Adaptive Numerical Integration Methods for Deterministic Analysis of Non-Stationary Noise in Dynamic Integrated Circuits

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Abstract—**This paper reports a new step-size control strategy for adaptive numerical integration in time-domain noise analysis of non-linear dynamic integrated circuits with arbitrary excitations. A non-stationary stochastic noise process is described as an Itô system of stochastic differential equations and a numerical solution for such a set of equations is found. Statistical simulation of dynamic circuits fabricated in 45 nm CMOS process shows that the proposed numerical methods offer an accurate and efficient solution.**

Keywords-integration methods, noise analysis, computed aided design, sotchastic differential equations, dynamic integrated circuits

I. INTRODUCTION

Noise limitations are a fundamental issue for robust circuit design and its evaluation has been subject of numerous studies [1]. Correspondingly, a number of CAD tools have been suggested [2]-[3]. The most important types of electrical noise sources (thermal, shot, and flicker noise) in passive elements and integrated circuit devices have been investigated extensively, and appropriate models have been derived [1] as stationary and in [4] as non-stationary noise sources. The noise performance of a circuit can be analyzed in terms of small-signal equivalent circuits by considering each of the uncorrelated noise sources in turn and separately computing their contribution at the output. Unfortunately, this method is only applicable to circuits with fixed operating points and is not appropriate for noise simulation of circuits with changing bias conditions. A widespread approach for noise simulation in the time domain is Monte Carlo analysis. However, accurately determining the noise content requires a large number of simulations, so consequently, the Monte Carlo method becomes very *cpu*-time consuming if the chip becomes large. As a result, several methods, such as variance reduction techniques including importance sampling, stratified sampling, correlated sampling, and regression sampling are reported to improve the precision of the statistical estimate with a smaller set of random simulations. Similarly, statistical regression techniques such as response surface modeling have been applied to further reduce the number of random simulations. Nevertheless, the computational cost still remains high for large-scale circuits.

In this paper, we treat the noise as a non-stationary stochastic process, and introduce an Itô system of stochastic differential equations (SDE) as a convenient way to represent such a process. We adapt model description as defined in [4], where thermal and shot noise are expressed as delta-correlated noise processes having independent values at every time point, modeled as modulated white noise processes. These noise processes correspond to current noise sources which are included in the models of the integrated-circuit devices. As numerical experiments suggest that both the convergence and stability analyses of adaptive schemes for SDEs extend to a number of sophisticated methods which control different error measures, we follow the adaptation strategy, which can be viewed heuristically as a fixed time-step algorithm applied to a time re-scaled differential equation. Additionally, adaptation also confers stability on algorithms constructed from explicit time-integrators, resulting in better qualitative behavior than for fixed time-step counter-parts [5]. Similarly, recognizing that the variance-covariance matrix when backward Euler is applied to such a matrix can be written in the continuous-time Lyapunov matrix form, we then provide a numerical solution to such a set of linear time-varying equations.

II. STOCHASTIC MNA FOR TIME-DOMAIN NOISE ANALYSIS

In general, for time-domain analysis, modified nodal analysis (MNA) leads to a nonlinear ordinary differential equation (ODE) or differential algebraic equation (DAE) system which, in most cases, is transformed into a nonlinear algebraic system by means of linear multistep integration methods [6]-[7] and, at each integration step, a Newton-like method is used to solve this nonlinear algebraic system. Therefore, from a numerical point of view, the equations modeling a dynamic circuit are transformed to equivalent linear equations at each iteration of the Newton method and at each time instant of the time-domain analysis. Thus, we can say that the time-domain analysis of a nonlinear dynamic circuit consists of the successive solutions of many linear circuits approximating the original (nonlinear and dynamic) circuit at specific operating points.

