the GEVD (in a fully connected network) [25], and a projection-based subspace estimation has been used in the LCMV beamforming [26]. Unfortunately, the idea behind these algorithms is to estimate the covariance matrices locally in nodes with the number of microphones greater than the required dimension of the subspace, then to use compression to reduce communications per node. This assumption is restricting in the ad hoc microphone arrays, since

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arbitrary nodes may have insufficient number of microphones.

The contribution of this paper is to employ the signal subspace into a distributed speech enhancement algorithm that does not rely on any restricting assumption on the graph. To this end, the maximum signal to interference-plus-noise (max-SINR) enhancement technique is implemented using the PDMM algorithm. The proposed algorithm, based on the generalized eigen-vectors (GEVC), has three advantages: 1) it employs the subspace structure in the distributed optimization problem, 2) the iterative algorithm is applicable to the synchronous or the asynchronous schemes, and to a free graph structure, and 3) it is ready to be turned into a fully blind algorithm, using a distributed GEVC estimator.

In the rest of this paper, we first introduce the distributed max-SINR speech enhancement technique. Section 2 establishes the theoretical foundations. The signal model is presented, and the max-SINR optimization problem is solved by the extended Karush-Kuhn-Tucker (KKT) condition. Next, the signal subspace of the generalized eigenvalue problem is discussed. Finally, the distributed optimization algorithm in which each node uses its respective element from the principal eigenvector is presented. The experiments are presented in Section 3, and the paper is concluded in Section 4.

2. DISTRIBUTED MAX-SINR FILTERING

2.1. Signal Model

We consider an ad hoc microphone array with M nodes (microphones) that constitute a random geometric graph. Our goal is to enhance the desired speech signal from a point source that is contaminated by noise and other interfering sources. The desired and the interference-plus-noise components are assumed to be uncorrelated, zero mean, stationary, broadband, and real. The acoustic enclosure is assumed to be reverberant. We study the problem in the STFT-domain, but we omit the frequency index k and time index t for simplicity, if there is no ambiguity.

In the following, S is the speech source, Y_m is the received signal at node m, with X_m and V_m being its desired and interference-plus-noise components. Then, the signal model using the acoustic transfer function (time-invariant or at least slowly varying) is

$$Y_m = G_m(k)S + V_m = X_m + V_m. \tag{1}$$

By stacking vectors $\mathbf{y} = [Y_1 \quad \cdots \quad Y_M]^T, \mathbf{x} = [X_1 \quad \cdots \quad X_M]^T$, $\mathbf{v} = [V_1 \quad \cdots \quad V_M]^T$, and $\mathbf{g}(k) = [G_1(k) \quad \cdots \quad G_M(k)]^T$, where the transcript ^{T} is the matrix transposition operator, we have

$$\mathbf{y} = \mathbf{g}(k)S + \mathbf{v} = \mathbf{x} + \mathbf{v}. \tag{2}$$

Then, the covariance matrices for the received signal, the desired signal, and the interference-plus-noise component respectively are

$$\begin{aligned} \mathbf{\Phi_x} &= E[\mathbf{x}\mathbf{x}^H] = \phi_S \mathbf{g}(k)\mathbf{g}^H(k), \\ \mathbf{\Phi_v} &= E[\mathbf{v}\mathbf{v}^H], \\ \mathbf{\Phi_y} &= E[\mathbf{y}\mathbf{y}^H] = \mathbf{\Phi_x} + \mathbf{\Phi_v} = \phi_S \mathbf{g}(k)\mathbf{g}^H(k) + \mathbf{\Phi_v}, \end{aligned} \tag{3}$$

where $\phi_S = E[SS^*]$ is the power spectral density of the source signal, and * and ^{H} are the complex conjugate and the Hermitian transpose operators.

