PRECODING TECHNIQUE FOR PEAK-TO-AVERAGE-POWER-RATIO (PAPR) REDUCTION IN MIMO OFDM/A SYSTEMS

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ABSTRACT

The paper develops a new technique to reduce the peak to average power ratio (PAPR) in OFDM modulation for a MIMO system. The proposed method exploits the eigen-beamforming mode (EM) in MIMO systems which is a common feature in 4th generation standards: WiMAX and LTE.These systems use the same beamforming weights for dedicated pilots and data so the weights are interpreted as a channel effect from the receiver perspective. There is no need to invert the weights at the receiver side since it is compensated for in channel equalization. Beamforming performance depends on the relative phase difference between antennas but is unaffected by a phase shift common to all antennas. In contrast, PAPR changes with the common phase shift . An effective optimization technique based on Sequential Quadratic Programming is proposed to compute the common phase shifts that minimize the PAPR.

***Index Terms*—** Orthogonal Frequency Division Multiplexing (OFDM), Multiple Input Multiple Output (MIMO), Beamforming, Optimization, WiMAX.

1. INTRODUCTION

A well-known drawback of OFDM is that the amplitude of the resulting time domain signal varies with the transmitted symbols in the frequency domain. If the maximum amplitude of the time domain signal is large, it may push the amplifier into the non-linear region which breaks the orthogonality of the sub-carriers and will result in a substantial increase in the error rate. PAPR reduction is a well-known signal processing topic in multi-carrier transmission and large number of techniques have been proposed in the literature during the past decades. PAPR reduction techniques are associated with costs in terms of bandwidth or/and transmit power. Also, most of them require modifications in both transmitter and receiver which makes it non-compliant to the existing communication standards. Multiple signal representation methods, such as partial transmit sequence (PTS) and selected mapping (SLM) are well-known techniques which reduce the peak amplitude of the OFDM signal by manipulating the phase of subcarriers. The phase weights are sent as a side information to the receiver to recover the original symbols[1].

A new Precoding PAPR reduction technique is proposed in [2], based on grouping the OFDM subcarriers in clusters and changing the phase of clusters in a manner similar to the PTS method but without the drawback of sending explicit side information. The proposed technique neither requires additional bandwidth nor power while delivering equal or better PAPR reduction gain compared to existing methods. This algorithm focuses on the practical case for a WiMAX base station with a single transmit antenna. In this paper we consider PAPR reduction techniques for multiple transmit antennas with Space Time Block Codes (STBC) in EM mode, which is the case for both WiMAX and LTE standards. Simulation result shows the probability of high PAPR increases for MIMO comparing to the single antenna. The beamforming weights also cause extra increase in PAPR; to avoid this, phase-only beamforming is usually used which limits the performance. This makes it more important to find a solution for PAPR, since MIMO-OFDM has become a popular technique for wireless communication in time-frequency selective channels. In a MIMO scenario, the peak amplitude needs to be searched and minimized jointly over all antennas which affects the PAPR characteristics compared to the single antenna system. Also, the coupling between several OFDM symbols on each antenna gives an extra degree of freedom in the minimization algorithm. An iterative phase optimization method based on SQP technique in [2] has been redefined and modified for multiple antenna system which finds the optimum weights by approximating and minimizing the quadratic objective function at each solution point. We show that the proposed technique keeps the PAPR in the same level as single antenna for EM-MIMO systems.

2. SYSTEM MODEL

Consider an OFDM system, where the data is represented in the frequency domain. The time domain samples s_n, $n = 1, 2, \ldots, N$, are calculated from the frequency domain symbols D_l using IFFT operation, where N denotes the IFFT size. Equivalently, the data block in time domain is denoted by vector $\mathbf{s}$ which is a result of multiplying the IFFT matrix $\mathbf{F}$ with the data block in frequency domain $\mathbf{D}$.

$$s_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} D_l e^{\frac{j2\pi ln}{N}} \;\Leftrightarrow\; \mathbf{s} = \mathbf{F}\mathbf{D}\,. \tag{1}$$

Note that the frequency domain signal D_k typically belongs to the digital modulation schemes including QAM, QPSK, 16QAM and 64QAM constellations, these are referred to as symbols while the s_ns are called OFDM samples or subcarriers.

