BLIND SPARSITY-AWARE MULTI-SOURCE LOCALIZATION

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ABSTRACT

We tackle the problem of localizing multiple sources in multipath environments using a recently proposed sparsity-aware correlation-based localization paradigm. It is shown in our previous studies that involving cross-correlations leads to a considerable improvement in terms of number of identifiable sources; however, the sources need to have *known* and *similar* statistics in order to construct a fingerprinting map and localize them. To surmount this constraint, we approach the problem of sparsity-aware localization from a frequency-domain perspective and propose a method which is *blind* to the statistics of the source signals. Moreover, we also show how this approach can further improve the performance in terms of number of identifiable sources. Our simulation results corroborate the efficiency of the proposed algorithm in terms of localization accuracy as well as detection capability.

Index Terms— Multi-source localization, multipath environments, RSS fingerprinting, sparse reconstruction.

1. INTRODUCTION

Precise localization of multiple sources is a fundamental problem which has received an upsurge of attention recently. A traditional wisdom in received signal strength (RSS)-based localization tries to extract distance information from the RSS measurements. However, this approach fails to provide accurate location estimates due to the complexity and unpredictability of the wireless channel. This has motivated another category of RSS-based positioning, the so-called location fingerprinting, which discretizes the physical space into grid points (GPs) and creates a map representing the space by assigning to every GP a location-dependent RSS parameter, one for every access point (AP). The location of the source is then estimated by comparing run-time phase measurements with a fingerprinting map (constructed in the *training phase*) at the source or APs, for instance using K-nearest neighbors (KNN) [1] or Bayesian classification (BC) [2].

A deeper look into the grid-based fingerprinting localization problem reveals that the source location is unique in the spatial domain, and can thus be represented by a 1-sparse vector. This motivated the use of compressive sampling (CS) to recover the location of the source using a few measurements by solving an ℓ_1 -norm minimization problem. This idea illustrated promising results for the first time in [3, 4] as well as in the following works [5, 6]. In [5], a two-step CS-based indoor localization algorithm for multiple targets is proposed where next to a coarse localization step (cluster matching) a fine localization step is considered in which CS is used to recover sparse signals. In [6], a greedy matching pursuit algorithm is proposed for RSS-based target counting and localization with high accuracy. In [7], we have proposed to reformulate the sparsity-aware localization problem so that we could make use of the cross-correlations of the signal readings at the different APs which had not been exploited before. An interesting by-product of the proposed approach in [7] is that under some conditions we could convert the given underdetermined problem to an overdetermined one and efficiently solve it using classical least squares (LS). A similar idea to incorporate cross-correlations of received signals at different elements of a linear array can also be found in the directionof-arrival (DOA) estimation context [8].

However, involving cross-correlations to gain more information constrains the source(s) in the run-time phase to have known (and similar in case of multiple sources) statistics in order to measure/compute a fingerprinting map in the training phase. This is not fulfilled in many practical passive localization scenarios and thus limits the range of applications of such algorithms. To overcome this important constraint permitting a wider variety of applications, we propose to approach the problem from a frequency-domain perspective by the help of a proper filter bank design leading to a method that is blind to the statistics of the source signals. Moreover, we also show that incorporating this information in the frequency domain improves the performance in terms of number of identifiable sources. The rest of the paper is organized as follows. In Section 2, the sparsity-aware network model is explained. Section 3 starts with a brief review of our recently proposed algorithm in [7] and next the idea of blind fingerprinting using frequency-domain information is presented. Simulation results in Section 4 corroborate our analytical claims. Finally, the paper is wrapped up in Section 5.