2.2. Centralized Maximum SINR Filtering

To establish the optimization problem, we consider a linear filter which provides the enhanced signal:

$$Z = \mathbf{h}^H \mathbf{y}, \tag{4}$$

where $\mathbf{h} = [H_1 \quad \cdots \quad H_M]^T$ is the array weight vector that may be found by solving different optimization problems. As indicated earlier, the objective of this paper is to investigate the maximum SINR filtering problem, so that the optimization problem is

$$\max_{\mathbf{h}} \quad \frac{\mathbf{h}^H \mathbf{\Phi_x} \mathbf{h}}{\mathbf{h}^H \mathbf{\Phi_v} \mathbf{h}} = \max_{\mathbf{h}} \quad \frac{\mathbf{h}^H \mathbf{\Phi_y} \mathbf{h}}{\mathbf{h}^H \mathbf{\Phi_v} \mathbf{h}} - 1. \tag{5}$$

The left and right hand sides of (5) are equivalent assuming that the desired and the interference-plus-noise components are uncorrelated.

The unconstrained optimization problem in (5) is equivalent to the constrained optimization problem of the form:

$$\max_{\mathbf{h}} \quad \mathbf{h}^H \mathbf{\Phi_y} \mathbf{h} \qquad \text{s.t.} \quad \mathbf{h}^H \mathbf{\Phi_v} \mathbf{h} = 1, \tag{6}$$

or, equivalently,

$$\min_{\mathbf{h}} \quad -\mathbf{h}^H \mathbf{\Phi_y} \mathbf{h} \qquad \text{s.t.} \quad \mathbf{h}^H \mathbf{\Phi_v} \mathbf{h} \leq 1, \tag{7}$$

in which $\mathbf{h}$ is constrained to be inside a closed Euclidean ellipsoid. The equivalency of the optimization problems in (6) and (7) is governed by two facts. Firstly, the global maximum of a convex maximization objective with a convex constraint occurs at an extreme point of the constraint set [27], so that substituting $\mathbf{h}^H \mathbf{\Phi_v} \mathbf{h} = 1$ in (6) by $\mathbf{h}^H \mathbf{\Phi_v} \mathbf{h} \leq 1$ does not alter the optimal solution. Secondly, maximization of a non-negative objective function is equivalent to minimizing its additive inverse.

Neither the problem in (6) nor the problem in (7) are standard convex minimization (or concave maximization) problems, to be solved for example with PDMM; however, as stated in [28], the first-order KKT conditions with an associated Lagrange multiplier $\gamma \geq 0$, complemented with the condition that $(-\mathbf{\Phi_y} + \gamma \mathbf{\Phi_v})$ is positive semidefinite, characterize a global solution to (7). "In such a case one can assert that, roughly speaking, the maximization problem is a convex problem in a hidden form." Therefore, we expect that PDMM and ADMM to be able to solve (7).

With this consideration, the optimal solution of (7) is founded by equating the gradient of its Lagrangian to zero:

$$\begin{aligned} \mathcal{L}(\mathbf{h}, \gamma) &= -(\mathbf{h}^H \mathbf{\Phi_y} \mathbf{h}) + \gamma(\mathbf{h}^H \mathbf{\Phi_v} \mathbf{h} - 1), \\ \vec{\nabla}_{\mathbf{h^H}} \mathcal{L}(\mathbf{h}, \gamma) &= -(\mathbf{\Phi_y} \mathbf{h}) + \gamma \mathbf{\Phi_v} \mathbf{h} = 0 \\ \mathbf{\Phi_y} \mathbf{h} &= \gamma \mathbf{\Phi_v} \mathbf{h}. \end{aligned} \tag{8}$$

Then, the optimal weight vector is equal to the eigenvector regarding the largest eigenvalue of the generalized eigenvalue problem in (8). In most cases, it is preferable to solve this problem using the generalized Schur algorithm. Alternatively, if $\mathbf{\Phi_v}$ is invertible, (8) can be transformed into the standard Hermitian eigenvalue problem, i.e.,

$$\big(\mathbf{\Phi_v}^{-1/2} \mathbf{\Phi_y} \mathbf{\Phi_v}^{-1/2}\big)\mathbf{h} = \gamma \mathbf{h}. \tag{9}$$

In this case, for numerical considerations, if M is up to a few thousands, $\mathbf{\Phi_v}$'s Cholesky decomposition can be used instead of its square root. In the sequel, we use the standard Hermitian eigenvalue decomposition of $\tilde{\mathbf{\Phi}}_\mathbf{y} = \mathbf{\Phi_v}^{-1/2} \mathbf{\Phi_y} \mathbf{\Phi_v}^{-1/2}$ in (9).

