# DOWNSTREAM VDSL CHANNEL TRACKING USING LIMITED FEEDBACK FOR CROSSTALK PRECOMPENSATED SCHEMES

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## ABSTRACT

With increasing bandwidths and decreasing loop lengths, crosstalk becomes the main impairment in VDSL systems. For downstream communication, crosstalk precompensation techniques have been designed to cope with this issue by using the collocation of the transmitters. These techniques naturally need an accurate estimation of the crosstalk channel impulse responses. We investigate the issue of tracking these channels. Due to the lack of coordination between the receivers, and because the amplitude levels of the remaining crosstalk channels after precompensation are very low, blind estimation schemes are inefficient in this case. So some part of the upstream or downstream bit rate needs to be used to help the estimation. In this paper, we design a new algorithm to try to limit the bandwidth used for the estimation purpose by exploiting the collocation at the transmitter side. The principle is to use feedback from the receiver to the transmitter instead of using pilots in the downstream signal. It is shown by simulations that the proposed estimation is more efficient, in terms of needed overhead, than a classical scheme using pilot symbols.

### 1. INTRODUCTION

Future DSL systems such as VDSL (very high bit rate digital subscriber line) evolve towards shorter loops thanks to the increasing development of optical fiber infrastructure. This allows the use of higher bandwidths, typically from 10 to as high as 30 MHz for very short loops. At these high frequencies and low attenuation channels, the FEXT (far end crosstalk) becomes the main degradation in the system, higher than additive noise. For this reason, a number of techniques have been designed to decrease the effect of FEXT [1, 2] using the coordination at the CO (central office). These schemes rely on a good estimation of the crosstalk channels between the different pairs of users (or equivalently pairs of lines) as well as the direct channels.

In this paper, we investigate the issue of tracking of these channel estimates. Copper wires generally have static channel impulse responses, but they can still vary slowly, for example due to temperature changes. So in order to guarantee a constant behavior of the crosstalk mitigation technique, some kind of tracking of the channel estimates is necessary. Due to the lack of coordination between the CPE (customer premise equipments, i.e. the users' receivers), the downstream channel estimation appears to be a much more complicate task than the upstream channel estimation. So we focus on downstream in this paper. Furthermore, due to the presence of the crosstalk mitigation techniques, the power of the signal corresponding to the other users becomes very low at the receiver of one user. In other terms, the crosstalk impulse responses that need to be tracked are of very low amplitude with respect to the noise. So the downstream channel estimation appears as the joint estimation of multiple channels of very low amplitude corresponding to multiple independent sources (the different users' signal). This is a very difficult issue and cannot be solved by blind techniques.

The easiest way to solve the problem would be to use a set of pilot symbols, sent periodically, to perform the tracking of the downstream channels at the CPE. Many solutions exist in this framework. Then, the information about the estimates needs to be sent back to the CO periodically to perform an update of the crosstalk mitigating transmission scheme. However, this may lead to a large amount of bandwidth usage since it is necessary to use part of the downstream for the pilots and part of the upstream to feedback the channel information. In order to try to limit the quantity of overhead needed for the tracking, we propose another method which takes advantage of the coordination that is present at the transmitter (CO).

The principle of the proposed algorithm is to send back to the CO some very limited amount of information about the signal received at the CPE. Now thanks to the coordination at the CO, all symbols transmitted to all different lines are known, and that additional information can be used for the estimation. Besides, since the estimation is performed at the CO itself, feedback of the estimation is no longer needed. The algorithm is presented in this paper and it is compared through simulations to a simple LMS solution using pilot symbols. It is shown that the proposed solution performs better for a given amount of bandwidth usage.

The issue of limiting the quantity of feedback for channel estimation has already been investigated in the MIMO wireless context in [3] and several other papers. However the problem considered here turns out to be very different. Indeed, in [3], the focus is on the feedback of the information to the transmitter. It is assumed that the estimation itself has been performed already. Here, the focus is on the estimation process and on limiting the total overhead (both pilots and feedback) associated with the estimation process.

<sup>&</sup>lt;sup>1</sup>This research was supported in part by the Commission of the EC under contract FP6 IST1-506790 (U-BROAD).