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2. SPARSITY-AWARE NETWORK MODEL

Consider that we have M APs distributed over an area which is discretized into N cells each represented by its central GP. We consider K source nodes (SNs) which are randomly located on these GPs. We assume that the APs are connected to each other in a wireless or wired fashion so that they can cooperate by exchanging their signal readings. Now, if the k-th SN broadcasts a time domain signal $s_k(t)$, the total received signal at the m-th AP can be expressed by

$$x_m(t) = \sum_{k=1}^{K} x_{m,k}(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} h_{l,m,k} s_k(t - \tau_{l,m,k}) + n_m(t).$$

where we consider an L-path channel for every SN with $h_{l,m,k}$ and $\tau_{l,m,k}$ respectively denoting the channel coefficient and time-delay of the *l*-th path from the *k*-th SN to the *m*-th AP; $n_m(t)$ is the additive noise at the *m*th AP. Our main assumptions on the signals and noises can be explained as follows. The signals $s_k(t)$ are assumed to be ergodic, mutually uncorrelated sequences, i.e., $\mathbb{E}\{s_k(t) \mid s_{k'}^*(t')\} = \eta_k r_k(t-t')\delta_{k-k'}$, with η_k being the k-th signal power, $r_k(\tau)$ the normalized signal correlation function with $r_k(0) = 1$, and δ_k the unit impulse function. Meanwhile, $\mathbb{E}\{.\}$ denotes the statistical expectation which is equal to temporal averaging due to the ergodic property of the signals. The noises $n_m(t)$ are assumed to be ergodic, mutually uncorrelated white sequences, i.e., $\mathbb{E}\{n_m(t) \ n_{m'}^*(t')\} = \sigma_n^2 \delta(t-t') \delta_{m-m'}, \text{ with } \sigma_n^2 = N_0 B$ the variance of the additive noise with density N_0 within the operating frequency bandwidth B, and $\delta(t)$ the Dirac impulse function. The transmitted signals are uncorrelated with the additive noise, i.e., $\mathbb{E}\{s_k(t) \mid n_m(t')\} = 0, \forall t, t' \text{ and } \forall m, k.$

Passive Localization: We consider $r_k(\tau) \neq r_{k'}(\tau)$ and assume they are unknown. Such a set-up fits into many passive localization scenarios, where we do not have any control over the source signals. Therefore, we need to develop an approach which is *blind* to the $r_k(\tau)$'s and uses the total received signals at the APs to localize the SNs simultaneously.

3. PROPOSED FREQUENCY-DOMAIN APPROACH

In [7] as well as in a subsequent extended work [9], in contrast to the existing studies in literature, we have proposed to consider $\tilde{\mathbf{y}} = \mathbb{E}\{\mathbf{x}(t) \otimes \mathbf{x}^*(t)\}$, with \otimes representing the Kronecker product and (.)* the complex conjugate, as the measurement vector. Accordingly, we have measured/computed an appropriate fingerprinting map $\tilde{\Psi}$ of size $M^2 \times N$ to parameterize the K-sparse $\tilde{\mathbf{y}}$ as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{\Psi}}\boldsymbol{\theta} + \tilde{\mathbf{p}}_n,\tag{1}$$

where $\tilde{\mathbf{p}}_n = \text{vec}(\sigma_n^2 \mathbf{I}_M)$ and $\boldsymbol{\theta}$ is an $N \times 1$ vector containing all zeros except for K non-zero elements with indices related to the locations of the K sources and values equal to the η_k 's.



Fig. 1. Frequency-domain filtering; $F^{(q)}(f)$ denotes the Fourier transform of $f^{(q)}(t)$.

The interesting advantage of this idea is that owing to the newly defined $\tilde{\Psi}$, situations could occur for which $M^2 > N$ and (1) could be solved using classical LS. The problem with this new approach (called sparsity-aware RSS localization via cooperative APs (SRLC)) as well as any other existing algorithm incorporating the cross-correlations such as [8] is that in order to form a linear system like (1) one has to make sure that all the SNs in the run-time phase have the same statistics as the training SN used to construct the fingerprinting map. This basically imposes some a priori knowledge on the problem which might be lacking in some practical situations, and thus we are also interested in an approach which is blind to the $r_k(\tau)$'s. We tackle this issue while we also try to take advantage of the large bandwidth of the received signal to gain some extra information and enhance the SRLC, by approaching the problem from the frequency domain (called SRLC-FD).