Consider MNA and circuits embedding, besides voltage-controlled elements, independent voltage sources, the remaining types of controlled sources and noise sources. Combining Kirchhoff's Current law with the element characteristics and using the charge-oriented formulation yields a stochastic differential equation of the form

$$A\frac{d}{dt}d(x(t)) + e(x(t),t) + f(x(t),t)\xi(t) = 0 \qquad (1)$$

where A is a constant singular incidence matrix determined by the topology of the dynamic circuit parts, the vector $d(x)$ consists of the charges of capacitances and the fluxes of inductances, and x is the vector of unknowns consisting of the nodal potentials and the branch currents through voltage-defining elements. The term $e(x,t)$ describes the impact of the static elements, $f(x,t)$ denotes the vector of noise intensities, and $\xi(t)$ is a vector of independent Gaussian white noise sources. The partial derivatives e_x, f_x, d_x, d_t, d_{xt} and d_{xx} are assumed to exist and to be continuous. At a first glance, the charge oriented system (1) seems to be disadvantageous since its dimension is significantly larger than the dimension of the classical MNA system [6]. However as numerical methods applied to such system require the differentiation of the charge and flux functions, solving the resulting system of nonlinear equations requires the second derivatives of these functions, i.e., more smoothness. This plays a significant role for the numerical solution since models are usually not twice differentiable. Additionally, it is computationally more expensive. Furthermore, charge and flux conservations are only fulfilled approximately.

Equation (1) represents a system of nonlinear stochastic differential equations, which formulate a system of stochastic algebraic and differential equations that describe the dynamics of the nonlinear circuit that lead to the MNA equations when the random sources ξ are set to zero. Solving (1) means to determine the probability density function P of the random vector x at each time instant t. However, generally it is not possible to handle this distribution directly. Hence, it may be convenient to look for an approximation that can be found after partitioning the space of the stochastic source variables ξ in a given number of subdomains, and then solving the equation in each subdomain by means of a piecewise-linear truncated Taylor approximation. Since the magnitude of the noise content in a signal is much smaller in comparison to the magnitude of the signal itself in any functional circuit, a system of nonlinear stochastic differential equations described in (1) can be piecewise-linearized; it is then possible to combine the partial results and obtain the desired approximated solution to the original problem.

We will interpret (1) as an Itô system of stochastic differential equations

$$Ad(X(s))\Big|_{t_0}^{t} + \int_{t_0}^{t} e(X(s),s)ds + \int_{t_0}^{t} f(X(s),t)dW(s) = 0 \qquad (2)$$

where the second integral is an Itô-integral, and W denotes an m-dimensional Wiener process. When considering a numerical solution of a differential equation, we must restrict our attention to a finite subinterval $[t_0,t]$ of the time-interval $[t_0,\infty]$ and, in addition, it is necessary to choose an appropriate discretization $t_0 < t_1 < \ldots < t_n < \ldots < t_N = t$ of $[t_0, t]$, due to computer limitations. The other problem is simulating a sample path from the Wiener process over the discretization of $[t_0,t]$: so considering an equally-spaced discretization, i.e. $t_n - t_{n-1} = (t -$ $t_0)/N = h$, $n = 1,\ldots,N$, where h is the integration stepsize, we have the following (independent) random increments $W_{tn} - W_{t(n-1)} \sim \mathcal{N}(0,h)$ of the Wiener process W_t.

Moreover, the sampling of normal variates to approximate the Wiener process in the SDE is achieved by computer generation of pseudo-random numbers. However, the use of a pseudo-random number generator needs to be evaluated in terms of statistical reliability. Nevertheless, most commonly used pseudo-random number generators have been found to fit their supposed distribution reasonably well, but the generated numbers often seem not to be independent as they are supposed to be: this is not surprising since, for congruent generators at least, each number is determined exactly by its predecessor [8].

III. ADAPTIVE NUMERICAL METHODS

A. Deterministic Euler-Maruyama Scheme

The adaptive methods control the time-step of a forward Euler deterministic step so that it deviates only slightly from a backward Euler step. This not only controls an estimate of the contribution to the time-stepping error from the deterministic step, but also allows the analysis of stability (large time) properties for implicit backward Euler methods to be employed in the explicit adaptive methods. Most simulation schemes for SDE's are derived using an Itô-Taylor expansion truncated after a finite number of terms, with the order of convergence depending on the number of terms considered in the truncation. Keeping only the first term on the deterministic grid $0 = t_0 < t_1 < \ldots < t_N = t$ end, yields the deterministic-implicit Euler-Maruyama scheme, which applied to (2) reads