#### 2. SYSTEM MODEL

We consider the estimation of the downstream crosstalk channels in a DSL environment. DMT modulation is assumed. It is also assumed that the cyclic prefix is long enough and the different users are transmitted synchronously from the CO so that the channel (including crosstalk) is free of intersymbol interference and intercarrier interference. Hence, for a given tone, the channel model is written as

$$\mathbf{y}' = \mathbf{H}' \, \mathbf{x} + \mathbf{n} \tag{1}$$

where  $\mathbf{x}, \mathbf{y}'$  are the vectors of transmitted and received samples<sup>1</sup>, respectively, for the different users (or equivalently, on the different lines),  $\mathbf{H}'$  is the channel matrix and  $\mathbf{n}$  is the vector of noise samples at the different receivers (CPE). In this paper, we focus on one fixed tone. The same developments can be done independently for each tone. The additive noise is assumed to be white. In the model (1), the diagonal elements of  $\mathbf{H}'$  correspond to the line transmission (also called direct channel later in this paper), the off-diagonal elements correspond to crosstalk. We assume N users, the channel matrix  $\mathbf{H}'$  is thus  $N \times N$ .

Because the receiver (CPE) are not collocated, each one of them can only use one received signal  $y_k$  for detection and/or estimation purposes. In order to mitigate the effect of FEXT, it is assumed that the CO uses some kind of precoder. We assume a linear precoder as presented in [1]. The CO designs a matrix **F** such that **H'F** is diagonal<sup>2</sup>, and sends

$$\mathbf{x} = \mathbf{F} \, \mathbf{u} \tag{2}$$

on the different lines, where  $\mathbf{u}$  are the transmitted information symbols for the different users. Thanks to the precoder design, the received samples for one user suffer from little interference from other users.

#### 3. POSSIBLE ESTIMATION SCHEMES

Now, because of the precoding, each user is only able to detect its own symbols. The power coming from the other users is very low. So considering the issue of channel estimation, any blind estimation of the FEXT channel coefficients would be impossible from the receiver point of view. As already pointed out, a first solution would be to use training sequences (or pilot symbols) to perform the channel estimation coefficients at each CPE and then send back the information to the CO. This is a very simple solution but it consumes a lot of bandwidth and/or time.

The second solution, which is investigated in this paper, is to allow a limited feedback from the different users about their received samples. This information is collected at the CO and the channel estimation is performed there. It is important to limit drastically the information that is sent back in order to keep an acceptable usage of the upstream bandwidth. Even with a limited amount of feedback, and since the CO knows perfectly what was sent on the different lines (the samples  $\mathbf{x}$  and the symbols  $\mathbf{u}$ ), the channel estimation is possible.

## 4. PROPOSED ALGORITHM DERIVATION

In this section, the proposed estimation algorithm is described in detail. It is first assumed that the direct channel coefficients are estimated perfectly at the receivers (this can be done easily with a decision-directed scheme). After detection, the contribution of the corresponding user's symbol is subtracted at the receiver, only remaining with the crosstalk interference and the noise. The receivers send back the *sign* of this quantity (crosstalk + noise), so that the smallest possible amount of information is used: 1 bit. We focus on real-valued symbols here. The extension to complex symbols is straightforward as the channel model for N users with complex symbols can be equivalently represented by a  $2N \times 2N$  matrix with real values.

Mathematically, we stack up K DMT blocks (still focusing on one tone only) in the following way.

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}^0 \dots \mathbf{x}^{K-1} \end{bmatrix}$$
(3)

where  $\mathbf{x}^k$  denotes the vector of transmitted samples for block k. Similarly, we build matrices **U**, **Y**' and **N**. The channel model and precoding operations are rewritten as

$$\mathbf{Y}' = \mathbf{H}' \,\mathbf{X} + \mathbf{N} \tag{4}$$

$$\mathbf{A} = \mathbf{F} \mathbf{U}. \tag{5}$$

At the different receivers, the diagonal elements of  $\mathbf{H'F}$ are assumed to be estimated perfectly, and the symbols transmitted to the corresponding users are also assumed to be detected perfectly. Their contribution is then subtracted to obtain

$$\mathbf{Y} = \mathbf{Y}' - \left\{ \mathbf{H}' \, \mathbf{F} \right\}_d \mathbf{U} \tag{6}$$

$$= \left(\mathbf{H}' \,\mathbf{F} - \left\{\mathbf{H}' \,\mathbf{F}\right\}_{d}\right) \mathbf{U} + \mathbf{N} \tag{7}$$

$$=\mathbf{H}\mathbf{U}+\mathbf{N}$$
(8)

where the last line defines a new channel matrix  $\mathbf{H}$  with zeros on the diagonal. That is the matrix that will be estimated at the CO by the algorithm. The notation  $\mathbf{A}_d$  represents the diagonal matrix formed by keeping only the diagonal elements of  $\mathbf{A}$ .