Let us start by explaining an appropriate filter bank design which plays an important role in the following analysis. Assume that we do not have any knowledge about the $r_k(\tau)$'s. Instead, at each AP we can efficiently estimate the bandwidth of the total received signal using appropriate spectrum estimation techniques; we call it B and it is assumed to be the same at the different APs. Besides, for a fixed network with known locations of the APs and GPs, the maximum delay difference between the received signals at the different APs $\Delta \tau_{\text{max}}$ can be computed during the training phase or from the knowledge of the network configuration. As a result, the maximum delay difference experienced by any signal from a multipath channel is $\Delta \tau_{\max} + \gamma$, where γ denotes the maximum delay spread of the multipath channel. Having estimated B and $\Delta \tau_{\max} + \gamma$, we use a set of (unit-energy) filters $\{f^{(q)}(t)\}_{q=1}^{Q}$ to divide B into $Q = \lceil B(\Delta \tau_{\max} + \gamma) \rceil$ adjacent subbands $\mathcal{B}^{(q)} = [(q-1)B/Q, qB/Q)$ with bandwidth B/Q. A schematic view of an arbitrary signal, channel and the filter bank is shown in Fig. 1. Notably, since $B/Q = B/[B(\Delta \tau_{\max} + \gamma)] < 1/\gamma$ with $1/\gamma$ representing the approximate coherence bandwidth of the channel, the output of the q-th filter at the m-th AP experiences a flat fading channel $H_{m,k}^{(q)}$ for every SN. Therefore, the related output signal can be written as

$$x_{m}^{(q)}(t) = \sum_{k=1}^{K} \left[s_{k}(t) * f^{(q)}(t) \right] H_{m,k}^{(q)} + n_{m}(t) * f^{(q)}(t)$$
$$= \sum_{k=1}^{K} s_{k}^{(q)}(t) H_{m,k}^{(q)} + n_{m}^{(q)}(t), \qquad (2)$$

where * denotes the convolution operator, and $s_k^{(q)}(t)$ and $n_m^{(q)}(t)$ respectively denote the filtered versions of $s_k(t)$ and $n_m(t)$. Further, by simply stacking the results for different APs, the total received signal vector can be expressed as $\mathbf{x}^{(q)}(t) = [x_1^{(q)}(t), \cdots, x_M^{(q)}(t)]^T$. Therefore, we have Q signals $\mathbf{x}^{(q)}(t)$ to compute $\tilde{\mathbf{y}}^{(q)} = \mathbb{E}\{\mathbf{x}^{(q)}(t) \otimes \mathbf{x}^{(q)*}(t)\}$ with its elements given by

$$y_{m,m'}^{(q)} = \mathbb{E}\left\{x_m^{(q)}(t) x_{m'}^{(q)*}\right\}$$

= $\mathbb{E}\left\{\sum_{k=1}^{K} s_k^{(q)}(t) H_{m,k}^{(q)} \sum_{k'=1}^{K} s_{k'}^{(q)*}(t) H_{m',k'}^{(q)*}\right\} + \mathbb{E}\left\{n_m^{(q)}(t)n_{m'}^{(q)}(t)\right\}$
= $\sum_{k=1}^{K} H_{m,k}^{(q)} H_{m',k}^{(q)*} \eta_k^{(q)} + \sigma_n^2 \delta_{m-m'},$ (3)

where $\eta_k^{(q)} = \mathbb{E}\{s_k^{(q)}(t) s_k^{(q)*}(t)\}\$ and where the second and last equalities follow from our assumptions on the signal and noise, as detailed in Section 2.

Now, let us ignore the effect of the noise in (3) for the time being, and discover the fingerprints. Interestingly, owing to our proposed filtering, the location-dependent fingerprints $H_{m,k}^{(q)}H_{m',k}^{(q)*}$ do not depend on the $r_k(\tau)$'s and the effect of the different $r_k(\tau)$'s appears in the $\eta_k^{(q)}$'s, which can be handled within the sparse vector of interest. Now, if we consider that the sources can only be located on N GPs, we can use any training source and measure the $r_k(\tau)$ -independent fingerprints for the q-th subband as

$$\tilde{\psi}_{m,m'}^{(q)} = \left[H_{m,1}^{(q)g} H_{m',1}^{(q)g*}, \cdots, H_{m,N}^{(q)g} H_{m',N}^{(q)g*} \right]^T, \quad (4)$$

where $(.)^g$ denotes values being measured/computed for the GPs. As a result, (3) can be rewritten as