The metric that is used to measure the peaks in the time-domain signal is PAPR which is defined as

$$\text{PAPR} = \frac{\max\limits_{0\le n\le N-1} |s_n|^2}{E\{|s_n|^2\}}. \tag{2}$$

Although not explicitly written in (2), it is well known that oversampling is required to accurately capture the peaks. In this paper, an oversampling of four times is used.

2.1. Partial Transmit Sequence (PTS)

Based on the PTS technique, the input data block $\mathbf{D}$ of length N is partitioned[1] into M disjoint sub-blocks

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[1]$\mathbf{D}$ represents a vector but it is in uppercase to denote the signal in frequency domain.

$\mathbf{D}_m = [D_{0,m}, D_{1,m}, \cdots, D_{N-1,m}]^T$, $m = 1, 2, \cdots, M$, such that $\sum_{m=1}^{M} \mathbf{D}_m = \mathbf{D}$ and at anytime n only one $D_{n,m}$, $m = 1, 2, \cdots, M$ is nonzero. The partitioning can be done in several ways, e.g. in adjacent sub-block partitioning, the nonzero elements of each sub-block are constructed from adjacent symbols. In contrast, in random partitioning the elements are selected randomly among symbols [1]. The time domain signal $\mathbf{b}_m = [b_{0,m}, b_{1,m}, \cdots, b_{N-1,m}]^T$ is obtained by taking an IFFT of length N from $\mathbf{D}_m$; these are called the partial transmit sequences (PTSs). Complex phase factors, $\psi_m = e^{j\phi_m}$, $m = 1, 2, \cdots, M$ are introduced to combine the PTSs. The time domain signal after combination is given by $s_n = \sum_{m=1}^{M} \psi_m b_{n,m}$.

2.2. MIMO-OFDM

A basic point to point MIMO communication system consists of M_t transmit antennas and M_r receive antennas. Space-time block codes (STBC) are designed to form the transmission blocks which exploit both diversity and multiplexing gain in MIMO. In MIMO-OFDM, the transmit sequences of multiple antennas are mapped to parallel symbols and then modulated by the IFFT operation to form the OFDM transmit blocks. Accordingly, the concept of time in the MIMO STBC is analogous to frequency in a MIMO-OFDM system and it is referred to as space frequency block codes (SFBC) in this paper.

As the transmit symbols are divided over different time slots in STBCs, in MIMO-OFDM the whole OFDM band is divided into several sub-bands and each sub-band is called a cluster. WiMAX and LTE standards specify two main modes of transmitting pilots: common pilots and dedicated pilots. Here, dedicated pilots allow per-cluster beamforming since channel estimation is performed per-cluster [2]. Note that the whole band is divided into C clusters while each cluster spans a portion of time and frequency [2]. The number of subcarriers in each cluster denoted by N_c and we assume M_t OFDM blocks are sent over M_t number of antennas in one time slot.

A discrete signal model for each cluster can be represented by the $\mathbf{Y}_c = \mathbf{H}_c \, \mathbf{S}_{\text{SFBC}_c} + \mathbf{n}_c$, $c = 1, \cdots, C$. Here transmit matrix $\mathbf{S}_{\text{SFBC}_c}$ of size $M_t \times N_c$, consists of subcarriers in cluster c over different antennas. The transmit matrix is multiplied with the corresponding channel matrix $\mathbf{H}_c$ and discrete noise is added to form the received matrix $\mathbf{Y}_c$ of size $M_r \times N_c$. The elements of matrix $\mathbf{H}_c$ of size $M_r \times M_t$ correspond to the complex channel gain between the transmit and receive antennas. The estimate of channel matrix is called the channel state information (CSI) which is used to form the beamforming weights. As explained before, each cluster is associated with one estimated channel or in other words the channel is invariant over the frequency band in one cluster. Generally, the EM MIMO-OFDM forms the eigen channels using CSI on the transmitter and receiver sides. Thus, a precoding matrix is formed by the right singular vectors of the channel matrix $\mathbf{H}_c$ and is referred to as a beamforming matrix [3]. The beamforming matrix $\mathbf{W}_c$ is a square matrix of size $M_t \times M_t$,

$$\mathbf{W}_c = [\mathbf{w}_c{}^{(1)}, \mathbf{w}_c{}^{(2)}, \cdots, \mathbf{w}_c{}^{(M_t)}] \,. \tag{3}$$