2.3. Signal Subspace

Assuming that the problem of interest is not under-determined, $\tilde{\mathbf{\Phi}}_\mathbf{y}$ can be decomposed into its eigen-space:

$$\tilde{\mathbf{\Phi}}_\mathbf{y} = \sum_{n=1}^{M} \tilde{\lambda}_n \tilde{\mathbf{q}}_n \tilde{\mathbf{q}}_n^H, \tag{10}$$

where non-zero eigenvalues are sorted in a descending order

$$\tilde{\lambda}_1 \geq \cdots \geq \tilde{\lambda}_M > 0. \tag{11}$$

Since matrix $\tilde{\mathbf{\Phi}}_{\mathbf{y}}$ is Hermitian symmetric, its eigenvectors will be orthonormal, i.e. $\tilde{\mathbf{q}}_n^H \tilde{\mathbf{q}}_n = 1$ and $\tilde{\mathbf{q}}_n^H \tilde{\mathbf{q}}_m = 0, \forall n \neq m$.

Here, we assume a desired point source for which, according to Cauchy-Schwarz inequality, we can write

$$|\mathbf{h}^H \tilde{\mathbf{q}}_1|^2 \leq \|\mathbf{h}\|_2^2 \|\tilde{\mathbf{q}}_1\|_2^2. \tag{12}$$

By constraining $\mathbf{h}^H \tilde{\mathbf{q}}_1 = 1$ and considering $\|\tilde{\mathbf{q}}_1\|_2^2 = 1$,

$$1 = |\mathbf{h}^H \tilde{\mathbf{q}}_1|^2 \leq \|\mathbf{h}\|_2^2. \tag{13}$$

A quadratic penalty term can be augmented to the objective of (7) to force $\mathbf{h}$ to take the lowest allowed ℓ^2 norm in (13), i.e. $\mathbf{h}^H \mathbf{h} = 1$. This implies that the convex optimization problem:

$$\min_{\mathbf{h}} \quad -(\mathbf{h}^H \mathbf{\Phi}_{\mathbf{y}} \mathbf{h}) + \epsilon \|\mathbf{h}\|_2^2 \qquad \text{s.t.} \quad \mathbf{h}^H \tilde{\mathbf{q}}_1 = 1 \tag{14}$$

is equivalent to (7). As discussed in Subsection 2.2, the optimization problem in (14) is feasible if $(-\mathbf{\Phi}_{\mathbf{y}} + \epsilon \mathbf{I}_{M \times M})$ is positive semidefinite which is governed by a proper selection of parameter ϵ. Provided that the principal component is calculable using power iteration and familiar approaches, and each node is aware of its respective element in the principal vector. In the following subsection, we provide algorithms that can be used to solve this optimization problem in a distributed manner.

2.4. Distributed Algorithms

If the optimization objective and the constraint in (14) are decoupled into local terms, distributed convex optimization algorithms can be used. Suppose that the ad hoc microphone array is mapped to the graph $G = (\mathcal{V}, \mathcal{E})$ with vertexes $\mathcal{V}$ (nodes) and edges $\mathcal{E}$. For practical reasons, each node $m \in \mathcal{V}$ may only be connected to a subset of nodes in its neighborhood, $\mathcal{N}(m)$. In order to decouple the objective function, we define auxiliary local terms at node m

$$Z_m(t-l) = \mathbf{h}^H \mathbf{y}(t-l) = \sum_{m \in \mathcal{V}} H_m^* Y_m(t-l), \tag{15}$$

where $\mathbf{y}(t-l), l \in \{0, \ldots, L-1\}$, are the stored received signal vector of the latest L time-frames.