$$y_{m,m'}^{(q)} = (\tilde{\psi}_{m,m'}^{(q)})^T \,\boldsymbol{\theta}^{(q)} + \sigma_n^2 \delta_{m-m'},\tag{5}$$

where $\theta^{(q)}$ is a K-sparse vector with only K non-zero elements equal to $\eta_k^{(q)}$'s. Next, similar to SRLC, we can stack the M^2 different $y_{m,m'}^{(q)}$'s and $\tilde{\psi}_{m,m'}^{(q)}$'s to form

$$ilde{\mathbf{y}}^{(q)} = ilde{\mathbf{\Psi}}^{(q)} \, oldsymbol{ heta}^{(q)} + ilde{\mathbf{p}}_n$$

where

$$\tilde{\mathbf{y}}^{(q)} = [y_{1,1}^{(q)}, \cdots, y_{1,M}^{(q)}, \cdots, y_{M,1}^{(q)}, \cdots, y_{M,M}^{(q)}]^T,$$

and

$$\begin{split} (\tilde{\Psi}^{(q)})^T &= [\tilde{\psi}_{1,1}^{(q)}, \cdots, \tilde{\psi}_{1,M}^{(q)}, \cdots, \tilde{\psi}_{M,1}^{(q)}, \cdots, \tilde{\psi}_{M,M}^{(q)}] \\ &= \begin{bmatrix} |H_{1,1}^{(q)g}|^2 & H_{1,1}^{(q)g}H_{2,1}^{(q)g*} & \cdots & |H_{M,1}^{(q)g}|^2 \\ |H_{1,2}^{(q)g}|^2 & H_{1,2}^{(q)g}H_{2,2}^{(q)g*} & \cdots & |H_{M,2}^{(q)g}|^2 \\ \vdots & \vdots & \ddots & \vdots \\ |H_{1,N}^{(q)g}|^2 & H_{1,N}^{(q)g}H_{2,N}^{(q)g*} & \cdots & |H_{M,N}^{(q)g}|^2 \end{bmatrix}. \end{split}$$

Now, based on this analysis, depending on the statistical properties of the received signals, i.e., the spectrum of the $s_k(t)$'s, the following three cases can be studied.

3.1. Flat spectrum

Looking at Fig. 1, we understand that if the spectrum of the sum of the $s_k(t)$'s is (almost) flat, the $\eta_k^{(q)}$'s will be (almost) the same in the different frequency bands $\mathcal{B}^{(q)}$. This basically makes it possible to construct an augmented version of the measurements as well as the fingerprinting maps, as $\eta_k^{(q)} \approx \eta_k$ and hence $\theta^{(q)} = \theta$, $\forall q$. This means θ will be a K-sparse signal with all elements equal to zero except for K elements equal to η_k . Thus, we construct the augmented version of the run-time measurements as $\tilde{\mathbf{y}}_{\text{FD}} = [(\tilde{\mathbf{y}}^{(1)})^T, \cdots, (\tilde{\mathbf{y}}^{(Q)})^T]^T$ and the augmented fingerprinting map as $\tilde{\mathbf{\Psi}}_{\text{FD}} = [(\tilde{\mathbf{\Psi}}^{(1)})^T, \cdots, (\tilde{\mathbf{\Psi}}^{(Q)})^T]^T$. Finally, we solve

$$\tilde{\mathbf{y}}_{\text{FD}} = \tilde{\boldsymbol{\Psi}}_{\text{FD}} \boldsymbol{\theta} + \mathbf{1}_Q \otimes \tilde{\mathbf{p}}_n, \tag{6}$$

As we explained, this time $\tilde{\Psi}_{\text{FD}}$ is a $QM^2 \times N$ matrix and thus (6) can be solved using LASSO $\hat{\theta} = \arg \min_{\theta} \|\tilde{\mathbf{y}}_{\text{FD}} - \tilde{\Psi}_{\text{FD}}\theta\|_2^2 + \lambda \|\theta\|_1$ with λ as a sparsity regularization parameter, if it is under-determined, or classical LS $\hat{\theta} = \tilde{\Psi}_{\text{FD}}^{\dagger} \tilde{\mathbf{y}}_{\text{FD}}$ if it is overdetermined. It is worth pointing out that even for the case where the signals have a partially flat spectrum, we can design a number of filters for that flat part of the spectrum and again construct (6) where in such a case we will have less subbands.