$$A(d(X_l) - d(X_{l-1})) + h_l e(X_l,t_l) + F(X_{l-1},t_{l-1})\Delta W_l = 0 \ (3)$$

where $h_l = t_l - t_{l-1}$, $\Delta W_l = W(t_l) - W(t_{l-1})$, and X_l denotes the approximation to $X(t_l)$. Realizations of ΔW are simulated as $\mathcal{N}(0, h_l)$-distributed random variables (the increments ΔW are generated as suggested in Section II). The errors are dominated by the deterministic terms as long as the step-size is large enough. In more detail, the error of the given methods behaves like $O(h^2 + \varepsilon h + \varepsilon^2 h^{1/2})$, when ε is used to measure the smallness of the noise ($f_r(x, t) = \varepsilon; f_r(x, t)$, $r = 1,\ldots, m$, $\varepsilon \ll 1$). The smallness of the noise also allows special estimates of the local error terms, which can be used to control the step-size. In [9] a stepsize control is given for the deterministic Euler scheme in the case of small noise that leads to adaptive step-size sequences that are uniform for all paths. The estimates of the dominating local error term are based on values of the deterministic term and do not cost additional evaluations of the coefficients of the SDE or their derivatives. Though having the lowest order of convergence, the Euler-Maruyama scheme completely avoids forming multiple stochastic integrals, noticeably improving the simulation speed, especially considering the large number of simulations needed to approximate small probabilities. However, as the order of the Euler-Maruyama method is low, the numerical results are inaccurate unless a small stepsize is used.

B. Deterministic Milstein Scheme

General stochastic Taylor schemes can be formulated compactly using hierarchical sets of multiply indices with iterated multiply stochastic integrals and iterated application of the differential operators to the coefficient function. The multiple stochastic integrals which they contain provide more information about the noise processes within discretization subintervals and this allows an approximation of higher order to be obtained. The Milstein scheme differs from the Euler scheme by an additional correction term for the stochastic part, which includes double stochastic integrals. The above procedure indicates the general pattern: the higher order schemes achieve their higher order through the inclusion of multiple stochastic integral terms; the coefficients of the scheme involve partial derivatives of the SDE coefficient functions; a scheme may have different strong and weak orders of convergence; and, the possible orders for strong schemes increase by a fraction ½, whereas possible orders for weak schemes are whole numbers. The higher order schemes require adequate smoothness of the deterministic and stochastic coefficients and sufficient information about the driving Wiener processes, which is contained in the multiple stochastic integrals. Additionally, in higher order strong Taylor approximations derivatives of the deterministic and stochastic coefficients have to be calculated at each step.

In this paper, to adapt the Milstein scheme to the SDE (2), we apply this method in such a way that it implicitly realizes a Milstein scheme for the inherent SDE. Except for higher order terms this is realized by

$$A(d(X_l) - d(X_{l-1})) + h_l e(X_l) + F(t_{l-1}, X_{l-1})\Delta W_l$$
$$- \sum_{j=1}^{k}((F_j)_x (Ad_x + he_x)^{-1} F(x_{l-1}, t_{l-1}))I_j^l = 0 \qquad (4)$$

where

$$I_j^l = (I_{j,i}^l)_{i=1}^k \quad I_{j,i}^l = \int_{t_{l-1}}^{t_l}\int_{t_{l-1}}^{s} dW_i(t)dW_j(s) \qquad (5)$$

In the last term the Jacobian $Ad_x + he_x$ of the previous iterate can be reused. An upper bound for the pathwise error of the Milstein method is determined using the Doss-Sussmann approach to transform the stochastic differential equation and the Milstein scheme to a random ordinary differential equation and a corresponding approximation scheme, respectively. The pathwise approximation of random ordinary differential equations is considered in [10], where the Euler and Heun methods are analyzed. Moreover, it is shown that the classical convergence rates of these schemes can be retained by averaging the noise over the discretization subintervals. In [11] it is shown that the explicit Euler-Maruyama scheme with equidistant step size $1/h$ converges pathwise with order ½-ε for arbitrary ε>0. Hence, the pathwise and the mean-square rate of convergence of the Euler method almost coincide.