The columns of $\mathbf{W}_c$ are denoted by $\mathbf{w}_c{}^{(q)}$ where $\mathbf{w}_c{}^{(q)} = [w_c{}^{(q,1)}, w_c{}^{(q,2)}, \cdots, w_c{}^{(q,M_t)}]^T$. This vector contains the coefficients by which the qth symbol are multiplied with, on antennas 1 to M_t. The $\mathbf{S}_{\text{SFBC}}$ is formed for different clusters separately and they are multiplied with $\mathbf{W}_c$ of the corresponding channel matrix. The result $\mathbf{S}_c = \mathbf{W}_c \, \mathbf{S}_{\text{SFBC}_c}$, $c = 1, \cdots, C$ is a $M_t \times N_c$ matrix representing the transmit subcarriers over M_t antennas in the cth cluster.

The transmit symbols in a cluster which are divided over different subcarriers and antennas in SFBCs are placed in $\mathbf{S}_{\text{SFBC}_c} = [\mathbf{\acute{s}}^c{}_1, \mathbf{\acute{s}}^c{}_2, \cdots, \mathbf{\acute{s}}^c{}_{M_t}]^T$, which is a matrix of size $M_t \times N_c$. The $\acute{\mathbf{s}}_i$ represents one vector of symbols which are transmitted over the ith antenna before applying the beamforming. As a result, the $\mathbf{\acute{s}}^c{}_i$ in $\mathbf{S}_{\text{SFBC}_c}$ are given by $\mathbf{\acute{s}}^c{}_i = [\acute{s}^c{}_{i,1}, \acute{s}^c{}_{i,2}, \cdots, \acute{s}^c{}_{i,N_c}]^T$, $i = 1, \cdots, M_t$, which is a column vector with length equal to the number of subcarriers in one cluster.

The complete final transmission block including beam formed OFDM samples over different antennas are collected as

$$\mathbf{S} = [\mathbf{S}_1, \cdots, \mathbf{S}_C] = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_{M_t}]^T \,, \tag{4}$$

where $\mathbf{s}_i$ is the final transmit sequence from ith antenna.

3. PROPOSED MIMO-PTS TECHNIQUE WITH CONTINUOUS PHASE WEIGHTS

The proposed PAPR technique in this paper uses the PTS setting to partition the OFDM block into several sub-blocks for phase shift. However, we do not follow the exhaustive search algorithm in the PTS technique to find the sub-optimum discrete phase weights. The essential point is that the sub-blocks are the clusters in our technique because the cluster is the smallest block that can be phase rotated without informing the receiver explicitly. Our algorithm relies on the channel equalization to compensate for the phase change at the receiver side. This allows for searching the optimum phase weights in a continuous interval between $[0, 2\pi)$. Furthermore, EM-MIMO combines several OFDM samples on multiple antennas to form the transmission signal on each antenna. This extra degree of freedom can be exploited to achieve better PAPR reduction gain combining the sub-blocks.

3.1. MIMO-PTS in Eigen-Beamforming Mode

in PTS, as explained in Sec.2.1, each OFDM block is divided into M disjoint sub-blocks, each of size $N \times 1$. Note that in our proposed technique sub-blocks are chosen as cluster units, so $M = C$. After taking IFFT of these sub-blocks, a big matrix of size $N \times M$ is formed for each OFDM block, with IFFT samples of sub-blocks in columns. This is referred to as $\mathbf{B}^{(q)}$ for qth OFDM block which is the left matrix in the following equation. Note, the summation of columns in $\mathbf{B}^{(q)}$ is equal to the IFFT on the qth OFDM block. The elements of $\mathbf{B}^{(q)}$ are denoted by $b_{n,m}^{(q)}$ which is the generalization of PTS in Sec.2.1 for M_t antennas.