Following the familiar approach in [22,23], the decomposed primal optimization problem equivalent to (14) is obtained

$$\begin{aligned} \min_{\boldsymbol{\omega}_m} \quad & \sum_{m \in \mathcal{V}} \boldsymbol{\omega}_m^H \mathbf{Q} \boldsymbol{\omega}_m \\ \text{s.t.} \quad & \sum_{m \in \mathcal{V}} (\mathbf{A}_m \boldsymbol{\omega}_m - \mathbf{b}) = 0, \end{aligned} \tag{16}$$

where

$$\begin{aligned} \boldsymbol{\omega}_m &= \Big[Z_m(t-L+1), \ldots, Z_m(t), h_m^*\Big]^T, \\ \mathbf{Q} &= \operatorname{diag}\Big(\frac{-1}{ML}, \ldots, \frac{-1}{ML}, \epsilon\Big), \\ \mathbf{A}_m &= \left[\begin{array}{ccc|c} 1 & & \mathbf{0} & -MY_m(t-L+1) \\ & \ddots & & \vdots \\ \mathbf{0} & & 1 & -MY_m(t) \\ \hline 0, & \cdots & ,0 & \tilde{q}_{1,m} \end{array}\right], \\ \mathbf{b} &= \Big[0, \cdots, 0, \frac{1}{M}\Big]^T, \end{aligned} \tag{17}$$

$\tilde{q}_{1,m}$ is the m-th element of $\tilde{\mathbf{q}}_1$, and $\operatorname{diag}(\cdot)$ produces diagonal matrix. As shown in [22, 23], the dual problem of (16) is solved with the PDMM in a systematic way. This targeted dual problem is

$$\begin{aligned} \min_{\boldsymbol{\mu}_m} \quad & \sum_{m \in \mathcal{V}} \Big(\boldsymbol{\mu}_m^H \mathbf{\Omega}_m \boldsymbol{\mu}_m - 2\Re\Big\{\mathbf{b}^H \boldsymbol{\mu}_m\Big\}\Big) \\ \text{s.t.} \quad & \boldsymbol{\mu}_m = \boldsymbol{\mu}_n \qquad \forall (m,n) \in \mathcal{E}, \end{aligned} \tag{18}$$

where $\boldsymbol{\mu}_m$ is the so-called primal-dual parameter at node m, and

$$\begin{aligned} \mathbf{\Omega}_m &= \mathbf{A}_m \mathbf{Q}^{-1} \mathbf{A}_m^H, \\ \boldsymbol{\omega}_m &= \mathbf{Q}^{-1} \mathbf{A}_m^H \boldsymbol{\mu}_m. \end{aligned} \tag{19}$$

At iteration i, PDMM updates the primal-dual variable $\boldsymbol{\mu}_m^{(i)}$ and its dual-dual variable $\boldsymbol{\nu}_{m|n}^{(i)}$, which is defined locally at node m with regard to its neighboring node $n \in \mathcal{N}$. The PDMM updates are

$$\begin{aligned} \boldsymbol{\mu}_m^{(i)} &= \Big(\mathbf{\Omega}_m + \sum_{n \in \mathcal{N}(m)} \mathbf{R}_{mn}\Big)^{-1} \\ & \qquad \Big(\mathbf{b} + \sum_{n \in \mathcal{N}(m)} \big(\mathbf{R}_{mn} \boldsymbol{\mu}_n^{(i-1)} + \frac{m-n}{|m-n|} \boldsymbol{\nu}_{n|m}^{(i-1)}\big)\Big) \\ \boldsymbol{\nu}_{m|n}^{(i)} &= \boldsymbol{\nu}_{n|m}^{(i-1)} - r \mathbf{R}_{mn} \Big(\boldsymbol{\mu}_m^{(i)} - \boldsymbol{\mu}_n^{(i-1)}\Big), \end{aligned} \tag{20}$$

where regularizing matrix $\mathbf{R}_{mn}$ should be optimized to achieve fastest convergence.