C. Noise Correlation Function

If X_t is a Gaussian stochastic process, then it is completely characterized by its mean and correlation function. From Itô's theorem on stochastic differentials, noting that X and dw are uncorrelated, variance-covariance matrix $K(t)$ of $X(t)$ with the initial value $K(0)=E[x\ x^T]$ can be expressed in differential Lyapunov matrix equation form as [12]

$$dK(t)/dt = E(t)K(t) + K(t)E^T(t) + F(t)F^T(t) \qquad (6)$$

Note that the mean of the noise variables is always zero for most integrated circuits. In view of the symmetry of $K(t)$, (6) represents a system of linear ordinary differential equations with time-varying coefficients. To obtain a numerical solution, (6) has to be discretized in time using an Euler-Maruyama or Milstein method. If backward Euler is applied to (6), the differential Lyapunov matrix equation can be written in a special form referred to as the continuous-time algebraic Lyapunov matrix equation

$$P_r K(t_r) + K(t_r)P_r^T + Q_r = 0 \qquad (7)$$

$K(t)$ at time point t_r is calculated by solving the system of linear equations in (7). Such continuous time Lyapunov equations have a unique solution $K(t)$, which is symmetric and positive semidefinite. Several iterative techniques have been proposed for the solution of the algebraic Lyapunov matrix equation (7) arising in some specific problems where the matrix P_r is large and sparse [13]-[14], such as the Bartels-Stewart method [15], and Hammarling's method [7], which remains the one and only reference for directly computing the Cholesky factor of the solution $K(t_r)$ of (7). Large dense Lyapunov equations can be solved by sign function based techniques [16] or by iterative approaches [17]. Krylov subspace methods, which are related to matrix polynomials have been proposed [18] as well.

In this paper, we apply a low rank version of the iterative method, which is related to rational matrix functions. The postulated iteration for the Lyapunov equation (7) is given by $K(0) = 0$ and

$$(P_r + \gamma_i I_n)K_{i-1/2} = -Q_r - K_{i-1}(P_r^T - \gamma_i I_n)$$
$$(P_r + \bar{\gamma}_i I_n)K_i^T = -Q_r - K_{i-1/2}^T(P_r^T - \bar{\gamma}_i I_n) \qquad (8)$$

for $i = 1,2,...$ This method generates a sequence of matrices K_i which often converges very fast towards the solution, provided that the iteration shift parameters γ_i are chosen (sub)optimally. For a more efficient implementation of the method, we replace iterates by their Chlesky factors, i.e., $K_i=L_iL_i^H$ and reformulate in terms of the factors L_i. The low rank Cholesky factors L_i are not uniquely determined. Different ways to generate them exist [19]. Note that the number of iteration steps i_{max} needs not be fixed a priori. However, if the Lyapunov equation should be solved as accurate as possible, correct results are usually achieved for low values of stopping criteria, which are slightly larger than the machine precision.

IV. EXPERIMENTAL RESULTS

The proposed method and both adaptive numerical methods have been implemented in Matlab. All the experimental results are carried out on a single processor Linux system with Intel Core 2 Duo CPUs with 2.66 GHz and 3 GB of memory. The proposed method solves the set of linear time-varying equations (6) including the noise content description to find the steady state value of the time-varying covariance matrix. This gives the variance at the output node and its cross-correlation with other nodes in the circuit, which makes it possible to evaluate the devices that most affect a particular performance, so that design efforts can be addressed to the most critical section of the circuit. The covariance matrix is periodic with the same period as either the input signal (e.g., translinear circuits) or the clock (in circuits such as dynamic logic).