Since the IFFT is a linear operation, the beamforming weights can be applied before or after IFFT summation. However, it is easier to explain when $\mathbf{W}_c$ are multiplied after sub-block partitioning and IFFT operation, as depicted in Fig.1. The PAPR weights are subsequently applied afterwards, and they are common between different antennas but different for contributing OFDM blocks on each antenna. Finally, sub-blocks of different weighted OFDM symbols are summed together to construct the final sub-blocks. The summation of final sub-blocks gives the transmit OFDM sequence. It is clear from Fig.1 that now the PAPR depends on the transmit sequences over all antennas rather than just one, as in single antenna case. To formulate the optimization problem, we can write $\mathbf{z}_{i,q} = \mathbf{B}^{(q)} \operatorname{diag}(\mathbf{w}^{(q,i)}) \boldsymbol{\psi}^{(q)}$ which is the contribution of the OFDM block q on the ith antenna output:

$$\mathbf{z}_{i,q} = \begin{bmatrix} b_{1,1}^{(q)} & b_{1,2}^{(q)} & \cdots & b_{1,M}^{(q)} \\ b_{2,1}^{(q)} & b_{2,2}^{(q)} & \cdots & b_{2,M}^{(q)} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N,1}^{(q)} & b_{N,2}^{(q)} & \cdots & b_{N,M}^{(q)} \end{bmatrix} \begin{bmatrix} w_1^{(q,i)} & \cdots & 0 \\ 0 & w_2^{(q,i)} & \vdots \\ \vdots & \ddots & 0 \\ 0 & \cdots & w_M^{(q,i)} \end{bmatrix} \begin{bmatrix} e^{j\phi_{q,1}} \\ e^{j\phi_{q,2}} \\ \vdots \\ e^{j\phi_{q,M}} \end{bmatrix}$$

[2]Both time and frequency can be considered in clustering but for simplicity reasons here each cluster spans only a portion of frequency band.

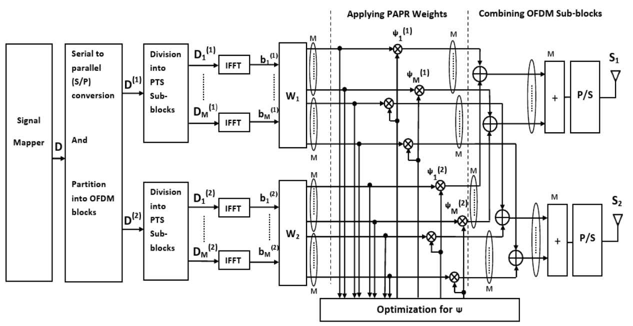

Fig. 1: Block diagram of MIMO-PTS technique for two antennas. The input data block $\mathbf{D}$ is partitioned into two M disjoint sub-blocks such that $\sum_{m=1}^{M} \mathbf{D}_m^{(q)} = \mathbf{D}^{(q)}$. Known beamforming weights $\mathbf{w}_1$ and $\mathbf{w}_2$ are multiplied with first and second OFDM sub-blocks, respectively to produce the contributing signal for the first (upper) and second (lower) antennas. Then, phase weights $\psi_m^{(q)} = e^{j\phi_{q,m}}$ are multiplied with all sub blocks. The relevant sub blocks are summed to form the final OFDM sequences on both antennas.

The output OFDM sequence after applying the beamforming weights and PAPR weights on ith antenna is $\mathbf{s}_i = \sum_{q=1}^{M_t} \mathbf{z}_{i,q}$. The output sample $s_{i,n}$ is given by

$$s_{i,n} = \sum_{q=1}^{M_t} \sum_{m=1}^{M} b_{n,m}^{(q)} \psi_m^{(q)} w_m^{(q,i)} \tag{5}$$

which shows the nth subcarrier of OFDM sequence on ith antenna.

3.2. Formulation of the Phase Optimization Problem

In order to minimize the PAPR, the phase weights are selected by minimizing the largest sample of OFDM sequence, according to (2), because the average power remains unaltered. The PAPR minimization now is performed over all antennas. The optimization problem can be formulated as

$$\mathbf{\Psi} = \arg \min_{\phi_{q,m}} \max_{i,n} |s_{i,n}|^2 , \tag{6}$$

where $\mathbf{\Psi} = [\boldsymbol{\psi}^{(1)}, \boldsymbol{\psi}^{(2)}, \cdots, \boldsymbol{\psi}^{(M_t)}]$ and $\boldsymbol{\psi} = \mathrm{vec}(\mathbf{\Psi})$. This is a minimax optimization problem when the objective function is $f_{\max}(\boldsymbol{\psi}) = \max_{i,n} |s_{i,n}|^2$,. The vector of OFDM samples over all antennas is defined as $\mathbf{s} = \mathrm{vec}(\mathbf{S})$ and the set of constraints are the square of the absolute values in vector $\mathbf{f}(\boldsymbol{\psi}) = |\mathbf{s}|^2$.