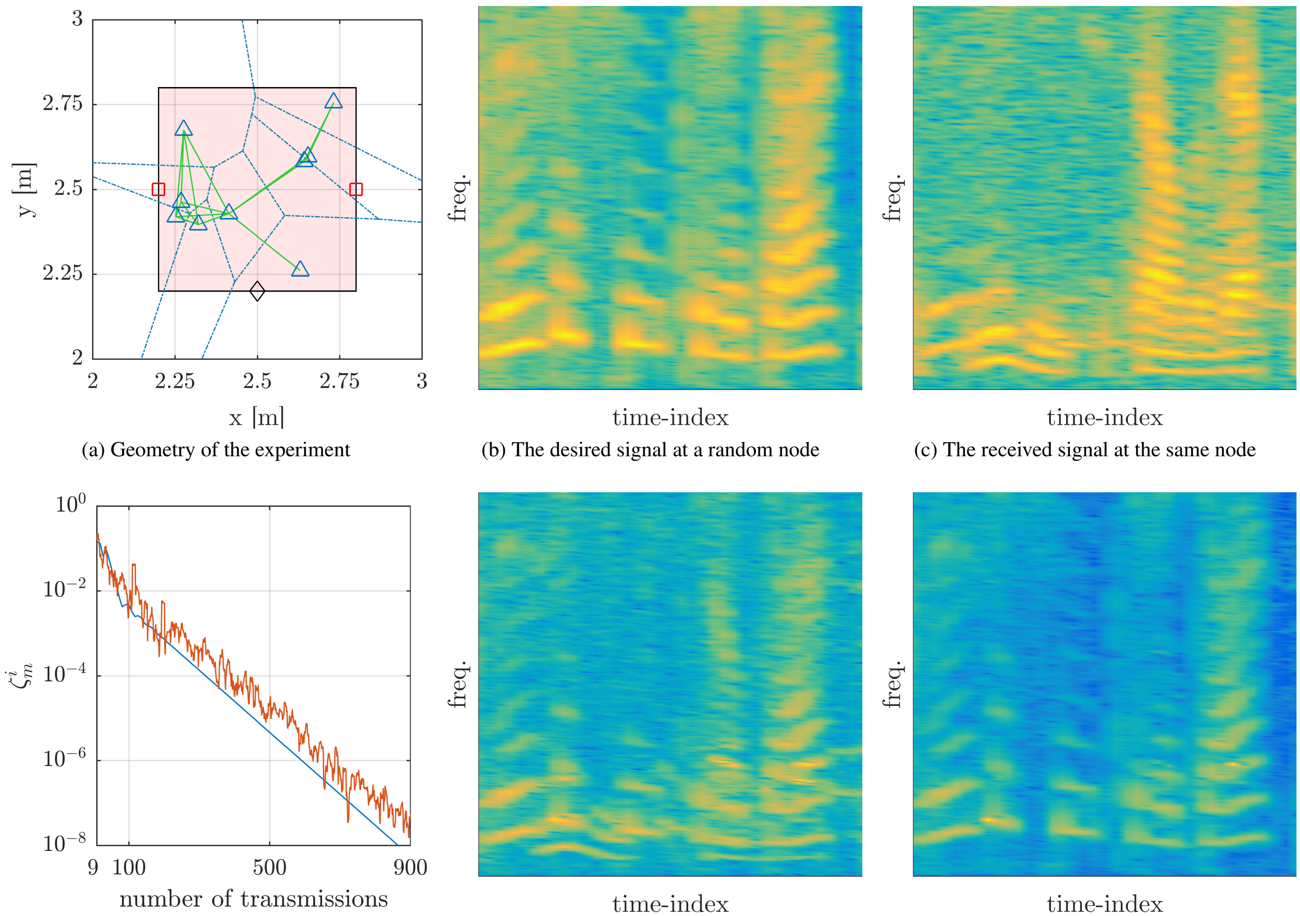
3. EXPERIMENTS

As proof-of-concept, we will here consider an illustrative example, by which the applicability and convergence of the proposed method is inspected; however not in dept, as it is done in previous theoretical and applied works on the PDMM algorithm, e.g. in [17, 20, 22, 23].