3.2. Varying spectrum; the simple solution (Q = 1)

In contrast to the case where the signals have a flat spectrum, for the non-flat case, we cannot construct augmented versions of the measurements and fingerprinting maps for a unique θ and solve a linear system similar to (6). Particularly, because of the different $\eta_k^{(q)}$'s in the different bands, the $\tilde{\Psi}^{(q)}$'s and $\tilde{y}^{(q)}$'s are related to different $\theta^{(q)}$'s. In this case, as a straightforward solution, we can simply take one of the bands, for instance the first band $\mathcal{B}^{(1)}$, and solve

$$\tilde{\mathbf{y}}^{(1)} = \tilde{\mathbf{\Psi}}^{(1)} \boldsymbol{\theta}^{(1)} + \tilde{\mathbf{p}}_n, \tag{7}$$

using LASSO or LS. This way, we at least have the same identifiability gain as SRLC, but more importantly, we are blind to the $r_k(\tau)$'s. However, we still have some important information present in the adjacent subbands which has not been exploited. This motivates the following subsection.

3.3. Varying spectrum; enhancing the identifiability gain

The question is how we can exploit the information present in all the subbands to attain an identifiability gain. An important observation which helps us to develop a solution is the fact that even though different subbands lead to different $\eta_k^{(q)}$'s for a non-flat spectrum, all the bands lead to linear models, similar to (7), where in all of them the sparse $\theta^{(q)}$'s share a common support. This motivates a group-LASSO (G-LASSO) type of solution to incorporate all the bands. However, different from classical G-LASSO, we have different maps $\tilde{\Psi}^{(q)}$ for different subbands. Similar cases occur in the framework of the multiple measurement vectors (MVV) problem [10]. To deal with this, we propose a modified version of G-LASSO as defined by

$$\hat{\boldsymbol{\Theta}} = \arg\min_{\boldsymbol{\Theta}} \sum_{q=1}^{Q} \|\tilde{\mathbf{y}}^{(q)} - \tilde{\boldsymbol{\Psi}}^{(q)}[\boldsymbol{\Theta}]_{:,q}\|_{2}^{2} + \lambda \sum_{n=1}^{N} \|[\boldsymbol{\Theta}]_{n,:}\|_{2},$$
(8)

where $\Theta = [\theta^{(1)}, \dots, \theta^{(Q)}]$. The first term on the right hand side of (8) is the LS part which minimizes the error for the different subbands and the second term enforces group sparsity. Based on the discussions presented in [10] for MMV, incorporating all the subbands within (8) will result in a gain in terms of identifiability compared to (7), as is also corroborated by our simulation results in the next section.

Identifiability and reconstruction quality: Detailed explanations on some necessary identifiability requirements as well as on the reconstruction quality (in terms of restricted isometry property (RIP)) of the SRLC and the SRLC-FD can be found in the extended version of this work [9].

4. SIMULATION RESULTS

In this section, we investigate the performance of the proposed algorithms in terms of probability of detection (P_d) and probability of false alarm (P_{fa}) against $1/\sigma_n^2$ as well as the number of existing SNs K. To this aim, we consider a wireless network of size 10×10 m² divided into N = 100GPs. We also consider up to K = 10 sources to be simultaneously localized. We consider a wideband BPSK signal with a rectangular pulse shape, 3dB bandwidth B = 10MHz and power $\eta = 1$. This means $r(\tau) = 1 - \frac{|\tau|}{B}$ for the baseband equivalent signal. We compute the autocorrelation and crosscorrelation functions during a time-slot of length T = 1ms. The multipath channel is assumed to be a complex Gaussian channel with L = 4 i.i.d. paths and an approximate coherence bandwidth of $1/\gamma = 1$ MHz. Instead of taking ideal expectations $\mathbb{E}\{.\}$ in the run-time phase, we work with discrete-time signals of limited length and hence the computations of the autocorrelations as well as the cross-correlations will not be ideal as in the derivations of Section 2. All simulations are averaged over 100 independent Monte Carlo (MC) runs where in each run the sources are deployed on different random locations. For all the reconstruction problems, we choose λ by cross-validation. For the sake of simplicity of our simulations, we concentrate on the *localization* performance, i.e., we are only interested to know which elements of the estimated θ correspond to a source and which elements are zeros and this is equivalent to determining the support vector $supp(\hat{\theta})$. Based on this, we define P_{err} , P_d and P_{fa} as follows:

- $-P_{err}$:= the probability that a source is detected when the source is in fact *not* present or it is *not* detected when it is in fact present.
- P_d := the probability that a source is detected when the source is in fact present.
- P_{fa} := the probability that a source is detected when the source is in fact *not* present.

Basically, P_d and P_{fa} specify all the probabilities of interest. However, we need a detection threshold to be able to compute them. To find the best threshold, we carry out a linear search within the range $[0, \max(\hat{\theta}))$ and select the value which minimizes P_{err} . Finally, note that here we only compare with the SRLC as its superiority to the other existing algorithms is already shown in [7, 9].

We consider the case where we have only M = 5 APs randomly distributed within the covered area. For such a case, $M^2 = 25 < N$ and thus it is expected that even the SRLC might not be capable of localizing all the K = 10 sources. In order to investigate the performance of the SRLC-FD, we assume that all the sources have different η_k 's with a uniform distribution $\mathcal{U}(0.8, 1.2)$ and we assume that $r_k(\tau) =$ $r(\tau)$ is unknown to SRLC-FD. We would like to emphasize that SRLC-FD can be employed even for cases where all the sources have different $r_k(\tau)$'s. However, since this cannot be handled by the SRLC, we omit those results here. We start by considering the case where K = 10 sources are randomly located on the GPs. Fig. 2 depicts a schematic view of localization for $1/\sigma_n^2 = 20$ dB. Clearly, SRLC-FD (Q = 1) denotes the idea of exploiting only one frequency band as explained in Subsection 3.2. As can be seen, while the SRLC and the SRLC-FD (Q = 1) are only capable of localizing K = 2 sources, the blind algorithm (SRLC-FD) could localize all the sources simultaneously.

We would also like to further assess the proposed algorithms in terms of P_d and P_{fa} . Fig. 3 compares the performance of the aforementioned algorithms against $1/\sigma_n^2$ for K = 4. As is clear from the figure, SRLC-FD (Q = 1) is performing the worst, but it is blind to $r(\tau)$. SRLC is performing acceptably, and attains the maximum P_d and almost a minimum P_{fa} for $1/\sigma_n^2$ values larger than 6dB. The same holds for the SRLC-FD which is even performing better than SRLC while it is blind. For the SRLC-FD, we have designed Q = 10filters and the proposed G-LASSO solution (explained in Subsection 3.3) is employed.

Let us get a more complete picture of the performance of the algorithms by taking a look at Fig. 4 where the detection and false alarm probabilities are depicted against K for



Fig. 2. Localization with M = 5, K = 10 and $1/\sigma_n^2 = 20$ dB



Fig. 3. Performance vs. $1/\sigma_n^2$ for M = 5 and K = 4

 $1/\sigma_n^2 = 20$ dB. As can be seen, the performance drops for the SRLC and the SRLC-FD (Q = 1) with K and thus P_d starts decreasing whereas P_{fa} rises for K > 4. Interestingly, for a large enough $1/\sigma_n^2$ (small enough noise), SRLC-FD attains an optimal performance even for K up to 10. This result illustrates the fact that our blind algorithm with no information about $r(\tau)$, by exploiting the information of the Q = 10frequency subbands could outperform SRLC in terms of the number of identifiable sources. Note that there is a major improvement in SRLC-FD compared to SRLC-FD (Q = 1).

5. CONCLUSIONS

This paper studies the problem of localizing multiple sources using a correlation-based localization paradigm. We have proposed a novel sparsity-aware fingerprinting localization which is blind to the statistics of the source signals by approaching the problem from a frequency-domain perspective. Our simulation results illustrate a good performance in terms of detection capability as well as localization accuracy.



Fig. 4. Performance vs. K for M = 5 and $1/\sigma_n^2 = 20$ dB

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