$$\begin{array}{ll} \underset{\boldsymbol{\psi}}{\text{minimize}} & f_{\max}(\boldsymbol{\psi}) \\ \text{subject to} & \\ \mathbf{f}(\boldsymbol{\psi}) \leq f_{\max}(\boldsymbol{\psi}) & \end{array} \tag{7}$$

The above expression implies that the phase weights are solved for all transmit antennas in an iterative optimization procedure which minimizes the largest sample among all antennas. The constraints guarantee that there is no larger sample than $f_{\max}(\boldsymbol{\psi})$, when the minimization is being done. Indeed, the $f_{\max}(\boldsymbol{\psi})$ is a fixed sample, for example nth sample. During the iterations which is indexed by k other samples which are functions of the same $\boldsymbol{\psi}$ vector, are pushed to stay below this maximum value.

4. SOLVING THE MINIMIZATION PROBLEM

In PTS the optimum weights are selected by performing an exhaustive search among the quantized set of phase options, which limits the number of sub-blocks and eventually the PAPR reduction gain. However, in the proposed technique, there is no restriction on phase coefficients so choosing the best phase coefficients is still challenging. An effective optimization algorithm must be used to extract the optimum phase weights. A practical gradient-based algorithm is proposed in [2] which is modified and adapted for the phase optimization problem of the PAPR reduction in multiple antenna system.

SQP is one of the most popular and robust algorithms for nonlinear constraint optimization. This is modified and simplified for the phase optimization problem of PAPR reduction. The algorithm proceeds based on solving a set of sub problems created as a quadratic model of the objective, subject to a linearization of the constraints. Accordingly, at each major iteration, a quadratic function is defined at the current solution. The Jacobian matrix of the constraints are used for linearization of the current constraints in original problem around $\boldsymbol{\psi}_k$. The minimization direction $\mathbf{d}$ is the optimal direction to move in order to minimize the largest sample.

$$\begin{array}{ll} \underset{\mathbf{d} \in \Re^n}{\text{minimize}} & q = \frac{1}{2}\mathbf{d}^T \mathbf{H}_k \mathbf{d} + \nabla f_{\max}(\boldsymbol{\psi}_k)^T \mathbf{d} \\ \text{subject to} & \\ \nabla \mathbf{f}(\boldsymbol{\psi}_k)^T \mathbf{d} + \mathbf{f}(\boldsymbol{\psi}_k) \leq 0 & \end{array} \tag{8}$$

A new convex optimization problem is formed in (8) and solving it gives the appropriate direction to move at each major iteration in the original problem. The quadratic objective function $q(\mathbf{d})$ reflects the local properties of the original objective function. The main reason to use a quadratic function is that such problems are easy to solve yet mimics the nonlinear behavior of the initial problem. This can be solved by quadratic programming. The Hessian of the Lagrange function $\mathbf{H}_k$ is required to form the quadratic objective function. Fortunately, it is not necessary to calculate this Hessian matrix explicitly since it can be approximated at each major iteration using a quasi Newton updating method [4]. The Active Set Strategy has been applied in the phase optimization algorithm. This method is particularly suitable for problems with a large number of constraints. The QP sub problem is also solved by iterations, A pseudo code is