The effectiveness of the proposed approaches was evaluated on several dynamic circuits exhibiting different distinctive features in a variety of applications. As a representative example of the results that can be obtained, we show an application of noise analysis to the characterization of dynamic logic gates and dynamic latch comparators fabricated in standard 45 nm CMOS technology (Figure 1 and Figure 2). Circuits designed using dynamic logic styles can be considerably faster and more compact than their static CMOS counterparts. Nevertheless, the absence of a static pull-up chain makes these dynamic circuits susceptible to input noise, power and ground bounce, leakage, and charge-sharing during the evaluate phase if the outputs are not being pulled down (Figure 1). Besides reducing gate noise margin due to possibly lowered supply voltage, the power and ground voltage mismatch between a driver gate and a receiver gate can translate to a *dc* noise at the input of the receiver. Noise presented at the inputs of a logic gate is primarily caused by the coupling effect among adjacent signal wires. Similarly, charge sharing reduces the voltage level at the dynamic node causing potential false switching of a dynamic logic gate. Without the feedback keeper in these circuits, the gates would have zero noise rejection and the dynamic nodes will discharge completely given enough time. The feedback keeper placed on the dynamic node maintains the charge on that node, giving the gate some degree of noise-rejection. The noise rejection capability of the circuit depends on the relative sizes of the transistors in the dynamic gate and the feedback keeper. However, note that if the dynamic node incorrectly discharges past a certain point, the result is irreversible and incorrect computation will result.

The concept of a dynamic comparator exhibits potential for low power and small area implementation and, in this context, is restricted to single-stage topologies without static power dissipation. A widely used dynamic comparator is based on a differential sensing amplifier [20] is shown in Figure 2a). In addition to the mismatch sensitivity, the latch is also very sensitive to an asymmetry in the load capacitance. This can be avoided by adding an extra latch or inverters as a buffering stage after the comparator core outputs. A fully differential dynamic comparator based on two cross-coupled differential pairs with switched current sources loaded with a CMOS latch is shown in Figure 2b) [21]. Because of the dynamic current sources together with the latch, connected directly between the differential pairs and the supply voltage, the comparator does not dissipate *dc*-power. Figure 2c) illustrates the schematic of the dynamic latch given in [22], where the dynamic latch consists of pre-charge transistors, a cross-coupled inverter, a differential pair and a switch.
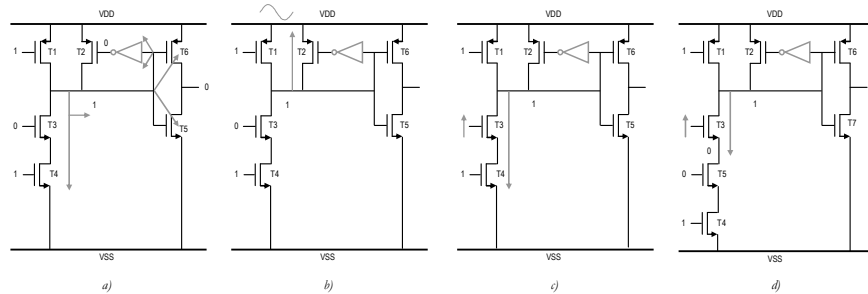

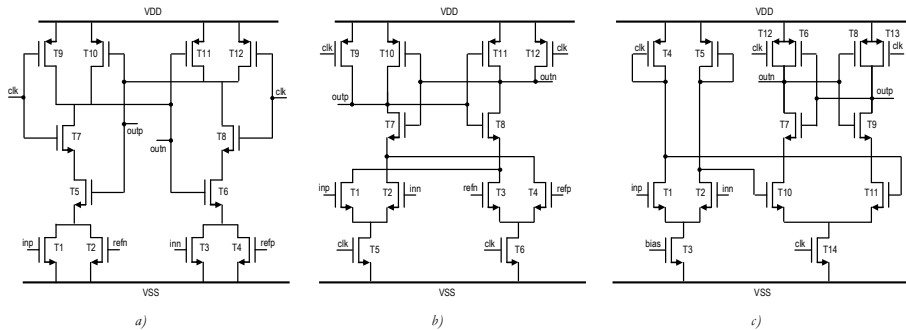Figure 1: Dynamic logic gate, a) Leakage currents, b) Supply noise, c) Input noise, and d) Charge sharing

Figure 2: Dynamic latch comparators, a) [20], b) [21], c) [22]