It is also assumed that the noise variance of each receiver is known at the CO. This will be necessary in the computation of the algorithm as shown later. The noise variance at receiver i is denoted by  $\sigma_{n,i}^2$ .

We denote by  $\mathbf{Z} = \operatorname{sign}(\mathbf{Y})$ , the set of received signs of the samples coming from the different lines. They are the observations on which the estimation will be based. The (sign) sample received from user *i* for block *k* is denoted by  $z_i^k = \operatorname{sign}(y_i^k)$ . The algorithm is based on the maximum likelihood principle. First, the likelihood of a set of channel coefficients **H** is written:

$$\Lambda(\mathbf{H}) = \prod_{k=0}^{K-1} \prod_{i=0}^{N-1} P(\operatorname{sign}(y_i^k) = z_i^k | \mathbf{H}, \mathbf{U})$$
(9)

where  $P(\operatorname{sign}(y_i^k) = z_i^k | \mathbf{H}, \mathbf{U})$  denotes the conditional probability on the value of some sign sample, given the transmitted symbols and given the set of channel coefficients. Note that the estimation can be performed independently for each line as the channel coefficients related to one line depend only on the received samples from the corresponding line. However, for the generality, we keep the matrix

<sup>&</sup>lt;sup>1</sup>The notations  $\mathbf{y}'$  and  $\mathbf{H}'$  are used here because the actual matrix to estimate  $\mathbf{H}$  and observations  $\mathbf{y}$  used will be a slightly modified version of this (see later)

 $<sup>^2 {\</sup>rm with}$  well-chosen entries on the diagonal to avoid an energy increase

formalism here. For one given sample, it can be shown easily that

$$P(\operatorname{sign}(y_i^k) = z_i^k | \mathbf{H}, \mathbf{U}) = Q\left(-z_i^k \frac{\mathbf{h}_i \mathbf{u}^k}{\sqrt{\sigma_{n,i}^2}}\right)$$
(10)

where  $\mathbf{h}_i$  is the *i*th row of  $\mathbf{H}$ ,  $\mathbf{u}^k$  is the *k*th column of  $\mathbf{U}$ , and where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^{2}/2} dt.$$
 (11)

So, finally,

$$\Lambda(\mathbf{H}) = \prod_{k=0}^{K-1} \prod_{i=0}^{N-1} Q\left(-z_i^k \mathbf{h}_i \mathbf{u}^k / \sigma_{n,i}\right).$$
(12)

The tracking algorithm is obtained by taking the derivative of the likelihood function, and performing a classical steepest descent procedure. The gradient of the maximum likelihood is given by

$$\frac{\partial \Lambda(\mathbf{H})}{\partial \mathbf{h}_{i}} = \frac{\Lambda(\mathbf{H})}{\sqrt{2\pi\sigma_{n,i}^{2}}} \sum_{k=0}^{K-1} (\mathbf{u}^{k})^{T} z_{i}^{k} G\left(-\frac{z_{i}^{k} \mathbf{h}_{i} \mathbf{u}^{k}}{\sigma_{n,i}}\right)$$
(13)

with

$$G(x) = \frac{e^{-\frac{x^2}{2}}}{Q(x)}.$$
 (14)

Now we build a basic tracking algorithm that computes the gradient for each new received sample (each block k) and adapts the channel in the direction of the gradient. So it realizes the sum over k in (13) by adapting progressively for each new coming sample (except that the channel estimate is changing slowly). It is important to keep the weightings that depend on the sample k (i.e. the factor  $G(\ldots)$ ) because it contains the information on the relative importance of each term of the gradient. The common factor can be removed of course, and incorporated in the stepsize. Finally, the following algorithm is provided:

$$\hat{\mathbf{h}_{i}}^{k+1} = \hat{\mathbf{h}_{i}}^{k} + \mu z_{i}^{k} \frac{e^{-\frac{(\hat{\mathbf{h}_{i}}^{k} \mathbf{u}^{k})^{2}}{2\sigma_{n,i}^{2}}}}{Q(-z_{i}^{k} \hat{\mathbf{h}_{i}}^{k} \mathbf{u}^{k} / \sigma_{n,i})} (\mathbf{u}^{k})^{T}$$
(15)

where  $\hat{\mathbf{h}}_{i}^{\kappa}$  denotes the current estimate at block k of row i of the channel matrix  $\mathbf{H}$ , and  $\mu$  is the stepsize. The tracking algorithm (15) appears to be similar to an LMS algorithm, or more precisely to the sign-LMS [4]. However it is still very different because there is no 'error' signal computed between the observation and the estimated version as it is the case for the sign-LMS algorithm. This would indeed require the knowledge of the received signal which is not available since only the sign is fed back. As can be seen, the 'error' signal is replaced here by some more complicated expression involving the sign of the received sample, the current estimation, and the known transmitted symbols. Consequently, the behavior and performance of this algorithm can be expected to be very different.