Algorithm 1 SQP

- Initialize the variables: $\boldsymbol{\psi}$, $\nabla f_{\max}(\boldsymbol{\psi}_k)^T$ and $\mathbf{H}_0 = \mathbf{I}$
- **for** $k = 1$ to K **do**
  - Calculate cost function $f_{max}(\boldsymbol{\psi}_k)$ and constraints $\mathbf{f}$
  - Calculate the Jacobian matrix, q and its constraints
  - Initialize the $\mathbf{d}_0$,$\mathbf{Q}$,$\mathbf{R}$, $\mathbf{Z}$ and initial search direction $\acute{\mathbf{d}}_0$
  - **while** optimal $\mathbf{d}_k$ found **do**
    - Compute projected gradient$\nabla q(\mathbf{d}_l)$
    - Find the distance to the nearest constrain α
    - Find $\acute{\mathbf{A}}_l$ as $\nabla \mathbf{f}(\boldsymbol{\psi}_k)^T \mathbf{d}_l + \mathbf{f}(\boldsymbol{\psi}_k) = 0$
    - Decompose the active set as $\mathbf{Q}^T \acute{\mathbf{A}}_l^T = \begin{bmatrix} R \\ 0 \end{bmatrix}$
    - Compute the subspace $\mathbf{Z}_l = \mathbf{Q}[:, P+1 : \acute{M}]$
    - Compute the QP search direction according to the Newton step criteria, $\acute{\mathbf{d}}_l = -\mathbf{Z}_l \left((\mathbf{Z}_l^T \mathbf{H}_k \mathbf{Z}_l)^{-1} (\mathbf{Z}_l^T \nabla q(\mathbf{d}_l)) \right)$
    - Update the search direction $\mathbf{d}_{l+1} = \mathbf{d}_l + \alpha \acute{\mathbf{d}}_l$,
    - **if** $\alpha = 1$ || length$(\acute{\mathbf{A}}_l) = M$ **then**
      - Calculate the eigen values of active constraints $\boldsymbol{\lambda}$
      - Check the optimality; $\boldsymbol{\lambda} > 0$ and return $\mathbf{d}_k$
      - Otherwise remove the constraints with $\lambda_i < 0$
    - **end if**
  - **end while**
  - Update the solution $\boldsymbol{\psi}_{k+1} = \boldsymbol{\psi}_k + \mathbf{d}_k$,
  - Update the Hessian $\mathbf{H}_k$ and make sure it is positive definite
- **end for**

provided at Algorithm 1. An active set constraints at lth QP iteration is denoted by $\acute{\mathbf{A}}_l$ and is used to set a basis for a QP search direc-

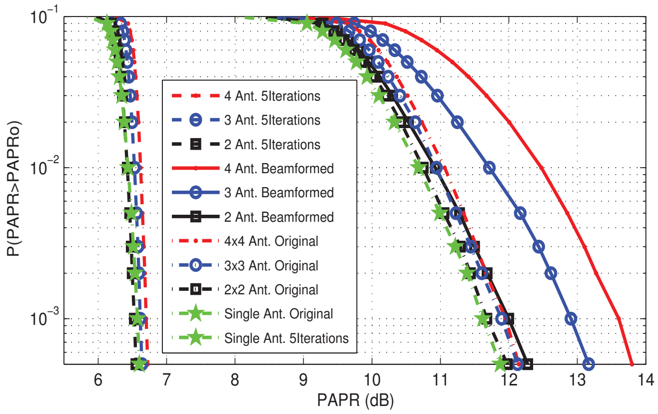

Fig. 2: CCDF curves of PAPR for 64QAM OFDM block with $N = 1024$ and $M = 60$. The curves are plotted for different number of antennas and $T = 2$. Corresponding PAPR reduced CCDFs are derived with 5 iterations.

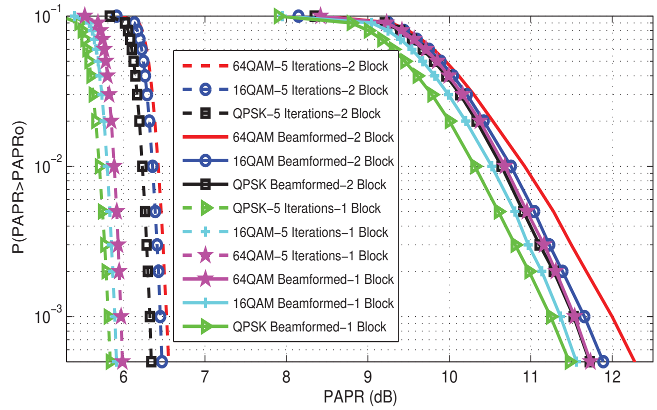

Fig. 3: CCDF curves of PAPR for 2×2 MIMO OFDM block with $N = 1024$ and $M = 60$. The curves are plotted for different constellations and $T = 1, 2$.