In the simulation we assumed that the time series x are composed of a smoothly varying function, plus additive Gaussian white noise ξ, and that at any point x can be represented by a low order polynomial (a truncated local Taylor series approximation). The amount of noise introduced for any electrical device in the circuit corresponds to the current noise sources, which are included in the models of the integrated-circuit devices

$$i_{th} = \sqrt{2kT/R}\,\xi(t) \quad i_{shot} = \sqrt{q_e I_D}\,\xi(t) \tag{9}$$

where T is the temperature, k is Boltzmann's constant, q_e is the elementary charge, and I_D is the current through junction. Figure 3 reports the point-by-point sample mean of the Euler-Maruyama solutions of the Itô SDE (2) and their empirical 95% confidence bands (from the 2.5th to the 97.5th percentile; outer bands, dashed lines). Figure 4 is similar as Figure 3 but refers to the Milstein solution of the Itô SDE (2). When the analytic solution of the SDE is known, the (average absolute) error at time T, depending on the desired number of simulations R, can be computed as [8]

$$\varepsilon = 1/2 \sum_{r=1}^{R} |X(t,r) - y(t,r)| \tag{10}$$

were $X(t,r)$ and $y(t,r)$ denote the value of the analytic solution at time t in the r-th trajectory and the value of the numerical solution for the chosen approximation scheme at time t in the r-th trajectory, respectively. Figure 5 compare the Euler-Maruyama solutions (dotted lines) of the Itô SDE (2) with the corresponding adapted Milstein solutions (solid lines) and the analytic solutions (dashed lines): the adapted Milstein and the analytic solutions are so close that they appear practically undistinguishable. For the calculation of the error, the analytic solution and the numerical solution must be computed on the same Brownian path (i.e. using the same sequence of pseudorandom numbers). At time T=1 the Euler-Maruyama method for the Itô SDE (2) implies an average error equals to 1.048×10^{-2}, while the adapted Milstein scheme for the Itô SDE implies an average error of 5.962×10^{-5}. These results show that the Milstein method is more accurate, although the Euler-Maruyama method is faster: 27% and 11% in comparison with classical Milstein method and proposed adapted Milstein method, respectively. Descriptive statistics are reported with respect to the simulated values at the endpoint t: e.g. for the Euler-Maruyama approximation of the Itô SDE we have, $E(X_t) \approx 1.161$ where $E(.)$ denotes expectation, $Var(X_t) \approx 0.367$, $Median(X_t)=1.029$, etc. One example of the estimated noise variance (obtained at the output node of the dynamic logic gate) is illustrated in Figure 6. In comparison with 1500 Monte Carlo iterations, at any of the circuit nodes, the difference is less then 1.1% and 3.2% for mean and variance, respectively, while achieving considerable *cpu*-time reduction (32.4 sec versus 2.1 sec). Similarly, for dynamic latch comparators [20]-[22], the difference is less then 1.1%, 1.0% and 1.1% for mean, and 2.9%, 3.1% and 3.0% for variance, respectively. Correspondingly, the achieved speed gain is 14, 16 and 15 times. For the adapted Milstein method, in comparison with

1500 Monte Carlo iterations, the difference for dynamic logic gate is less then 0.2% and 0.8% for mean and variance, respectively, with 14 times *cpu*-time reduction. Similarly, the achieved speed gains for dynamic latch comparators [20]-[22] are 12, 14 and 13 times, while the precision is within 0.3%, 0.2% and 0.3% for mean, and 0.7%, 0.9% and 0.8% for variance. Consequently, the adapted Milstein method realizes three times speed increase in comparison with classical Milstein method.

For the recursive algorithm presented here it is observed that a faster lowest level kernel solver (with suitable block size) leads to an efficient solver of triangular matrix equations. For models with large dimension usually the matrix P_r has a banded or a sparse structure and applying the Bartels-Stewart type algorithm becomes impractical due to the Schur decompositions (or Hessenberg-Schur), which cost expensive $O(N^3)$ flops. In comparison with the standard Matlab function *lyap.m*, the *cpu*-time shows that computing the Cholesky factor directly is faster by approximately N flops.
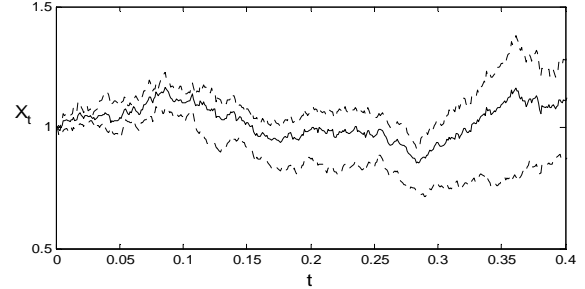