Finally, the ultimate goal is to adapt the precoder to the changes in the channel. To achieve this, the diagonal coefficients of the matrix  $\mathbf{H'F}$  (direct channel coefficients) have to be sent back periodically as well. This allows the CO to reconstruct  $\mathbf{H'F}$  and hence  $\mathbf{H'}$ , and then to compute the new precoder.



Figure 1. Results for a noise variance of  $10^{-6}$ ;  $\mu = 50 \times 10^{-9}$ . Respective SNR corresponding to all channel coefficients are -19 dB, -21 dB, -27 dB, -44 dB

#### 5. SIMULATION RESULTS

We present results for N = 5 users, and hence 4 interfering users. This is sufficient for many practical cases as most of the crosstalk power usually comes from 2 or 3 dominant sources. The channel values used here come from a set of measurements performed at FTRD (France Telecom R&D) and are complemented with standard specifications [5]. The figures show the estimation algorithm for one specific user, that is one line of the matrix **H**. The evolution of the estimation of the different channel coefficients is shown in number of samples received. Figures 1 and 2 show the results for different noise variances  $(10^{-6} \text{ and } 10^{-9})$  and different channel situations. The symbol variance is always normalized to 1. In all cases, the proposed algorithm is compared to an LMS algorithm that would use the full precision observations  $y_i^k$ . This actually corresponds to what would be obtained with the use of pilot symbols. Obviously, the results for a given number of samples must be higher in that case since the information available is higher, but the corresponding bandwidth usage for the transmission of the pilot symbols can be up to 4 or 5 times higher, depending on the constellation sizes used.

As a matter of fact, it appears that for high noise (Fig. 1), the proposed algorithm performs almost as well as the LMS algorithm using the full precision samples. So this suggests that the use of pilot symbols will demand a higher bandwidth usage for similar performances. It is confirmed in Fig. 3 by comparing the results obtained with the same number of bits used for feedback and for pilots (for a slightly lower noise variance of  $2 \times 10^{-7}$ ). When the noise becomes too small, the algorithm does not perform well (Fig. 2). This is because the high quantization used for feedback (sign only) contributes to a significant loss in precision, and becomes unacceptable when the noise is low. In conclusion, the algorithm is well suited when the noise is higher than the interfering signal. So it is perfectly suited



Figure 2. Results for a noise variance of  $10^{-9}$ ;  $\mu = 15 \times 10^{-9}$ . Respective SNR corresponding to all channel coefficients are 0 dB, -23 dB, 6 dB, 8 dB



Figure 3. Results for a noise variance of  $2 \times 10^{-7}$ ;  $\mu = 30 \times 10^{-9}$ . Comparison for the same number of bits used in feedback or pilots. The users all have constellations of size 16.

to the issue of interest (the standard value of the noise corresponds to a variance of  $10^{-6}$ ).

Finally, figure 4 shows the performance of the updated precoder using the proposed algorithm for channel estimation. It shows the obtained SNIR (signal to noise and interference ratio) – bottom curve – as well as the SIR (signal to interference ratio) – upper curve – as a function of the noise variance and for 3 of the 5 users. The interference corresponds here to the crosstalk from the different users. As shown by the figure, the remaining crosstalk is much lower than additive noise after the new precoding (20 to 30 dB in most cases). Once again, the performance of the





algorithm is getting poor for low noise and the interference becomes significant. Only user 1 has good performance for low noise. This is due to the fact that, in the channel configuration used here, the crosstalk to user 1 is initially very low already.

#### 6. CONCLUSIONS

We have proposed a new method to estimate the downstream channel in a DSL environment using crosstalk precompensation. The method uses a limited amount of feedback of the received samples from the receivers to the transmitter (CO) where the lines are collocated and where the estimation is performed. It is intended as a way to limit the use of bandwidth for the estimation purpose. It is shown that for practical situations (noise higher than crosstalk after precompensation), the proposed method is indeed more efficient; at least than the simple LMS algorithm investigated.

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