tion $\acute{\mathbf{d}}_l$. This constitutes an estimate of the constraint boundaries at the solution point $\mathbf{d}_l$. When a new constraint joins the active set, the dimension of the search space is reduced as expected. The possible subspace for $\acute{\mathbf{d}}_l$ is built from a basis $\mathbf{Z}_l$, whose columns are orthogonal to the active set $\acute{\mathbf{A}}_l$, such that $\acute{\mathbf{A}}_l\mathbf{Z}_l = 0$. Therefore, any linear combination of the $\mathbf{Z}_l$ columns constitutes a search direction, which is assured to remain on the boundaries of the active constraints. Here, P is the number of active constraints and $\acute{M} = M_t M$ shows the number of design parameters in the optimization problem, which is the number of total sub-blocks in 1.

The complexity evaluation is not straightforward, an explicit expression for number of operations is given by,

$$K\Big((0.65\acute{M}^3 + 2.7\acute{M}^2 + 6\acute{N}\acute{M} + 2\acute{N})L + (22\acute{N}\acute{M} + 9\acute{M} + \acute{N})\Big)$$

which is derived in [2] with details. The parameter K denotes the maxim number of iterations for original problem in (7) and L is the number of iterations to solve the QP sub-problem. The parameter $\acute{N} = M_t N$ is the number of all OFDM samples over all antennas.

5. SIMULATION RESULTS

In WiMAX, one cluster spans 14 sub-carriers over two OFDM symbols in time, containing four pilots and 24 data symbols. For a 10MHz system, there are a total of 60 clusters. In agreement with WiMAX setting, the proposed PAPR reduction technique is simulated for an OFDM block of size 1024 with 840 data subcarriers and 92 guard subcarriers at the both end. The block is divided into 60 clusters and appropriate phase weights are looked for within 5 iterations of SQP algorithm. The complementary CDF (CCDF) is used here to evaluate different methods, which denotes the probability that the PAPR of a data block exceeds a given threshold and is expressed as CCDF $= 1 -$ CDF. The total number of 10000 OFDM block is randomly generated to produce the PAPR curves. The beamforming weights are chosen to be the right singular vector of generated complex channel matrices for each block. In EM-MIMO system, by introducing more phase weights as an extra degree of freedom to the optimization algorithm, more PAPR reduction gain is provided. It is clear from Fig 2 that the probability of getting high peaks increased by putting more antennas at the transmitter. The beamforming weights cause extra growth in PAPR probability as well. In WiMAX setting with two OFDM symbol in a cluster the number of OFDM symbols are increased by two by adding one extra antenna. However the PAPR-reduced curves are at the same range for $1, 2, 3$ and 4 antennas which indicated more gain for more antenna, expectedly. The PAPR reduction gain is about 7.3 dB for 4 antennas which is 1 dB more than single antenna system in probability of 5×10^{-4}. It can be seen from Fig 3 that the PAPR is higher for clusters with two OFDM block in time but the PAPR reduction gain is 6 dB for both settings of one and two OFDM blocks. So the algorithm does not affect by the number of OFDM blocks in time, since they are rotated by the same phase coefficients. As expected, the PAPR is higher for more complex constellations as 64QAM but the gain of PAPR reduction algorithm is the same.

6. CONCLUDING REMARKS

A novel precoding PAPR reduction technique has been developed for a multiple transmit antenna system, exploiting the cluster beamforming weights which is a general feature in 4G communication systems. The proposed technique comes with interesting unique properties, making it a very appealing method especially for standard constrained applications as LTE and WiMAX. The PAPR reduction gain is significant compared to other techniques while no side information is sent to the receiver, so the throughput is not affected. The transmitted power and bit error rate does not increase. This PAPR technique allows using the optimal eigenvectors for beamforming without any performance degradation nor PAPR increase.

An optimization technique for finding the best weights was proposed. The PAPR reduction problem was formulated as a minimax problem that was solved by deriving the gradient and modifying the SQP algorithm to solve the optimization. The proposed algorithm minimizes the PAPR over all antennas and time slots in a STBC-MIMO system resulting in a PAPR reduction of more than 7dB for a four antenna MIMO system.

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