Figure 3: Itô SDE: normalized mean and 95% confidence bands of the Euler-Maruyama approximation
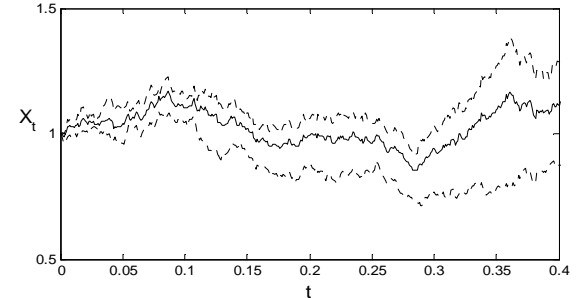

Figure 4: Itô SDE: normalized mean and 95% confidence bands of the Milstein approximation
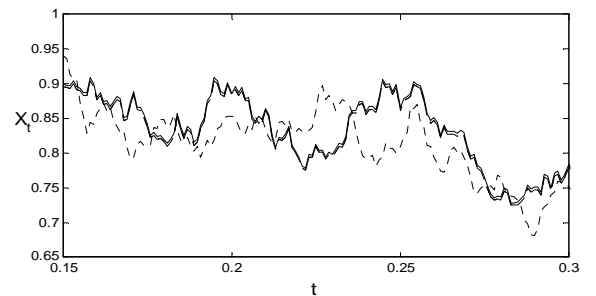

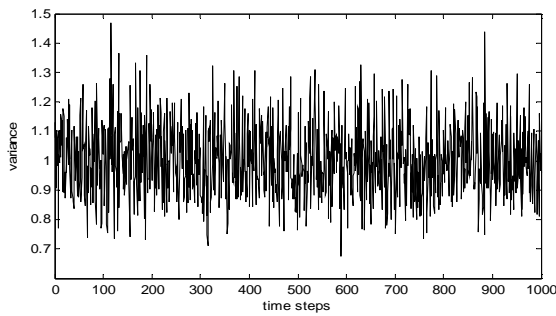Figure 5: Euler-Maruyama vs Milstein vs analytic solution
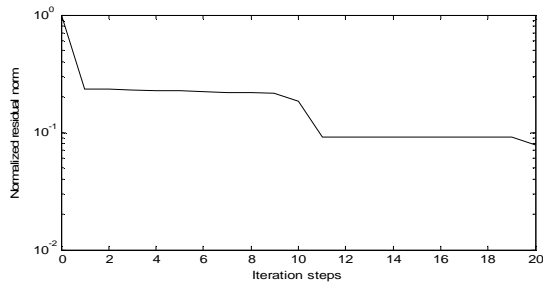
Figure 6: Estimation of noise variance.

Figure 7: Stopping criterion: maximal number of iteration steps.

Similarly, when the original matrix equation is real, using real arithmetic is faster than using complex arithmetic. Hence, we resort to iterative projection methods when the matrix is large. The approximate solution of the Lyapunov equation is given by the low rank Cholesky factor L, for which $LL^H \sim K$. L has typically fewer columns than rows. In general, L can be a complex matrix, but the product LL^H is real. More precisely, the complex low rank Cholesky factor delivered by the iteration is transformed into a real low rank Cholesky factor of the same size, such that both low rank Cholesky factor products are identical. However, doing this requires additional computation. The iteration is stopped after a priori defined iteration steps (Figure 7).

V. CONCLUSIONS

Statistical simulation is one of the foremost steps in the evaluation of successful high-performance IC designs due to circuit noise that strongly affect devices behavior in today's deep submicron technologies. As circuit noise is modeled as non-stationary process, Itô stochastic differentials are introduced as a convenient way to represent such a process. Two adaptive deterministic numerical integration methods, namely, the Euler-Maruyama and adapted Milstein schemes, are proposed to find a numerical solution of Itô differential equations. Additionally, an effective numerical solution for a set of linear time-varying equations defining the variance-covariance matrix is found. The effectiveness of the proposed approaches was evaluated on several dynamic circuits. As the results indicate, the suggested numerical method provides accurate and efficient solutions of stochastic differentials for noise analysis.

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