An ad hoc microphone array with 9-node random geometric graph is used to enhance the contaminated signal in the setup shown in Fig. 1a. A $5\text{m} \times 5\text{m} \times 3\text{m}$ room is simulated with $\mathrm{T}_{60} = 150$ ms, using the image method [29]. All microphones are omnidirectional. A sampling frequency of 8000 Hz is used with 1024-point DFT and 75% overlapping Hanning window. Speech signals from the TSP speech database are used. A mixture from a desired source (black diamond), and two interferers (marked with red squares), with equal power, is received by randomly distributed microphones (blue triangles). Spatially white Gaussian noise is added to the received signals, about 6 dB bellow the desired components. The desired signal received by a random node m is shown in the STFT-domain in Fig. 1b. The contaminated received signal by node m is shown in Fig. 1c. As observed in the figure, the signal is degraded a lot because of closer interfering sources, which resulted in a powerful interfering-plus-noise component. In the simulations, it is assumed that each node is only aware of the one element in the principal generalized eigenvector, which is regarding itself. The PDMM algorithm is then used to solve the optimization problem in (14), albeit in a distributed manner, to obtain the solution of the max-SINR optimization problem originally formulated in (5). Here, nodes update their local variables without explicit knowledge of the raw data at other nodes. Both synchronous and asynchronous update schemes are simulated. For the asynchronous update scheme a randomly picked node is updated at each iteration, then the dual variables are broadcasted.

To study convergence of the proposed method, and to compare different updating schemes, first the residual of the primal-dual variable, $\boldsymbol{\mu}_m$, is calculated at iteration i:

$$\zeta_m^i = \big\|\boldsymbol{\mu}_m^{(i)} - \boldsymbol{\mu}_m^{(i-1)}\big\|, \quad m \in \mathcal{V}, \tag{21}$$



(a) Geometry of the experiment

(b) The desired signal at a random node

(c) The received signal at the same node

(d) Convergence curves, for synchronous (blue) and asynchronous (red) schemes

(e) The enhanced signal at 45 transmissions

(f) The enhanced signal at 900 transmissions

Fig. 1. Max-SINR speech enhancement with ad hoc microphone arrays exploiting a random geometric graph

Figure 1d shows the convergence for a few hundred transmissions. On the one hand, the synchronous updating scheme results in very little ripples which are vanishing with increased number of transmissions. On the other hand, the asynchronous updating scheme causes steady fluctuations; however it is also converging "on average" with a similar convergence rate to the convergence rate of the synchronous scheme. A question is how one can be sure that variables are converging toward a desirable point.

As indicated earlier, the theoretical background on PDMM, e.g. in [17], has been verified in previously proposed algorithms, e.g., by using the relative convergence plots, by studying some variables that have known global values, or by studying converging point of the array gain as in [23]. In this work, the enhanced signals at two different iterations are used. As shown in Fig. 1e, just with 45 transmissions, which is equivalent to 5 iterations in synchronous updating scheme, most unwanted parts of the received signal have been suppressed while preserving the structure of desired component. Using 900 transmissions, equivalently 100 synchronous iterations, the signal is enhanced to a greater extent as can be seen in Fig. 1f. However, it is also observable that some desired components are also reduced at both stages, but more heavily with increased number of iterations. This is, of course, inherent to the max-SINR speech enhancement, and would also be visible in the centralized enhancement. Moreover, there are certain methods to overcome this drawback, e.g., using variable span filtering techniques. Another point of problem in both Fig. 1e and Fig. 1f is the few isolated points at which the PDMM algorithm is diverging. This can in fact be a result of unfulfilled assumptions, e.g., from problematic matrix rank deficiency. Again, this is not a problem solely for the proposed distributed algorithm; however, as this experience suggests, selection of some regularizing parameters such as ϵ and $\mathbf{R}_{mn}$ can be critical at some point, which of course can be solved with better selections.

4. CONCLUSIONS

In this paper, we have proposed a distributed algorithm, which employs the signal subspace to obtain the optimal solution with respect to the max-SINR criterion. We have shown that such an optimization problem can be solved successfully with the PDMM algorithm. The convergence of the algorithm has been compared under synchronous and asynchronous updating schemes, and the results suggest similar behavior on average. Moreover, some practical considerations regarding the distributed max-SINR algorithm are discussed within an experiment.

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